OPEN ACCESS

Vol 4, No. 1, 2017, 19-26

Geoplanning
Journal of Geomatics and Planning
E-ISSN: 2355-6544
http://ejournal.undip.ac.id/index.php/geoplanning
doi: 10.14710/geoplanning.4.1.19-26

COORDINATE TRANSFORMATION USING FEATHERSTONE AND VANÍČEK PROPOSED APPROACH - A CASE STUDY OF GHANA GEODETIC REFERENCE NETWORK

Y. Y. Ziggah [a, b], **J. Ayer** [c], **P. B. Laari** [d], **E. Frimpong** [b]

[a] *Department of Surveying and Mapping, China University of Geosciences, Wuhan, 430074, P.R. China.*
[b] *Department of Geomatic Engineering, University of Mines and Technology, Tarkwa, Western Region, Ghana.*
[c] *Department of Geomatic Engineering, Kwame Nkrumah University of Science and Technology, Kumasi, Ashanti Region, Ghana.*
[d] *Department of Environment and Resource Studies, University for Development Studies, Tamale, Ghana.*

Article Info:
Received: 30 July 2016
in revised form: 18 September 2016
Accepted: 26 January 2017
Available Online: 26 March 2017

Keywords:
Coordinate transformation, Global Navigation Satellite Systems, Geodetic Reference Network, Featherstone and Vaníček model

Corresponding Author:
Yao Yevenyo Ziggah
Geomatic Engineering Department, Faculty of Mineral Resource Technology, University of Mines and Technology, Tarkwa, Western Region, Ghana
Email: yyziggah@umat.edu.gh

Abstract: *Most developing countries like Ghana are yet to adopt the geocentric datum for its surveying and mapping purposes. It is well known and documented that non-geocentric datums based on its establishment have more distortions in height compared with satellite datums. Most authors have argued that combining such height with horizontal positions (latitude and longitude) in the transformation process could introduce unwanted distortions to the network. This is because the local geodetic height in most cases is assumed to be determined to a lower accuracy compared with the horizontal positions. In the light of this, a transformation model was proposed by Featherstone and Vaníček (1999) which avoids the use of height in both global and local datums in coordinate transformation. It was confirmed that adopting such a method reduces the effect of distortions caused by geodetic height on the transformation parameters estimated. Therefore, this paper applied Featherstone and Vaníček (FV) model for the first time to a set of common points coordinates in Ghana geodetic reference network. The FV model was used to transform coordinates from global datum (WGS84) to local datum (Accra datum). The results obtained based on the Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) in both Eastings and Northings were satisfactory. Thus, a RMSE value of 0.66 m and 0.96 m were obtained for the Eastings and Northings while 0.76 m and 0.73 m were the MAE values achieved. Also, the FV model attained a transformation accuracy of 0.49 m. Hence, this study will serve as a preliminary investigation in avoiding the use of height in coordinate transformation within Ghana's geodetic reference network.*

How to cite (APA 6th Style):
Ziggah, Y. Y, et al. (2017). Coordinate transformation using Featherstone and Vaníček proposed approach - a case study of Ghana geodetic reference network. *Geoplanning: Journal of Geomatics and Planning, 4*(1), 19-26. doi:10.14710/geoplanning.4.1.19-26

1. INTRODUCTION

The last decade has witnessed the ascendancy in the application of Global Navigation Satellite Systems (GNSS) for geospatial works in developed and developing countries. However, most developing countries like Ghana which is yet to migrate onto a geocentric datum cannot apply directly GNSS positional measurement without transforming the data into its local coordinate system. In line with this, several research works have been carried out to ascertain the applicability and capability of transformation methods such as three parameter, Bursa-Wolf, Molodensky-Badekas, Standard Molodensky, Iterative Abridged Molodensky, Veis, 3D projective, 12 parameter linear affine in Ghana geodetic reference network (Ayer & Fosu, 2008; Ayer & Tiennah, 2008; Ayer, 2008; Dzidefo, 2011; Poku-Gyamfi & Schueler, 2008; Ziggah et al., 2013a; Ziggah et al., 2013b). However, the general insight gathered from these studies in Ghana showed varying coordinate transformation results and accuracy among the various authors even though the same dataset is utilized. Upon careful observation, it is noticed that this phenomenon of inconsistencies in the results could mainly be attributed to the estimated local geodetic height used in the coordinate transformation process. This is because the iterative Abridged Molodensky technique utilized to estimate the ellipsoidal height correction and in return used to estimate the ellipsoidal height is difficult to reach convergence for the ellipsoidal height correction factor. In so doing, researchers in Ghana applied different geodetic height correction values to estimate the local geodetic height thus obtaining different coordinate transformation results. It therefore confirms the assertion made by Dzidefo (2011) and Kotzev (2013) that there exist no ideal transformation parameters to be utilized in Ghana. Hence, this has contributed to the users' adoption of different transformation parameters in the GNSS data processing in Ghana.

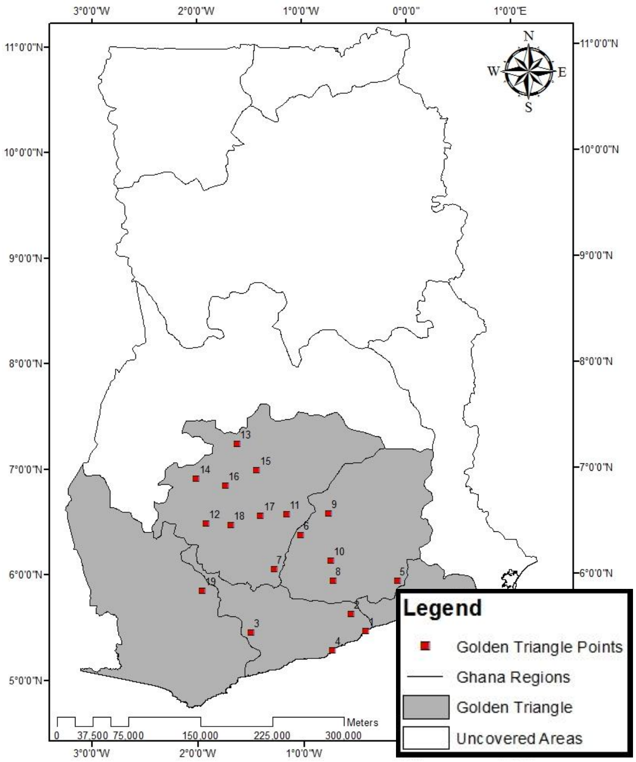
It is important to note that the effect of height distortions in horizontal geodetic datum transformation has been duly investigated (Featherstone & Vanicek, 1999; Vaníček & Steeves, 1996). In order to minimize these distortions, Vaníček and Steeves (1996) proposed a four-parameter transformation model in which geodetic height in both global datum and local datum is not used and only three translations and one rotation parameter are needed to carry out coordinate transformation. Hence, the transformation parameters determined are free from geodetic height distortions. Similarly, Featherstone and Vanicek (1999) extended the four-parameter model to six-parameters on the premise that the orientation of the local geodetic system with respect to the geocentric system could possibly be done without using the local astronomic system at the origin of the network. The proposition thereof lies in coordinate transformation containing either four or six parameters.

On the basis of the above related issues, the present authors were motivated to apply for the first time the Featherstone and Vanicek (1999) coordinate transformation model (FV model) in Ghana's geodetic reference network. We deem it appropriate to apply such a method because Ghana's national coordinate system is a two-dimensional projected grid coordinates. Applying the FV model will most importantly eliminate distortions related to the height component in the coordinate transformation of Ghana. This study will serve as guide on the importance of not using the local geodetic height since our national coordinate system is a two-dimensional projected grid coordinates.

2. DATA AND METHODS

A secondary data of 19 geodetic common point coordinates was acquired from the Ghana Survey and Mapping Division of Lands Commission. These data sets were in World Geodetic System 1984 (WGS84) and War Office 1926 ellipsoid. The War Office 1926 is the reference ellipsoid for Accra datum. The obtained data sets covered five out of ten regions in Ghana. These regions form the Ghana geodetic reference network known as the Golden Triangle. Figure 1 shows the study area and common point's distribution.

Figure 1. Area of study showing data distribution



doi: 10.14710/geoplanning.4.1.19-26

2.1. Conversion of Geodetic Coordinate to Cartesian Coordinate

The forward conversion of geodetic coordinate to Cartesian coordinate was the first step employed in the coordinate transformation process. To accomplish this task, the standard forward equation (Heiskanen & Moritz, 1967) defined in Equation (1) was used.

$$\begin{aligned} X &= (N+h)\cos\varphi\cos\lambda \\ Y &= (N+h)\cos\varphi\sin\lambda \\ Z &= [N(1-e^2)+h]\sin\varphi \end{aligned} \quad [1]$$

where φ, λ and h is the geodetic latitude, geodetic longitude and ellipsoidal height while X, Y, Z is the cartesian coordinates to be estimated. N in Equation (1) is the radius of curvature in the prime vertical defined by Equation (2) as

$$N = \frac{a}{\sqrt{1-e^2\sin^2\varphi}} \quad [2]$$

Here, e is the first eccentricity expressed in Eq. (3) as

$$e = \frac{\sqrt{a^2-b^2}}{a} \quad [3]$$

where a and b are the semi-major axis and semi-minor axis.

On the basis of the concept proposed by Featherstone and Vanicek (1999); Vaníček and Steeves (1996) the heights for WGS84 and War Office 1926 should not be used. Hence, Equation (1) was modified into Equation (4) as

$$\begin{aligned} X &= N\cos\varphi\cos\lambda \\ Y &= N\cos\varphi\sin\lambda \\ Z &= N(1-e^2)\sin\varphi \end{aligned} \quad [4]$$

Equation (4) was then applied to convert all the 19 geodetic coordinates of common points designated in this study as $(\varphi,\lambda)_{WGS84}$ and $(\varphi,\lambda)_{War}$ into cartesian coordinates. The transformed cartesian coordinates for WGS84 and War Office 1926 are represented in this study as $(X, Y, Z)_{WGS84}$ and $(X, Y, Z)_{War}$ respectively.

2.2. Coordinate using Featherstone and Vaníček (FV) Model

The Featherstone and Vaníček (FV) model is a six parameter transformation that combines three rotation axis and three origin-shifts in a mathematical model which presents a relationship between points in two different Cartesian coordinate systems (Featherstone & Vanicek, 1999). This study applied the FV model to determine three rotational and three translational parameters for transforming coordinates from global datum (WGS84) to Ghana local geodetic datum (War Office 1926). The FV model (Equation (5)) (Featherstone & Vanicek, 1999) could be represented mathematically as

$$Q_{War} - Q_{WGS} = -M_i R - T \quad [5]$$

Here, Q_{War} is the cartesian $(X, Y, Z)_{War}$ coordinates, Q_{WGS} is the cartesian $(X, Y, Z)_{WGS84}$, M is a three-by-three matrix containing each Q_{WGS} positions, R is the rotation matrix and T is the translation vector. M in Equation (5) is defined by Equation (6) as

$$M = \begin{bmatrix} 0 & -Z_i & Y_i \\ Z_i & 0 & -X_i \\ -Y_i & X_i & 0 \end{bmatrix}, i = 1,\ldots,n \quad [6]$$

where n is the number of observation points. Equation (5) could be rewritten in a simplified form expressed in Equation (7) as

$$\forall i = 1,\ldots n: \ \Delta Q_i = A_i \ P \quad [7]$$

where $A_i = [-M, I]$ is the design matrix, I is the identity matrix, ΔQ_i is the observation vector and $P = [R\ , T]^t$ is the transpose of the vector of the unknown transformation parameters to be determined. In this study, the least squares approach defined by Equation (8) was used to estimate the unknown parameters.

$$P = (A^T C_{\Delta Q}{}^{-1} A)^{-1} \ A^T C_{\Delta Q}{}^{-1} \Delta Q \quad [8]$$

where A^T is the transpose of the design matrix, $C_{\Delta Q} = C_{Q_{WGS}} + C_{Q_{WAR}}$ represent the sum of the variance covariance matrix of the cartesian coordinates of WGS84 and War Office 1926 system.

2.3. Estimating New War Office Cartesian coordinates

Here, the six transformation parameters calculated in Section 2.2 was used to transform the WGS84 cartesian coordinates into the War Office 1926 system to obtain new War Office cartesian coordinates denoted as (X_{nwar}, Y_{nwar}, Z_{nwar}). These coordinates were obtained by rewriting Equation (7) as Equation (9).

$$Q_{War} = Q_{WGS} - (A_i * P) \quad [9]$$

2.4. Converting Cartesian Coordinates to Geodetic Coordinates

The (X_{nwar}, Y_{nwar}, Z_{nwar}) (Section 2.3) was converted into geodetic coordinates. This conversion was necessary so that the obtained geodetic coordinates could be projected on to the Transverse Mercator to obtain two-dimensional projected grid coordinates which is the national mapping coordinate system utilized for surveying and mapping purposes in Ghana. To achieve this, Paul's method (Paul, 1973) was used in the reverse conversion. The choice of this method was based on a study carried out by Kumi-boateng and Ziggah (2016) where the Paul's method performed slightly better than other six reverse conversion techniques evaluated for the Ghana geodetic reference network. In that respect, having (X_{nwar}, Y_{nwar}, Z_{nwar}), the geodetic latitude (φ) was obtained using Equation (10) given as

$$p\tan\varphi = \varsigma + \frac{Z}{2} \quad [10]$$

where $p = \sqrt{X^2 + Y^2}$ and $\varsigma = \sqrt{t_1} + \sqrt{\frac{Z^2}{4} - \frac{\beta}{2} - t_1 + \frac{\alpha Z}{4\sqrt{t_1}}}$.

To get ς , the variables $(\alpha, \beta, q, u_1, t_1, \varsigma)$ should be calculated in an orderly manner using Equation (11) to (15) respectively.

$$\alpha = \frac{p^2 + a^2 e^4}{1 - e^2} \quad [11]$$

$$\beta = \frac{p^2 - a^2 e^4}{1 - e^2} \quad [12]$$

$$q = 1 + \frac{27Z^2(\alpha^2 - \beta)}{2(\beta + Z^2)^2} \quad [13]$$

$$u_1 = \tfrac{1}{2}\left\{\sqrt[3]{q + \sqrt{q^2 - 1}} + \sqrt[3]{q - \sqrt{q^2 - 1}}\right\} \quad [14]$$

$$t = \left(\frac{\beta + Z^2}{6}\right)u + \frac{Z}{12} - \frac{\beta}{6} \quad [15]$$

Detailed derivation of the Paul's method can be found in Paul (1973). In this study, the conversion from ellipsoidal coordinates to plane coordinates (Transverse Mercator Projection); equations given in Dzidefo (2011) were used.

2.5. Accuracy Assessment

The accuracy of the FV model utilised was analysed using statistical indices. This was done by quantifying the residuals obtained when the FV model results were subtracted from the existing projected grid coordinates. The statistical indices used are the root mean square error (RMSE), mean absolute error (MAE), horizontal position error (HE) and standard deviation (SD). They are defined by Equation (16) to (19) respectively as

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(O_i - P_i)^2} \quad [16]$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|O_i - P_i\right| \quad [17]$$

$$HE = \sqrt{\Delta E^2 + \Delta N^2} \quad [18]$$

$$SD = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(e - \bar{e})^2} \quad [19]$$

where n is the number of observations, O_i and P_i represents the existing projected grid coordinates and computed projected grid coordinates. e is the error between O_i and P_i with $\bar{e}$ as its mean.

3. RESULTS AND DISCUSSION

3.1. Transformation Parameters Determined

Table 1 shows the transformation parameters and their related standard deviations for transforming coordinates from WGS84 (US Department of Defense, 1984) to War Office 1926 system using Featherstone and Vaníček (1999) model. With reference to Table 1, ΔX, ΔY, ΔZ is the translation parameters while Rx, Ry and Rz represent the rotational parameters. These translation parameters (Table 1) signify the degree of shift in origins of WGS84 and War Office 1926 along the three axes in three-dimension space. The rotational parameters around each of the X, Y and Z axes relate the orientation of the WGS84 and War Office 1926 systems.

Table 1. Parameters from FV model

Parameters	Value
ΔX (m)	164.585 ± 0.098
ΔY (m)	-4.781 ± 1.326
ΔZ (m)	-21.902 ± 1.003
Rx (arc seconds)	-1.613E-06 ± 1.180E-06
Ry (arc seconds)	-4.736E-05 ± 1.56E-07
Rz (arc seconds)	4.335E-06 ± 1.55E-07

3.2. Analysis of Transformed Coordinates Results

The shifts in coordinates (ΔE, ΔN) between the existing coordinates and transformed coordinates by Featherstone and Vaníček (1999) (FV model) are presented in Table 2. The SD values and HE for the control points are also shown. These results (Table 2) indicate the extent at which each of the transformed coordinates produced by the FV model varies with respect to the existing coordinates. Figure 2 gives an illustration on how the residuals generated oscillate along the ideal zero residual (horizontal line) with respect to the observation points. From Figure 2, a fairly consistent rise and fall was noticed for the Easting coordinates whereas a sharp rise and fall was observed for the Northing coordinates respectively.

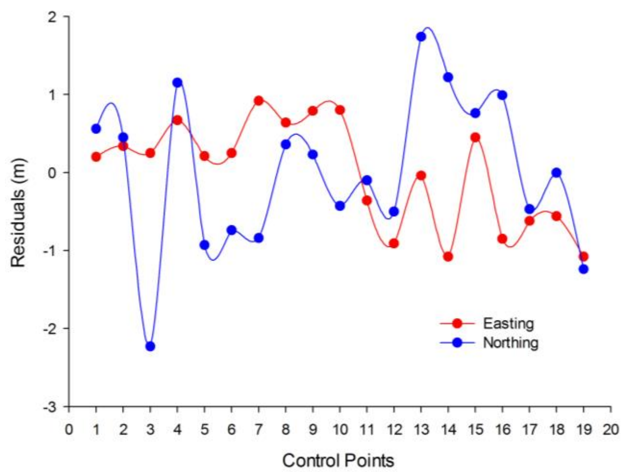
Overall analysis of Figure 2 indicates that the residuals in Northings were higher than the Eastings. These residuals incurred by the FV model clearly depict the limitation in most mathematical models that they could only produce results approximating the existing data. Although the height component was not applied in this transformation, the residuals obtained suggest that the FV model could not completely absorb the horizontal coordinate distortions mainly contributed by the astro-geodetic network (War Office

1926) system. These factors however, have contributed to the inability of the FV model to notice its potential of providing higher (sub-metre or even sub-centimetre) accuracy even though it is a rigorous model. Hence, the ideal condition of obtaining zero residuals could not be achieved in this study. In order to mitigate these residual effects, we are proposing that distortion modelling should be carried out after coordinate transformation in Ghana geodetic reference network. However, the fact still remains that the FV model will serve as a preliminary step that will facilitate a viable consensus in selecting applicable transformation parameters in Ghana. This could be achieved because the height component which has created such inconsistencies in the transformation results among researchers in Ghana is not applicable in the Featherstone and Vaníček (1999) model.

Table 2. Deviation of Transformed Coordinates from Existing Coordinates

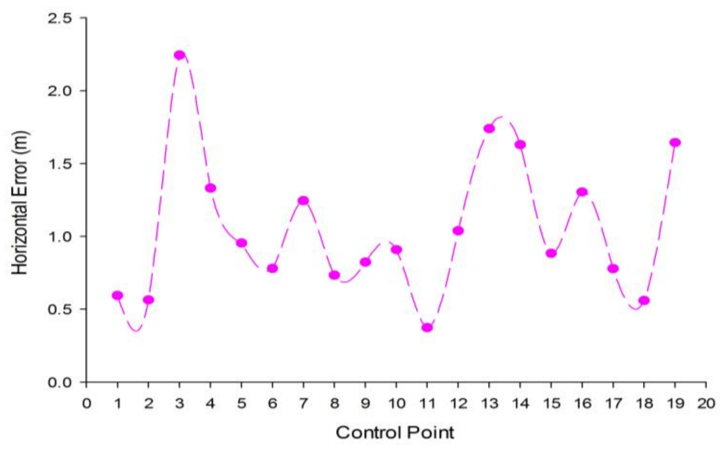
Point ID	ΔE	ΔN	HE
1	0.20	0.56	0.59
2	0.34	0.45	0.56
3	0.25	-2.23	2.24
4	0.67	1.15	1.33
5	0.21	-0.93	0.95
6	0.25	-0.74	0.78
7	0.92	-0.84	1.25
8	0.64	0.36	0.73
9	0.79	0.23	0.82
10	0.80	-0.43	0.91
11	-0.36	-0.10	0.37
12	-0.91	-0.50	1.04
13	-0.04	1.74	1.74
14	-1.08	1.22	1.63
15	0.45	0.76	0.88
16	-0.85	0.99	1.30
17	-0.62	-0.47	0.78
18	-0.56	0.00	0.56
19	-1.08	-1.24	1.64
SD	**0.68**	**0.98**	**0.49**

Figure 2. Residuals in Easting and Northing Coordinates



In order to ascertain the horizontal positional accuracy of the transformed coordinates this study applied Equation (18). The obtained estimated HE values (Table 2) show that a maximum horizontal uncertainty of 2.24 m was observed for Point 3. This high HE value for Point 3 was mostly contributed by the Northing coordinate which had an error of -2.23 m while an error of 0.25 m was gotten for Easting. Hence, because the HE is dependent on both values to get its estimates the value for Point 3 became higher. Nonetheless, this could be attributed to observational error or the point is located in a mountainous region. The FV model produced 0.374 m as the minimum HE. A graphical illustration of the HE (Table 2) is shown in Figure 3. The SD value of 0.49 m (Table 2) realised for HE indicate the transformation accuracy of the FV model utilised.

Figure 3. Horizontal errors of the transformed coordinates



The validity of the FV model was further assessed using the RMSE, MAE and SD respectively. In relation to the RMSE (Table 3), 0.66 m and 0.96 m were obtained for the Easting and Northing coordinates. These RMSE values quantify how close the FV model transformed coordinates differs from the observed data. That is, the FV model deviates from the most probable value (zero) by not more than 0.66 m and 0.96 m in Eastings and Northings respectively. Moreover, the MAE (Table 3) in Eastings and Northings were 0.58 m and 0.79 m respectively. This gives an indication on the magnitude of how close the FV model transformed coordinates is to the existing coordinates on average. The SD values (Table 3) for the coordinate differences in Easting and Northing show how wide the transformed coordinates are dispersed from the most probable value. Hence, signifying the precision of the data used.

Table 3. Model Performance Assessment

Statistical Indicators	Eastings (m)	Northings (m)
RMSE	0.66	0.96
MAE	0.76	0.73
SD	0.68	0.98

4. CONCLUSION

Coordinate transformation is an active research area especially in countries that still use astro-geodetic datum for their surveying and mapping purposes. It is well understood that astro-geodetic networks were established as a horizontal datum based on local astronomical coordinates and thus lacked ellipsoidal height. However, in Ghana most coordinate transformation has been done by including height estimated using the Abridged Molodensky model to the horizontal positions (latitude and longitude). Conversely, this estimated height for the non-geocentric datum is irrelevant in horizontal geodetic datum transformation.

Rather, the height introduces more distortions into the local geodetic network. For the purpose of avoiding the use of height in coordinate transformation process, this study applied Featherstone and Vaníček model for the first time in Ghana geodetic reference network. The results revealed a RMSE error of 0.66 m and 0.96 m in the Eastings and Northings with their corresponding MAE values at 0.76 m and 0.73 m respectively. A transformation accuracy of 0.49 m realised by the FV model showed that it could be utilized to transform coordinate in Ghana geodetic reference network.

In line with the results, it was noticed that although the height component was not used in the transformation process the residuals produced between the existing and some FV transformed coordinates were high. The conclusion drawn here was that Ghana geodetic network is highly inherent with distortions that could not be more absorbed by the FV model. Hence, we agree with Featherstone and Vaníček (1999) that distortion modelling should be carried out right after transformation is done. This will help improve the coordinate transformation results because most distortions will be modelled out. Hence, for future studies in Ghana, we recommend that the distortion modelling should be adopted as part of the transformation process.

5. ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their time and effort to improve the quality of this paper. Our sincere appreciation also goes to the Ghana Survey and Mapping Division of Lands Commission for providing us with the necessary data set.

6. REFERENCES

Ayer, J. (2008). Transformation models and Procedures for Framework Integration of the Ghana National Geodetic Network. *The Ghana Surveyor, 1*(2), 52–58.

Ayer, J., & Fosu, C. (2008). Map Coordinate Referencing and the use of GPS Datasets in Ghana. *Journal of Science and Technology (Ghana), 28*(1). [CrossRef]

Ayer, J., & Tiennah, T. (2008). Datum transformation by the iterative solution of the abridging inverse Molodensky formulae. *The Ghana Surveyor, 1*(2), 59–66.

Dzidefo, A. (2011). Determination of transformation parameters between the World Geodetic System 1984 and the Ghana geodetic network. Kwame Nkrumah University of Science & Technology. Retrieved from http://ir.knust.edu.gh/bitstream/123456789/298/1/THESIS.pdf

Featherstone, W., & Vanicek, P. (1999). The role of coordinate systems, coordinates and heights in horizontal datum transformations. *Australian Surveyor, 44*(2), 143–150. [CrossRef]

Heiskanen, W. A., & Moritz, H. (1967). Physical geodesy. *Bulletin Géodésique (1946-1975), 86*(1), 491–492.

Kotzev, V. (2013). Consultancy service for the selection of a new projection system for Ghana. Draft Final Reports, World Bank Second Land Administration Project (LAP-2), Ghana.

Kumi-boateng, B., & Ziggah, Y. Y. (2016). Accuracy Assessment of Cartesian (X , Y , Z) to Geodetic Coordinates (ϕ , λ , h) Transformation Procedures in Precise 3D Coordinate Transformation – A Case Study of Ghana Geodetic Reference Network. *Journal of Geosciences and Geomatics, 4*(1), 1–7. [CrossRef]

Paul, M. K. (1973). A note on computation of Geodetic coordinates from geocentric (Cartesian) coordinates. *Bulletin Géodésique (1946-1975), 108*(1), 135–139. [CrossRef]

Poku-Gyamfi, Y., & Schueler, T. (2008). Renewal of Ghana's geodetic reference network. In 13th FIG Symposium on Deformation Measurement and Analysis, 4th IAG Symposium on Geodesy for Geotechnical and Structural Engineering, LNEC, LISBON (pp. 1–9).

US Department of Defense. (1984). *World Geodetic System 1984 (WGS84).*

Vaníček, P., & Steeves, R. R. (1996). Transformation of coordinates between two horizontal geodetic datums. *Journal of Geodesy, 70*(11), 740–745. [CrossRef]

Ziggah, Y. Y., et al. (2013a). Determination of GPS coordinate transformation parameters of geodetic data between reference datums—a case study of ghana geodetic reference network. *International Journal of Engineering Sciences and Research Technology, 2*(4), 965–971.

Ziggah, Y. Y., et al. (2013b). Accuracy Assessment of Centroid Computation Methods in Precise GPS Coordinates Transformation Parameters Determination-A Case Study, Ghana. *European Scientific Journal, 9*(15), 200–220.