

ACCURACY ASSESSMENT OF HEIGHTS OBTAINED FROM TOTAL STATION AND LEVEL INSTRUMENT USING TOTAL LEAST SQUARES AND ORDINARY LEAST SQUARES METHODS

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Abstract: Spirit levelling has been the traditional means of determining Reduced Levels (RL's) of points by most surveyors. The assertion that the level instrument is the best instrument for determining elevations of points needs to be reviewed; this is because technological advancement is making the total station a very reliable tool for determining reduced levels of points. In order to achieve the objective of this research, reduced levels of stations were determined by a spirit level and a total station instrument. Ordinary Least Squares (OLS) and Total Least Squares (TLS) techniques were then applied to adjust the level network. Unlike OLS which considers errors only in the observation matrix, and adjusts observations in order to make the sum of its residuals minimum, TLS considers errors in both the observation matrix and the data matrix, thereby minimising the errors in both matrices. This was evident from the results obtained in this study such that OLS approximated the adjusted reduced levels, which compromises accuracy, whereas the opposite happened in the TLS adjustment results. Therefore, TLS was preferred to OLS and Analysis of Variance (ANOVA) was performed on the preferred TLS solution and the RL's from the total station in order to ascertain how accurate the total station can be relative to the spirit level.

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1. INTRODUCTION

Measurements through field surveying procedures are one of the essential tasks in all areas of the geoscientific disciplines. The three fundamental measured quantities in surveying and mapping are distances, angles and heights. These quantities form the basis for estimating coordinates of positions concerning a particular datum (either horizontal or vertical). It is important to note that the angles and distances provide coordinates in the horizontal datum. However, in establishing a vertical datum, the classical approach widely used is the levelling.

Levelling is the most widely used method for obtaining the elevations of ground points. It is the art and science of determining altitudes of points on or beneath the surface of the earth relative to a reference datum and is usually carried out as a separate procedure from that used for fixing planimetric position (Schofield & Breach, 2007). It is also done to obtain data for mapping, engineering design, construction, setting out (Ghilani & Wolf, 2014) and among others. The mean sea level is the surface adopted as the reference datum for vertical control surveys because it is assumed to have an equipotential surface (Schofield & Breach, 2007; Uren & Price, 2010). Therefore, it is pertinent to say that height above the surface adopted as a datum is known as the reduced level.

Reduced levels (RLs) could be determined directly by spirit levelling (using level instrument) or through indirect techniques such as trigonometric levelling via a total station, photogrammetry or through modern positioning systems such as Global Navigation Satellite System (GNSS). Conversely, there is a common belief that reduced levels obtained through spirit levelling are superior to the indirect methods as mentioned above (Lee & Rho, 2001). To investigate this assertion, total station and a level instrument were used to determine the reduced levels of three survey control stations starting from a known station A, at the height of 76.080 m.

The measured RLs in the level net, like any other survey measurement, contain errors and hence must be adjusted. Usually, level networks are adjusted using Ordinary Least Squares (OLS) approach (Okwuashi & Eyoh, 2012b); but OLS, which is based on regression analysis considers only the observations to be stochastic (Acar et al., 2006) and thus account for errors only in the observation vector. Conversely, there exist errors in both the design and observation matrices which ought to be modelled out. To solve the defects, a more robust adjustment method based on the Errors-in-Variable (EIV) known as Total Least Squares (TLS) was considered in this study. In other words, TLS can incorporate errors in both the observation vector and data matrix (Jin, Tong, & Li, 2011).

This study analyzed the residuals obtained from both OLS and TLS. A test of hypothesis at 5% level of significance on the results obtained from the level instrument and the total station was carried out. It was done to ascertain their difference in accuracies further providing statistical meaning to the results attained.

2. DATA AND METHODS

2.1. Data Acquisition

This study applied primary data through spirit and trigonometric levelling. From a known station A, at an elevation of 76.080 m, the reduced levels of unknown stations U, V and W were each determined using a level instrument and a total station. The reduced levels of the stations are tabulated in **Table 1**, the differences in elevation between the stations are summarised in **Table 2**. **Figure 1** is a schematic view of the level network.

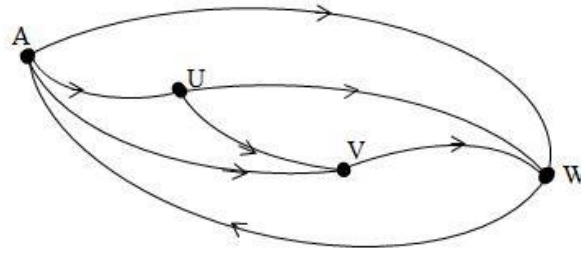
Table 1. Reduced levels of stations (units in metres) (Author, 2016)

Stations	Spirit Levelling	Trigonometric Levelling
A	76.080	76.080
U	73.355	72.417
V	73.831	73.836
W	72.966	73.004

Table 2. Elevation differences between stations (units in metres) (Author, 2016)

Height	From	To	ΔH
ΔH_{AU}	A	U	-3.725
ΔH_{AV}	A	V	-2.249
ΔH_{AW}	A	W	-3.114
ΔH_{UV}	U	V	+1.502
ΔH_{UW}	U	W	+0.663
ΔH_{VW}	V	W	-0.389

Figure 1. Schematic view of the level network (Author, 2016)



2.2. Ordinary Least Squares (OLS) and Total Least Squares (TLS)

Consider a system of equations in the form of **Equation 1** to be solved by least squares:

$$AX \approx L, \text{ where } A \in R^{m \times n}, X \in R^{n \times d}, L \in R^{m \times d}, \text{ and } m \geq n; \quad [1]$$

where m is the number of rows and n is the number of columns.

The solution of the matrix of unknown parameters X , by OLS approach is given by:

$$X = (A^T A)^{-1} (A^T L) \quad [2]$$

The corresponding error vector V is obtained by:

$$V = AX - L \quad [3]$$

On the other hand, solution of unknown parameters \hat{X} , by TLS approach is obtained through:

$$L + V_L = (A + V_A) \hat{X} \text{ rank } (A) = m < n \quad [4]$$

where V_L is the error vector of observations and V_A is error matrix of the data matrix; with the assumption that both have independently and identically distributed rows with zero mean and equal variance (Akyilmaz, 2007).

Golub and van Loan (1980) came up with TLS to rectify the inefficiency of Ordinary Least Squares (OLS) thus, accounting for perturbations in data matrix and observation matrix. TLS is an algorithm that gives a unique solution, in analytical form in terms of the Singular Value Decomposition (SVD) of the data matrix (Markovsky & Van Huffel, 2007). From Golub and van Loan (1980) and Okwuashi and Eyoh (2012a), the TLS algorithm is an iterative process which seeks to minimise the errors in **Equation 4** such that

$$\min \|[A, L] - [\hat{A}, \hat{L}]\|_F, [\hat{A}, \hat{L}] \in R^{n(m+1)} \quad [5]$$

The optimisation process goes on until a minimizing $[\hat{A}, \hat{L}]$ is obtained; any \hat{X} that satisfies $\hat{A}\hat{X} = \hat{L}$ is the TLS solution. To obtain the solution of $A\hat{X} = L$, we write the functional relation:

$$[A, L] [\hat{X}^T, -1]^T \approx 0 \quad [6]$$

The TLS problem can be solved using SVD. The SVD of the augmented matrix $[A, L]$ is required to determine whether or not it is rank deficient. Matrix $[A, L]$ can be represented by SVD as;

$$[A, L] = USV^T \quad [7]$$

where $U =$ real valued $m \times n$ orthonormal matrix, $UU^T = I_m$

$V =$ real valued $n \times n$ orthonormal matrix, $VV^T = I_n$

$S = m \times n$ matrix with diagonals being singular values, off-diagonals are zeros

The rank of matrix $[A, L]$ is $m+1$, and must be reduced to m using the Eckart-Young Mirsky theorem.

The TLS solution after the rank reduction is given by:

$$\left[\hat{X}^T, -1 \right]^T = \frac{1}{V_{m+1, m+1}} V_{m+1} \quad [8]$$

If $V_{m+1, m+1} \neq 0$, then $AX = L = -1/(V_{m+1, m+1}) \cdot A[V_{1, m+1}, \dots, V_{m, m+1}]^T$ belongs to the column space of \hat{A} ; hence X solves the basic TLS problem (Okwuashi and Eyoh, 2012a). The corresponding TLS correction is given by:

$$[\Delta \hat{A}, \Delta \hat{L}] = [A, L] - [\hat{A} - \hat{L}] \quad [9]$$

2.3. Application of OLS and TLS

Observation equations were deduced from **Table 2** to obtain **Equation 10**:

$$\left. \begin{aligned} \Delta H_{AU} &= H_U - H_A \Rightarrow H_U = +72.355 \\ \Delta H_{AV} &= H_V - H_A \Rightarrow H_V = +73.831 \\ \Delta H_{AW} &= H_W - H_A \Rightarrow H_W = +72.966 \\ \Delta H_{UV} &= H_V - H_U = +1.502 \\ \Delta H_{UW} &= H_W - H_U = +0.663 \\ \Delta H_{VW} &= H_W - H_V = -0.839 \end{aligned} \right\} \quad [10]$$

Finding the partial derivatives of **Equation 10** with respect to H_U , H_V and H_W will generate the data matrix A . The observation matrix L , and solution matrix X , are also given below:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \quad L = \begin{bmatrix} 72.355 \\ 73.831 \\ 72.966 \\ 1.502 \\ 0.663 \\ -0.839 \end{bmatrix} \quad X = \begin{bmatrix} H_U^* \\ H_V^* \\ H_W^* \end{bmatrix}$$

where H_U^* , H_V^* and H_W^* are the adjusted reduced levels at stations U , V and W respectively.

2.4. Analysis of Variance (ANOVA)

ANOVA is an inferential statistical tool for comparing the means of two or more groups (Jackson, 2013). It is a method that tests the groups to find out whether their sample means could have been determined from populations with the same true mean (Brown & Berthouex, 2002). It compares variations between groups with variations within groups by making use of the F distribution even though its sample variances have a Chi-squared (χ^2) distribution. The sample variances are distributed according to the F -distribution. The F is a skewed distribution whose exact shape depends on the degrees of freedom. Since the variable of interest (reduced levels) is one, the one-way randomized ANOVA was used. ANOVA assumes that (Rutherford, 2001);

- expected values of the errors are zero (absence of outliers)
- variances of all errors are equal to each other (homogeneity of variance)
- errors are independent (independence of errors)
- Means are normally distributed (normality of sampling distribution of means).

3. RESULTS AND DISCUSSION

The results for the unknowns by OLS and TLS approach is given in **Table 3** below. It can be observed from **Table 3** that OLS approximated the values of the unknowns which decreases accuracy, whereas TLS

gave more accurate results. The residuals obtained from the two methods of adjustment are also given in **Table 4**.

Table 3. Solution of unknown parameters (units in metres) (Analysis, 2016)

Adjusted Height	OLS	TLS
H [*] _u	72.33550000000000	72.335504614988100
H [*] _v	73.83100000000000	73.831004638665500
H [*] _w	72.98550000000000	72.985504625279500

Table 4. Residuals from OLS and TLS (units in metres) (Analysis, 2016)

Height	OLS	TLS
HU	0.0195000000000008	0.019495385011936
HV	0.0000000000000014	-0.000004638665473
HW	-0.0195000000000008	-0.019504625279495
HV – HU	0.0065000000000007	0.006499976322592
HW – HU	0.0129999999999994	0.012999989708579
HW - HV	0.0064999999999987	0.006500013385987
Sum	0.0260000000000003	0.025986100484125

The TLS gave better estimates of the unknowns, as well as marginally better accuracy of 0.025986100484125 against 0.0260000000000003. Therefore, the TLS was preferred to that of the OLS. ANOVA was then applied on the TLS results and that from the total station instrument. The F ratio computed is compared with its theoretical value $F_{(v_1, v_2, \alpha)}$, read from the F distribution table at a 5% level of significance. The null hypothesis is rejected for the alternative if the computed value is greater than the theoretical value. The null hypothesis (H_0) states that the means of both techniques are the same; thus there is no significant difference between the two techniques. The alternative hypothesis (H_1) states that their means are different and hence there is a significant difference. These can be expressed mathematically as:

$$\left. \begin{array}{l} H_0 : \mu_1 = \mu_2 \\ H_1 : \mu_1 \neq \mu_2 \end{array} \right\} \quad [11]$$

where μ_1 and μ_2 represent the means of the TLS adjusted RL's and the total station RL's respectively. The results from the ANOVA have been summarised in **Table 5**.

$$F \text{ ratio} = \frac{\text{Estimate of Between Groups Variance}}{\text{Estimate of Within Groups Variance}} = \frac{\text{Mean Square Between } (MS_B)}{\text{Mean Square Within } (MS_W)} \quad [12]$$

where $MS_B = \frac{\text{Sum of Squares Between Groups } (SS_B)}{\text{Degrees of Freedom Between Groups } (df_B)}$ and

$$MS_W = \frac{\text{Sum of Squares Within Groups } (SS_W)}{\text{Degrees of Freedom Within Groups } (df_W)}$$

Table 5. Variations between and within groups (Analysis, 2016)

Variation	Sum of Squares (SS)	Degrees of Freedom (df)	Mean Square Error (MS)	F Ratio
Between Groups	0.001837014269446	1	0.001837014269446	0.0034
Within Groups	2.141414868944560	4	0.535353717236139	
Total	2.143251883214000	5		

The critical value $F_{(1,4,0.025)}$ is 12.22, which is much greater than the computed value of 0.0034. Hence, the null hypothesis is accepted, and the alternative is rejected. This finding concludes that, at 95 % confidence level, the means of the two levelling techniques are equal. Thus, the reduced levels from the total station are almost the same as those from the level instrument.

4. CONCLUSION

The conclusion drawn from this study is that the total station can be used in the absence of the level for survey works such as volume estimation and topographical surveys which demand less accuracy in heights. However, it is recommended by the authors that for very high accuracy demanding works such as deformation monitoring, geoid determination and engineering surveys, for example, processing plant set-outs, bridges, dam constructions and railways will demand the use of the level instrument.

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