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WAVE RUN-UP ON ROCK SLOPES OF A COASTAL STRUCTURE

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ABSTRACT

A wave run-up height on a coastal structure with sloping face is one of the considerations in the planning and design of sea-walls or breakwaters. In this paper, a wave run-up height on rock slopes of a coastal structure is presented using the empirical approach based on incident waves entering coastal structure. A height and increasing level of wave run-up from incident wave amplitude are presented on rock slopes of a coastal structure with various angle of structure slopes and various water depths. The influence of incident wave height and wave period on the wave run-up height are described through the correlation analysis, and the probability distribution of wave run-up height is described as a Weibull distribution.

Keywords: Wave run-up, periodic wave, coastal structure, Weibull distribution, probability distribution.

I. INTRODUCTION

A wave run-up height on a coastal structure with sloping face is one of the considerations in the planning and design of sea-walls or breakwaters. Over–topping of a protective structure can have most serious consequences, both with respect to damage to the structure itself and the flooding of areas on the land ward side. Wave run-up on a coastal structure is defined as the vertical height above still-water level to which a wave will rise on the slope structure. A phenomenon of the wave run-up on rock slopes of coastal structure is illustrated in Figure 1. In order to explain the wave run-up process, it is necessary to describe the characteristic of waves as they enter shallow water, break, and finally run up on the sloping face. As the waves enter shallow water, they are influenced by a sloping bed. From this point both the speed of travel and wave length decrease, and also the wave height decrease slightly.

Reviews on wave run-up on a coastal structure have been written by various researcher. Madsen and White (1976) have described a theoretical approach in the problem of wave run-up on the seaward slope by analyzing the associated energy dissipation on a rough impermeable slope through an unknown wave friction factor. Van der Meer and Stam (1992) have derived qualitative analysis for wave run-up influenced by various parameters on run-up, and the formula for the assessment of various run-up levels as a function of the surf similarity or breaker parameter. Mustafid and Hargono (in press, 2001) have extended the theoretical approach for predicting wave run-up on a coastal impermeable structure based on data measurements as the parameters of incident waves entering to coastal structure, and described a statistical model of wave run-up height to determine the probability distribution. of wave run-up height.

The main objective of this paper is to determine a wave run-up height on rock

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slopes of coastal impermeable structure with various angle of structure slopes and various water depths based on data of periodic incident waves entering to coastal structure, which is derived from data measurements. The wave run-up on a rock slopes of coastal structure is presented in Figure 1, and the sketch of wave run-up is presented in Figure 2. We also present a statistical model to determine the influence of incident wave height and wave period on the increasing level of wave run-up height, and also to derive the probability distribution of wave run-up height with a Weibull distribution.

The method of wave run-up height prediction is an empirical approach to derive wave run-up height based on data measurements. Then, statistical approach with correlation analysis is done to determine the influence of incident wave parameters on the increasing wave run-up height, and with distribution analysis to derive a statistical model of wave run-up height. Furthermore, the probability distribution of wave run-up height is presented as a Weibull distribution. The sampling of wave data were obtained from the coast of Klidanglor, Batang on 25 October 2000 from 0:00 A.M. until 1:00 A.M. on 26 October, 2000 (Figure 3). The location of data measurements is about 200 meter north of east breakwater with a water depth of 5 meters.

II. WAVE RUN-UP

In this section, we describe wave run-up heights on rock slopes of coastal structure with various angle of structure slopes and various water depths based on data of periodic incident waves entering to coastal structure. The wave run-up is describe on a rock slope structure that are assumed to be impermeable slope with bottom friction. The depth of water before the sloping face is also a constant. Waves are assumed to be relatively long waves of small amplitude normally happen on rough impermeable slope and are unbroken in the vicinity of the structure toe. Wave are considered to be monochromatic.

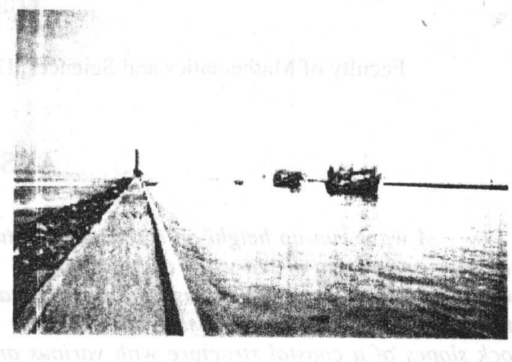

Figure 1. The phenomena of wave run-up on a rock slope of coastal structure, coast of Klidanglor, Batang, Central Java (Photo taken in October, 2000).

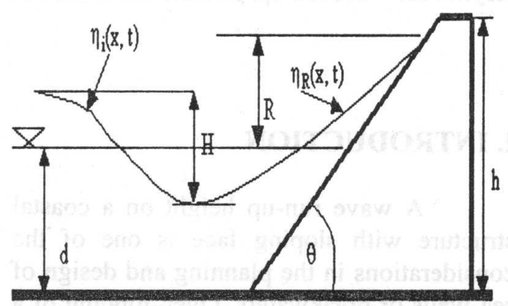

Figure 2. A sketch of wave run-up on a slope of coastal structure

Figure 2 shows parameters involved in describing wave run-up on slope structure. The notation $\eta_i(x, t)$ is the free surface elevation of incident wave above the still water level as a periodic incident wave with wave height H and wave periodic T, and the notation $\eta_R(x, t)$ is the free wave run-up height above the still water level. Furthermore, d is constant depth of water before the sloping face, and θ is the angle of structure slope.

An incident wave equation with small amplitude can be expressed from continuity equation as Laplace equation:

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$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \qquad (1)$$

where φ is a potential velocity. From equation (1), the free surface elevation of $\eta_i(x, t)$ can be expressed by:

$$\eta_i(x,t) = -\frac{1}{g}\frac{\partial \varphi}{\partial t}\Big|_{y=0}$$

$$= \frac{H}{2}\cos(kx - \omega t) \qquad (2)$$

where k is wave number, and ω is a frequency of wave (Dean and Dalrymple, 1984). The relation of wave number k and frequency ω can be expressed by equation:

$$k = \frac{\omega}{\sqrt{gd}} \qquad (3)$$

where g is the acceleration due to gravity (Graper, 1984).

Based on the parameter of incident wave $\eta_i(x,t)$, a wave run-up height can be expressed numerically in the equation:

$$R = H\frac{a\xi}{1 + b\xi} \qquad (4)$$

where H is an incident wave height, a and b are empirical coefficient. The empirical coefficient in equation (4) are given by the value of a = 1,022 and b = 0,247 (Van der Meer and Stam, 1992, Goda 1988)). The run-up in equation (4) is predicted as a nonlinear function of surf parameter ξ which is determined by equation:

$$\xi = \frac{\tan\theta}{\sqrt{H/L}} \qquad (5)$$

where L is a deepwater wavelength. Since the incident wave is periodic, relation of wave length L and the number wave can be expressed by L = 2π / k, then from equation (3), the wavelength L with depth of water d can be expressed by:

$$L = T\sqrt{gd} \qquad (6)$$

where T is an incident wave period.

In the next section, we discuss the influence of incident wave height H and wave period T on the increasing level of wave run-up height based on data observations.

III. STATISTICAL ANALYSIS

In this section, we present a statistical analysis of wave run-up height on rock slope structure. The relationship between incident wave parameters and wave run-up height are measured by a correlation coefficient, and the probability distribution of wave run-up height are described as a Weibull distribution.

The relationship between incident wave parameters x values and wave run-up height y values to know whether or not variable x and y are correlated measured by a correlation coefficient r. From sample with n observations of variable x and y, correlation coefficient r is defined by:

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\ \sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}} \qquad (7)$$

where $\bar{x}$ is the mean of the observation of x values. The correlation coefficient r indicates the proportional reduction in the variability of y attained by the use of information about x. The estimation of the population correlation is denoted by the symbol ρ. We might want to test the hypothesis that the variables x and y are not correlated, that is, to test the hypothesis H_o : ρ = 0 against H_1 : ρ ≠ 0. We use t-test with a degree of freedom of n - 2 :

$$t = \frac{r\sqrt{n-2}}{\sqrt{1 - r^2}} \qquad (8)$$

to find the rejection region at level of significance α. Test of the hypothesis H_o is rejected if $|t| > t_{\alpha/2,\, n-2}$. If H_o is rejected,

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we conclude that variable x and y are correlated at level of significance α

Furthermore, a statistical distribution with Weibull distribution is applied to predict the wave run-up height. The cumulative Weibull distribution of particular value of significant wave height $\hat{H}$ is given by:

$$P(H \le \hat{H}) = 1 - \exp\left[-\left(\frac{\hat{H} - B}{A}\right)^f\right] \quad (9)$$

where H is the significant wave height, f is shape parameter, A is scale parameter, and B is location parameter. The statistical model of wave run-up height is described in terms of linear regression:

$$H_m = \hat{A}\, y_m + \hat{B} \quad (10)$$

where H_m is m^{th} value in the ranked significant heights, m is rank of the significant height value (Tucher 1991; Goda 1988). The variable y_m is given by the following formulae:

$$y_m = \left[-\{1 - \ln P(H \le H_m)\}\right]^{1/f} \quad (11)$$

The probability P in equation (11) is given by:

$$P(H \le H_m) = 1 - \frac{m - 0{,}2 - 0{,}27/\sqrt{f}}{N + 0{,}2 + 0{,}23/\sqrt{f}} \quad (12)$$

where N is the total number of observation of significant heights. The sum of the squares of residual is denoted by:

$$\sum_{m=1}^{N}\left[H_m - (\hat{A}\, y_m + \hat{B})\right]^2 \quad (13)$$

This is selected to provide a good fit to the linear regression models (10). The parameter f, A and B in the Weibull distribution (9) can be derived by a least squares fit based on the linear regression (10) with data measurements of incident wave. Based on a least squares fit of the linear regression (10) and a least of the sum of the squares of residual (13), the parameter A and B can be estimated by:

$$\hat{A} = \frac{N\sum_{i=1}^{N} H_{m,i}\, y_{m,i} - \sum_{i=1}^{N} H_{m,i} \sum_{i=1}^{N} y_{m,i}}{N\sum_{i=1}^{N} (y_{m,i})^2 - \left(\sum_{i=1}^{N} y_{m,i}\right)^2}$$

$$\hat{B} = \bar{H}_m - \hat{A}\, \bar{y}_m \quad (14)$$

where H_m is given from data measurements of incident wave (Figure 3), and the variable y_m is given in equation (11).

IV. ANALYSIS OF WAVE RUN-UP HEIGHT

In this section, we present an analysis of wave run-up height on rock slopes of coastal structure with various angle of structure slopes and various water depths based on wave height and wave period of the incident wave. Data measurements of wave height and wave period of incident wave are presented in Figure 3. Based on incident wave heights, the value of average, average significant (H_{33}), and maximum of incident wave height are presented in Table 1.

To determine the wave height with empirical approach (4), first, compute wave length L with equation (6) based on incident wave period with the water depth of 5 meter. Then, compute the surf parameter ξ given in equation (5) based on incident wave height and wave length (6). Next, compute the wave run-up height with equation (4), where the value of the empirical coefficient a and b are given by a = 1,022 and b = 0,247. Based on the amplitude of incident wave with water depth d = 5 meter, and angle of structure slopes of $\theta = 10^o$, 20^o,

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and 30°, the computation results of wave run-up height are given in Table 2.

We extend the result of wave run-up height prediction to describe the influence of incident wave on the wave run-up heights for various water depth with the same of incident height. Based on the incident wave parameters (Table 1), the prediction of wave run-up height and increasing level from incident wave amplitude with various water depths are given in Table 2. The wave run-up height prediction based on incident wave with water depth d = 5 meter for angle of structure slopes of $\theta = 10^o$, $\theta = 20^o$, and $\theta = 30^o$, and wave run-up height prediction with the angle of structure slopes of $\theta = 30^o$ and water depth d = 5 meter, d = 3 meter and d = 1 meter are presented in Table 2.

Table 2 shows the value and increasing level of wave run-up heights. The increasing level of wave run-up height from the amplitude of incident wave (Table 1) is smaller than the increasing level of average significant run-up height (H_{33}) or the increasing level of average run-up height.

Figure 3 shows the comparison of wave height and wave period associated from data measurements. The relationship between incident wave height and wave run-up height, wave period and wave run-up height are measured by the coefficient of correlation r given in (7). The computation of correlation coefficient r from data measurements for a wave run-up on a rock slopes of coastal structure with angle of a structure slope of $\theta = 30^o$ and with water depth d = 5 meter are given in Table 3.

From Table 3, the correlation of incident wave height and wave run-up height is measured by the correlation coefficient r = 0,993 with the sample t-test t = 5,522, and the correlation of incident wave period and wave run-up height is measured by the correlation coefficient r = -0,113 with the sample t-test t = - 0,209. From the Table of the Student t distribution with level of significant $\alpha = 0{,}05$ and degree of freedom of n - 2, we derived the statistic $t_{0{,}025,\ 24}$ = 2,06. Then, the result of the test is reject H_o : $\rho = 0$ for r = 0,993, means that incident wave height and wave run-up height are correlated with significant at the 5 percent, and accept H_o : $\rho = 0$ for r = -0,113, means that incident wave period and wave run-up height are not correlated with significant at the 5 percent.

To determine the probability distribution of wave run-up height with Weibull distribution, first, compute the probability of the mth significant height not being exceeded as in (12) and the variable y_m with equation (11). Then compute the parameter f, A and B as in the linear regression (10) by a least square method. Parameter f, A and B are estimated by computing with a least squares fit based on the linear regression (10) and a least of sum of square of residuals (SSR) as given in (13). The result of the value of parameter of f, and parameter A and B in the form of the linear regression (10) are given in Table 4.

The probability distribution of wave run-up height with angle of sloping face $\theta = 30^o$ with water depth d = 1 m, d = 3 m, and d = 5 m are given in Figure 4. The value of probability is the probability of exceedance of wave run-up height particular value on the wave run-up height. From the incident wave amplitude of data measurement (Table 1), the average wave run-up height is 0,713 meter with probability of exceedance 0,440, the average significant of run-up height (H_{33}) is 1,012 meter with probability of exceedance 0,129, and the maximum of wave run-up height is 1,209 with probability of exceedance 0,057.

Table 1. The incident wave height.

	Wave height (in meter)
Minimum	0,150
Average	0,296
Significant (H_{33}) average	0,556
Maximum	0,600

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Table 2. The value and the increasing level of wave run-up heights from incident wave amplitude.

	Run-up height (in meter)			Average increasing level (%)		
	d = 1 m	d = 3 m	d = 5 m	d = 1 m	d = 3 m	d = 5 m
Run-up ($\theta = 10^o$)						
Average Height	0,276	0,338	0,370	100,92	145,11	167,46
Average Height (H_{33})	0,363	0,450	0,495	63,48	102,31	122,36
Maximum height	0,403	0,505	0,558	34,41	68,26	86,03
Run-up ($\theta = 20^o$)						
Average Height	0,457	0,537	0,575	228,03	282,93	308,95
Average Height (H_{33})	0,621	0,739	0,797	178,14	230,56	256,04
Maximum height	0,710	0,857	0,930	136,56	185,53	209,87
Run-up ($\theta = 30^o$)						
Average Height	0,593	0,675	0,713	320,99	376,58	401,78
Average Height (H_{33})	0,824	0,952	1,012	267,98	324,23	350,39
Maximum height	0,964	1,130	1,209	221,39	276,73	303,05

Table 3. The correlation coefficient r, and test the hypothesis H_o : $\rho = 0$ against H_1 : $\rho \neq 0$, α = 0,05.

	Run-up : $\theta = 30^o$, d= 5m	H_o : $\rho = 0$
Wave Height	r = 0,993	Accepted
Wave Period	r = - 0,113	Rejected

Table 4. Statistical model of run-up height for angel of structure slope $\theta = 30^o$ with various water depth d = 1 m, d = 3 m, and d = 5 m.

	Parameter f	Run-up height model (linear regression)	SSR
Incident wave height	1,4	$H_{sm} = 0,209\ y_m + 0,106$	0,021
Run-up height, d = 1 m	1,8	$H_{sm} = 0,385\ y_m + 0,250$	0,045
Run-up height, d = 3 m	1,7	$H_{sm} = 0,437\ y_m + 0,286$	0,064
Run-up height, d = 5 m	1,7	$H_{sm} = 0,471\ y_m + 0,293$	0,074

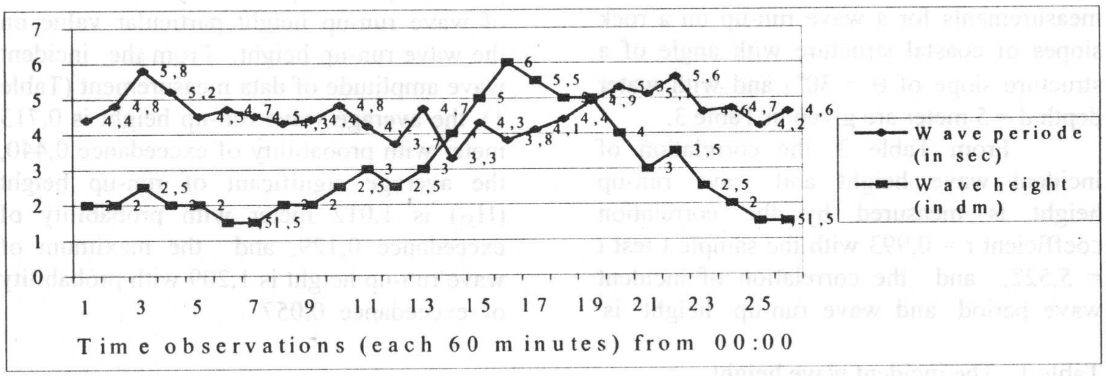

Figure 3. Wave height (in dm) and wave period associated (in sec) of data measurements of incident wave.

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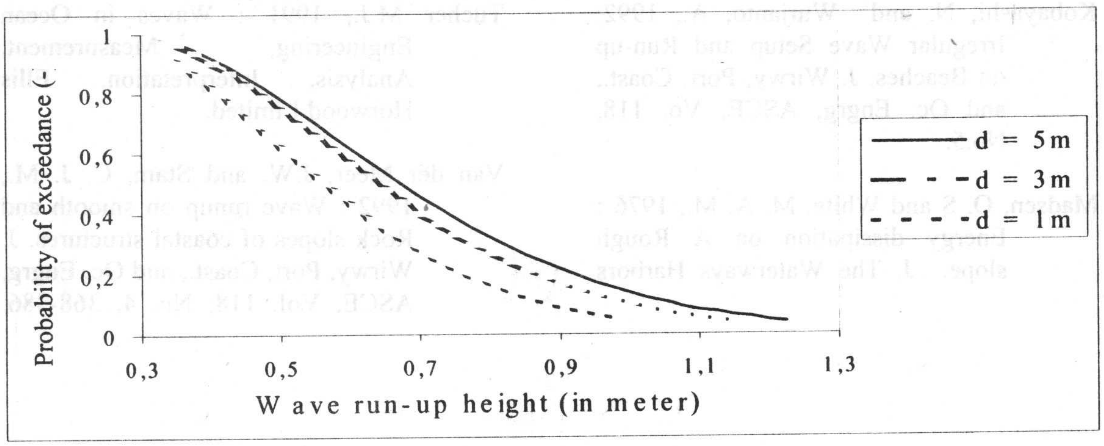
Figure 4. The probability distribution of run-up height on a coastal structure with angel of slope θ = 30°.



V. CONCLUSIONS

The prediction of wave run-up height on a rock slopes of a coastal structure are described with various angle of structure slopes and various water depths based on data measurements. The wave run-up height is described based on data measurements as the parameters of periodic incident waves entering a coastal structure. Based on data measurements, the incident wave height and wave run-up height are correlated, but incident wave period and run-up height are not correlated significantly at 5 percent. The formulation of wave run-up height equation is expressed as a function of the surf parameter ξ, and the surf parameter ξ is described as a function of incident wave height. Therefore, the wave run-up height depends on an incident wave height, and the increasing level of run-up height is influenced by the incident wave height.

For a coastal structure with angle of structure slope θ = 30°, the increasing level of the maximum wave run-up height from incident wave amplitude (0,5 x wave height) are 221 % for water depth d = 1 meter, 277% for water depth d = 3 meter, and 303 % for water depth d = 5 meter. The increasing level of the maximum wave run-up height from the incident wave amplitude is smaller than the increasing level of average significant run-up height (H_{33}), but the value of maximum wave run-up height depends on the maximum of incident wave height. Furthermore, the probability distribution of wave run-up height is described as a Weibull distribution. Parameters of Weibull distribution are estimated by computing with a least squares fit based on the linear regression with a least of sum of square of residuals. Based on a incident wave amplitude, the average significant run-up height (H_{33}) is 1,012 meter with probability of exceedance 0,129, and the maximum wave run-up height is 1,209 with probability of exceedance 0,057.

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