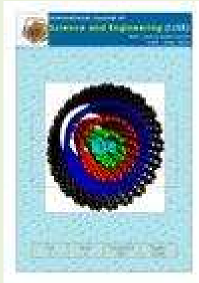
International Journal of Science and Engineering(IJSE)
Home page: http://ejournal.undip.ac.id/index.php/ijse



Reliability Based Design Optimization of Concrete Mix Proportions Using Generalized Ridge Regression Model

Rachna Aggarwal [1)], Maneek Kumar[2)], R.K.Sharma[3)], M.K.Sharma[3)]

[1)] Department of Mathematics, M.L.N. College, Yamunanagar, Haryana, India
[2)] Department of Civil Engineering, Thapar University, Patiala, India
[3)] School of Mathematics and Computer Applications, Thapar University, Patiala, India

Email: rachnaaggarwal1976@gmail.com

Abstract *- This paper presents Reliability Based Design Optimization (RBDO) model to deal with uncertainties involved in concrete mix design process. The optimization problem is formulated in such a way that probabilistic concrete mix input parameters showing random characteristics are determined by minimizing the cost of concrete subjected to concrete compressive strength constraint for a given target reliability. Linear and quadratic models based on Ordinary Least Square Regression (OLSR), Traditional Ridge Regression (TRR) and Generalized Ridge Regression (GRR) techniques have been explored to select the best model to explicitly represent compressive strength of concrete. The RBDO model is solved by Sequential Optimization and Reliability Assessment (SORA) method using fully quadratic GRR model. Optimization results for a wide range of target compressive strength and reliability levels of 0.90, 0.95 and 0.99 have been reported. Also, safety factor based Deterministic Design Optimization (DDO) designs for each case are obtained. It has been observed that deterministic optimal designs are cost effective but proposed RBDO model gives improved design performance.*

Keywords— Concrete; Compressive strength; Reliability; Optimization; Ridge regression

Submission: 10 October 2014 Corrected : 25 November 2014 Accepted: 10 December 2014

Doi: 10.12777/ijse.8.1.26-37

[***How to cite this article***: Aggarwal R., Kumar M., Sharma R.K. and Sharma M.K. *(2015).* Reliability Based Design Optimization of Concrete Mix Proportions Using Generalized Ridge Regression Model, *International Journal of Science and Engineering,* , 8(1),36-47, Doi: 10.12777/ijse.8.1.26-37

I. INTRODUCTION

Sustainable development while conserving the environment with an objective of welfare and safety of the people has been a subject of increasing concern during last few decades. At the same time, optimal allocation of available natural and financial resources is considered very important. Therefore methods of risk and reliability analysis developed during the last few decades are becoming more and more important as decision support tools in civil engineering applications (Sorenson, 2004).

Concrete is the most widely used man made construction material. Every year billion tons of cement is converted into concrete world-wide. Concrete is a mixture of cement, water, fine aggregate, coarse aggregate and admixtures. A good amount of work has been done by researchers to optimally allocate the ingredients proportions for concrete mixes while satisfying specific requirements related to compressive strength, slump, tensile strength etc. Yeh (1999, 2003, 2007, and 2009) determined optimal concrete mix compositions with lowest cost and required performance using nonlinear programming technique. Karihaloo and Kornbak (2001) optimized tensile strength and ductility, simultaneously, for a given compressive strength in the design of fiber reinforced concrete mixes. Lim *et al*. (2004) used genetic algorithm to find appropriate concrete mix proportions for high performance concrete under specified requirements. Optimal concrete mix proportions for maximum compressive strength of concrete using Taguchi method and genetic algorithm were determined by Őzbay *et al*. (2006). Jayaram *et al*. (2009) proposed elitist genetic algorithm models for the optimization of high volume fly ash concrete. Lee *et al*. (2009) used convex hull approach to define effective region constrained by the domain defined by limited data base and then, genetic algorithm was used to find optimal concrete mix parameters in the effective region. Baykasoğlu *et al*. (2009) solved a multi-objective optimization model for high strength concrete parameters using genetic algorithm with prediction models based on regression analysis and Gene Expression Programming (GEP).

The formulation of a structural optimization problem that ignores the scattering of various design parameters is termed as Deterministic Design Optimization (DDO). A numerically feasible optimum design, according to the deterministic formulation, once applied in a real physical system, may lose its feasibility due to the unavoidable dispersion on the values of structural parameters (material properties, dimensions, loads, etc.). Performance of the applied design may be far worse than expected. As such, in real world applications, if uncertainties are not taken into account, the significance of the optimum solutions would be limited (Lagaros *et al*., 2008). In DDO, the uncertainties are usually accounted for by the introduction of safety factors as described by the design codes of practice *IS 456* (2000). As a matter of fact, these safety factors are calibrated for average design situations and cannot ensure consistent reliability levels for specific design conditions. In this sense, the Reliability Based Design Optimization (RBDO) becomes a very powerful tool for robust and cost-effective designs. Contrary to traditional DDO, RBDO process modulates the safety margins within the optimization process taking into account uncertainty effect for each variable. In this sense, the safety factors are optimally defined within the system, compared to deterministic design where the safety factors are set before undergoing the optimization process. Thus, optimizing concrete mix proportions based on reliability is of great practical importance in comparison to deterministic optimization (Chateauneuf, 2008).

Despite the evident advantages of RBDO over deterministic design procedures, its application to engineering problems can be quite challenging due to high numerical cost involved in its solution (Valdebenito & Schuëller, 2010). Considering these issues, several tools are developed by researchers to solve RBDO problems efficiently problems (Thanedar & Kodiyalam, 1992; Enevoldsen & Sorensen, 1993; Wang & Grandhi, 1995; Luo & Grandhi, 1995; Chen *et al*., 1997; Royset *et al*., 2001; Aggarwal, 2004; Du & Chen, 2004; Zou & Mahadevan, 2006). SORA method developed by Du and Chen (2004) is an efficient decoupling approach to solve RBDO problem. It employs a single loop strategy with a serial of cycles of deterministic optimization and reliability assessment. In the present work, SORA method is used to achieve reliable optimal concrete mixture proportions.

In optimization process, simplified mathematical models are needed that could provide efficient representation of various concrete mix parameters. Cost of concrete is a linear function of its constituents but compressive strength of concrete might be a nonlinear function of its constituents as it is known only through its discrete outcomes. So, form and degree of the model for compressive strength is not known. The success of prediction model depends both on proper forms of the model and on the proper values of the parameters of the model. The parameters are usually estimated from the experimental data. The purpose of the parameter estimation in these cases is to not just to fit experimental data, but to find parameters as close to the true ones as possible. Scientists and engineers traditionally rely on different variants of the method of ordinary least square regression for estimating model parameters. This method leads to unbiased estimators. The unbiased property is meaningful only if the fitted model is the true model, and most often this may not be guaranteed and as such unbiased property should not be over emphasized (Ngo *et al*., 2004). Also, Hoerl and Kennard (1970) argued that in multiple linear regression, parameter estimates based on minimum residual sum of squares have a high probability of being far away from true parameter values, if prediction variables are not orthogonal. They proposed Ridge Regression (RR) technique that belongs to the class of biased estimators. This method leads to smaller values of Mean Square Error (MSE) function (which is the measure of goodness of estimators) for estimating parameters of linear models using non-orthogonal predictor variables.

In the present work, linear, pure quadratic (without interaction terms) and full quadratic models for compressive strength of concrete are developed using Ordinary Least Square Regression (OLSR), Traditional Ridge Regression (TRR) and Generalized Ridge Regression (GRR) techniques. The performance of developed models is compared on the basis of prediction accuracy. The full quadratic GRR model that has best prediction power is used in RBDO model. The RBDO model formulated based on the selected models for cost and compressive strength of concrete is then solved using SORA method. RBDO results are obtained for a wide range of target compressive strength with target reliability levels of 0.90, 0.95 and 0.99. Also, safety factor based DDO designs for each case are obtained to compare the performance of proposed RBDO model.

II. Experimental Dataset

The Compressive strength data explored in this study was generated in controlled laboratory conditions by Kumar (2002). The concrete mixes were proportioned using four basic ingredients, namely, water, cement, coarse aggregate and fine aggregate. The proportions of materials for concrete mixes were determined by DoE method of mix design (Gambhir, 1995). Ordinary Portland cement of 43 grade having specific gravity of 3.14 was used. The 7 and 28 days compressive strength of cement was 35.6 MPa and 45.5 MPa, respectively. The fine aggregate was river bed sand with a fineness modulus of 2.09 and specific gravity of 2.54. Three types of coarse aggregate viz., CA-I, CA-II and CA-III, were used in different proportions in order to increase the density of resulting mix. Table 1 contains the salient properties of these aggregates. The coarse aggregates were divided into three zones, namely, A, B and C, based on the percentage of different types of aggregates used. Table 2 summarizes details of these zones. Also, the water content variation for each zone of aggregate is shown in Table 2. A set of 49 concrete mixes was prepared by varying water-cement ratio, cement contents and aggregates fractions (Kumar, 2002). Water-cement content ratio was kept between 0.42 and 0.55. Out of these 49 mixes, 18, 17 and 14 mixes were prepared using zone A, zone B and zone C of coarse aggregates, respectively. For each mix, 15 cubes of 150 *mm* size were cast and were tested at 28 days of curing period. Thus, a sufficiently large data bank was generated and the same has been used in the present work for analyzing compressive strength of concrete. Also, unit cost of each material is determined by taking into account the price rates in India. Based on the prices, cost of 1 m^3 of concrete is calculated for each mixture and is measured in Indian rupee (*Rs*.).

Table 1.Properties of coarse aggregates

Type of aggregate	Unit mass (compact) (kg/m^3)	Specific gravity	Percentage absorption (%)
CA-I	1.58	2.68	1.80
CA-II	1.48	2.68	1.18
CA-III	2.15	2.60	1.20

Table 2. Zones of aggregates

Zone	Percentage passing 20 mm sieve and retained on 10 mm sieve (CA-I)	Percentage passing 10 mm sieve and retained on 4.75 mm sieve (CA-II)	Percentage passing 4.75 mm sieve and retained on 2.36 mm sieve (CA-III)	Water content requirement (kg/m^3)
A	67	33	-	180 – 210
B	50	50	-	190 – 220
C	-	50	50	200 – 230

III GENERALIZED RIDGE REGRESSION

In matrix notation, the multiple linear regression model can be expressed as:

$$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon} \quad (1)$$

where $\boldsymbol{Y}$ is a $n \times 1$ vector for response variable, $\boldsymbol{X}$ is a $n \times (p+1)$ matrix. First column of $\boldsymbol{X}$ consists of ones and remaining p columns are for explanatory variables or predictors, $\boldsymbol{\beta}$ is a $(p+1) \times 1$ vector for unknown regression coefficients and $\boldsymbol{\varepsilon}$ is a $n \times 1$ vector of experimental errors with mean 0 and variance σ^2. OLSR estimators of regression coefficients are given as:

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{X'X})^{-1}\boldsymbol{X'Y} \quad (2)$$

A unique solution of Eq. (2) may exist even when the predictor variables are non orthogonal, i.e., the matrix $\boldsymbol{X'X}$ is ill conditioned. However, in such nearly singular cases, the solution is very unstable. Also, OLSR estimates have large variances in such cases (Rawling *et al.*, 1998). This greatly affects the prediction accuracy of OLSR model. Prediction accuracy is an important aspect of model development when the model is to be used for further analysis. Ridge regression technique proposed by Hoerl and Kennard (1970) is a biased regression technique that shrinks regression coefficients and hence reduces the variance of the regression coefficients. This technique produces stable regression coefficients and improves the prediction accuracy of the model.

Ridge regression estimates are given as:

$$\widehat{\boldsymbol{\beta}}_{\boldsymbol{RR}} = (\boldsymbol{X'X} + k\boldsymbol{I})^{-1}\,\boldsymbol{X'Y} \quad (3)$$

where $k \geq 0$ is called the ridge parameter and $\boldsymbol{I}$ is the identity matrix of order $(p+1) \times (p+1)$. Regression estimates given by Eq. (3) are termed as TRR estimates. Hoerl and Kennard (1970) also proposed an improvement on TRR in the form of GRR. GRR estimates of the regression coefficients are given as:

$$\widehat{\boldsymbol{\beta}}_{\boldsymbol{GRR}} = (\boldsymbol{X'X} + \boldsymbol{K})^{-1}\,\boldsymbol{X'Y} \quad (4)$$

where $\boldsymbol{K}$ is a $(p+1) \times (p+1)$ diagonal matrix. Diagonal entries of $\boldsymbol{K}$ are called ridge parameters. GRR gives the user some flexibility regarding the shrinkage of each regression coefficient, as it may be desirable to treat the coefficients differently (Ryan, 1996).

If all the ridge parameters are taken to be equal to k, then ridge estimates obtained using Eq. (4) are same as TRR estimates. For $\boldsymbol{K} = \boldsymbol{0}$, the ridge regression coefficients are identical to OLSR coefficients.

It is worth mentioning here that ridge regression is a linear regression technique and exact form of relationship between compressive strength of concrete and its ingredients is still not known. In the present work, first and second order approximation models for compressive strength of concrete have been developed. Cross validation criterion employed by Yan (2008) has been used to find optimal ridge parameters for TRR and GRR models that minimizes the mean square prediction error (MSE) of validation set given in Eq. (5).

$$MSE = (\boldsymbol{y_t} - \widetilde{\boldsymbol{y}}_{\boldsymbol{t}})'(\boldsymbol{y_t} - \widetilde{\boldsymbol{y}}_{\boldsymbol{t}})/n_t \quad (5)$$

where $\boldsymbol{y_t}$ denotes vector of dependent variable for validation set, $\widetilde{\boldsymbol{y}}_{\boldsymbol{t}}$ denotes predicted values of $\boldsymbol{y_t}$ and n_t denotes number of observations in validation set.

IV PREDICTION MODELS FOR COMPRESSIVE STRENGTH OF CONCRETE

Factors influencing compressive strength of concrete for determining predictor variables

Concrete mixes used in this study are composed of water (w), cement (c), fine aggregate (fa) and coarse aggregate (ca), all measured in kg/m^3 and 28 days compressive strength ($st28$) is measured in MPa. Basic descriptive of these parameters is given in Table 3. Two statistical analyses have been conducted to decide the predictor variables for compressive strength of concrete using correlation approach. In first analysis, $st28$ has been considered with absolute content values of w, fa, ca, c and in second analysis, this strength is considered with ratio of water and cement contents (w/c), ratio of fine aggregate and cement contents (fa/c), ratio of coarse aggregate and cement contents (ca/c) and cement content (c). The results of these analyses are given in Tables 4 (a) and (b). These results suggest that potential predictors for $st28$ are w/c, fa/c, ca/c and c as the numerical values of coefficient of correlations of these variables with $st28$ is more than 0.500.

Linear, pure quadratic and full quadratic models for estimating compressive strength of concrete are developed in the present work. Interaction terms $w/c * c$, $fa/c * c$ and $ca/c * c$ are not considered in the development of full models since these terms will represent absolute values of water, fine aggregate and coarse aggregate contents, respectively.

Table 3.Descriptive statistics

Variable	Minimum (kg/m^3)	Maximum (kg/m^3)	Mean (kg/m^3)	Standard deviation (kg/m^3)	Specific gravity
w	180.00	230.00	202.44	12.69	1.00
fa	416.93	642.18	535.64	57.29	2.54
ca	798.48	1252.05	1064.85	133.42	2.65
c	350.00	475.00	424.49	37.32	3.12
$st28$	31.66	54.49	45.80	5.42	-

Table 4(a). Correlation matrix for analysis 1

Parameter	w (kg/m^3)	fa (kg/m^3)	ca (kg/m^3)	c (kg/m^3)	$st28$ (MPa)
w (kg/m^3)	1.000	0.805	-0.305	0.541	0.000
fa(kg/m^3)		1.000	0.102	0.026	-0.462
ca(kg/m^3)			1.000	-0.375	-0.214
c(kg/m^3)				1.000	0.821

Table 4(b). Correlation matrix for analysis 2

Parameter	w/c	fa/c	ca/c	c (kg/m^3)	$st28$ (MPa)
w/c	1.000	0.960	0.517	-0.734	-0.968
fa/c		1.000	0.546	-0.637	-0.900
ca/c			1.000	-0.776	-0.581
c(kg/m^3)				1.000	0.821

Sample data analysis

To analyze multicollinearity among the sample data, Ryan (1996) suggested to examine the correlations between the pairs of predictor variables and the Variance Inflation Factor (VIF) of predictor variables. A pair wise correlation matrix of predictor variables might be insufficient to identify collinearity problem because linear dependencies may exist among combinations of predictors. Hence, it is necessary to examine VIFs also. Following Ryan (1996), the VIF_i of i^{th} predictor variable x_i (say) has been considered as:

$$VIF_i = \frac{1}{1-R_i^2} \qquad (6)$$

where R_i^2 is the squared multiple correlation coefficient that results from regression of x_i against all other predictors. It is clear that if x_i has a strong linear relationship with other predictor variables, R_i^2 is close to 1 and VIF value tends to be very high.

In the absence of linear relationship among predictor variables, R_i^2 is zero and VIF equals 1. As a rule of thumb, multicollinearity is said to exist if VIF value for a predictor variable is more than 10.

The pair wise correlation between the selected predictor variables listed in Table 4(b) shows that all the pair wise correlations are numerically greater than 0.500. Three pairs of variables have very high degree of correlation. The pair w/c and fa/c has highest correlation (0.960). The other two pairs with high correlation are that of ca/c and c (-0.776) and that of w/c and c (-0.734). These results indicate that the given data set suffers from multicollinearity. Also, it can be noted from the Table 5 that VIF values for w/c and fa/c exceed 10 and thus provide an evidence for presence of multicollinearity. Further, in quadratic models, strong multicollinearity is present because of form of the models.

Table 5. VIF values

Parameter	VIF value
w/c	33.444
fa/c	25.863
ca/c	4.532
c	7.877

Development of prediction models for compressive strength of concrete

The total sample set consists of 49 concrete mix composition observations. Total sample set is randomly divided into training set of 33 observations and validation set of 16 observations. In order to illustrate the performance of developed models, MSE_p for validation set is defined as:

$$MSE_p = MSE(\boldsymbol{K_{opt}}) \qquad (7)$$

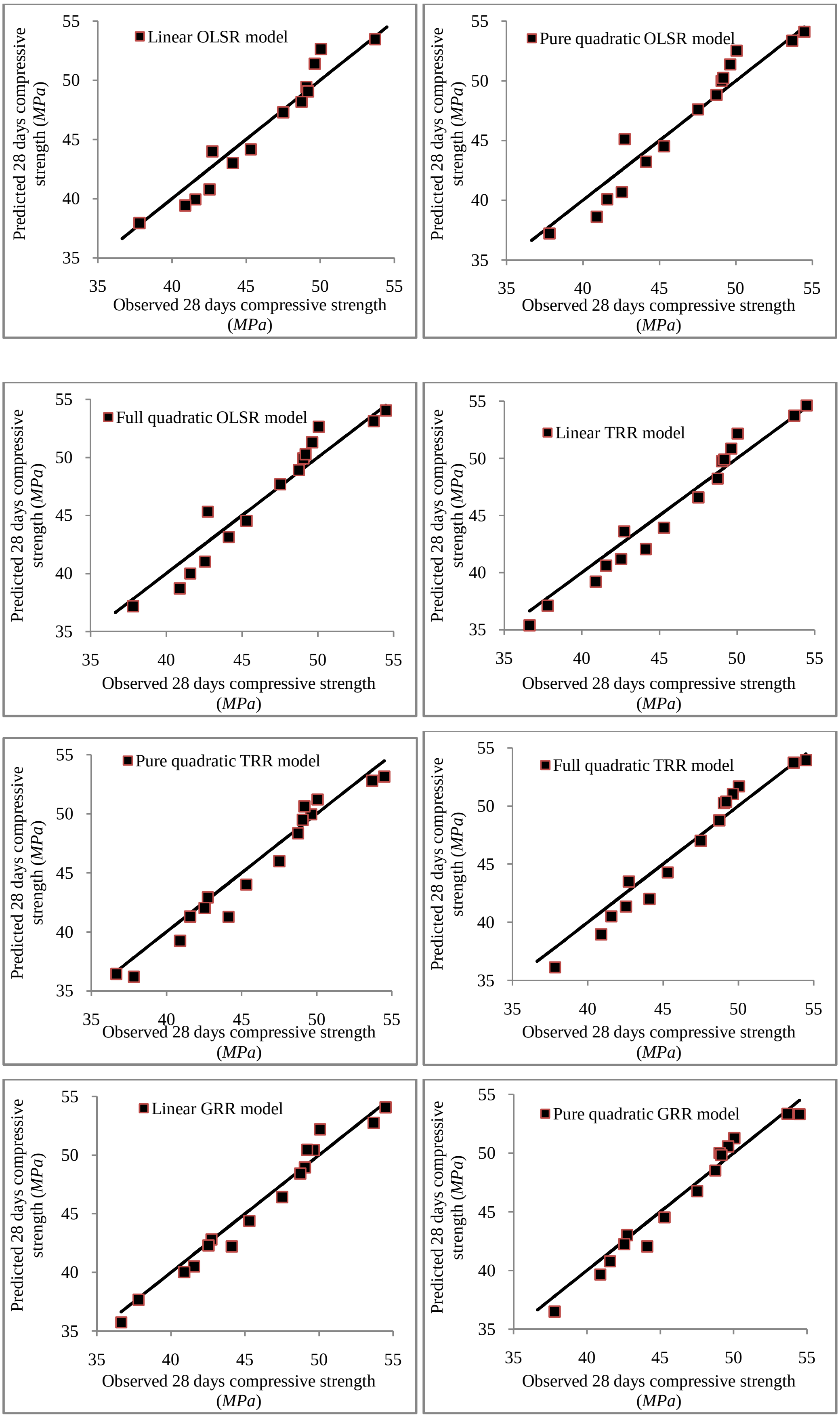
where $MSE(\boldsymbol{K_{opt}})$ is the value of mean square error evaluated from Eq. (5) at optimal value of $\boldsymbol{K}$ which is obtained using Differential Evolution (DE) algorithm. However, for OLSR models, $\boldsymbol{K} = \boldsymbol{0}$. In order to obtain the optimal ridge parameters for TRR and GRR models, DE algorithm was employed using parameters $N_p = 50$, $Cr = 0.9$, $F = 0.85$ and $g_{max} = 500$. Here, N_p symbolizes the number of individuals in a population, $Cr \epsilon [0,1]$ is the crossover probability, $F \epsilon [0,2]$ denotes the mutation probability, maximum number of generations are denoted by g_{max}. DEMAT, a MATLAB program developed by Price *et al*. (2005) is used to carry out DE algorithm.

For linear, pure quadratic and full quadratic TRR models, the optimal ridge parameters obtained by DE are 0.000690, 1 and 0.021556, respectively. For linear GRR model, five diagonal elements of optimal diagonal matrix $\boldsymbol{K}$ are 0, 0.008037, 1, 0.737931 and 1, respectively. For pure quadratic GRR model, nine diagonal elements of optimal diagonal matrix $\boldsymbol{K}$ are 0.000374, 0.007582, 1, 1, 1, 1, 1, 0 and 0.921217, respectively. For full quadratic GRR model, twelve diagonal elements of optimal diagonal matrix $\boldsymbol{K}$ are 0.999999, 1, 0, 1, 1, 1, 1, 1, 0.997760, 1, 0.002325 and 0, respectively. The regression coefficients of the developed models are summarized in Table 6.

To demonstrate the performances of nine developed compressive strength models, the predicted compressive strength values for the validation set are plotted against the observed values for the validation set. The graphs obtained are shown in Fig. 1. The MSE_p values of each of the nine developed models are listed in Table 7.

Table 6. Regression coefficients for developed models

	Regression Coefficients								
	Linear OLSR model	Pure Quadratic OLSR model	Full Quadratic OLSR model	Linear TRR model	Pure Quadratic TRR model	Full Quadratic TRR model	Linear GRR model	Pure Quadratic GRR model	Full Quadratic GRR model
w/c	-189.13975	476.20913	561.35812	-103.55820	-0.76121	-5.36516	-80.14997	-52.69489	-0.110144
fa/c	12.05470	-22.21296	-113.09515	-3.23475	-2.57674	-3.42644	-2.80901	-1.39377	-35.27014
ca/c	-1.21449	13.08796	52.26107	0.67563	1.15347	11.46859	0.84905	0.59404	0.83211
c	0.01453	-0.12024	-0.09109	0.04807	0.17823	0.19839	0.06636	0.40038	0.38787
$(w/c)^2$	-	-650.43865	-677.48681	-	-0.70444	-5.39211	-	-0.38515	-0.11413
$(fa/c)^2$	-	10.18659	-66.87370	-	-6.23284	-9.84671	-	-3.53504	-0.39153
$(ca/c)^2$	-	-2.78011	-2.62999	-	0.18649	-3.06588	-	0.20384	0.34431
c^2	-	0.00016	0.00014	-	-0.00011	-0.00017	-	-0.00039	-0.00041
$w/c * fa/c$	-	-	356.09713	-	-	-9.12190	-	-	-0.20540
$w/c * ca/c$	-	-	-194.69196	-	-	-8.81293	-	-	-35.14388
$fa/c * ca/c$	-	-	43.56722	-	-	7.34365	-	-	11.91443
Intercept	117.95049	-14.03388	-37.12946	77.31834	-0.12678	-0.27211	57.36012	-22.81334	-0.034569


Linear OLSR model
Pure quadratic OLSR model
Full quadratic OLSR model
Linear TRR model
Pure quadratic TRR model
Full quadratic TRR model
Linear GRR model
Pure quadratic GRR model
Predicted 28 days compressive strength (MPa)
Observed 28 days compressive strength (MPa)
35
40
45
50
55

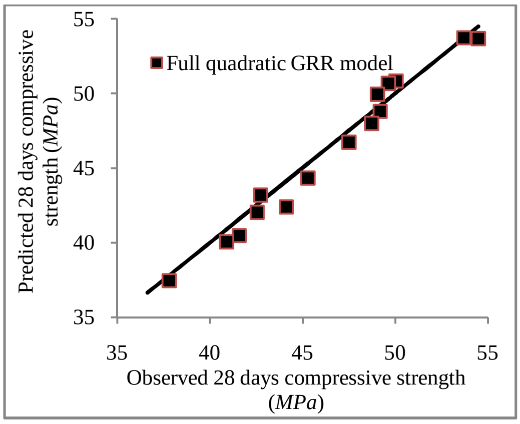


Figure 1. Performances of nine compressive strength models

Table 7. Mean square error for test sample data set

Name of the model	MSE_p
Linear OLSR model	1.78793
Pure Quadratic OLSR model	3.05464
Full Quadratic OLSR model	3.02470
Linear TRR model	1.42587
Pure Quadratic TRR model	1.52168
Full Quadratic TRR model	1.80699
Linear GRR model	1.02357
Pure Quadratic GRR model	1.12640
Full Quadratic GRR model	0.89794

It can be seen from Fig. 1 that predicted values of GRR models distribute closer along the diagonal in comparison to OLSR and TRR models indicating that prediction power of these models is better than that of OLSR and TRR models. The above observation is also supported by the MSE_p values for each of the nine developed models given in Table 7. It can be noted from the Table 7 that values of MSE_p of GRR models are lesser than that for OLSR and TRR models and least value of MSE_p is for full quadratic GRR model.

So, full quadratic GRR model for 28 days compressive strength of concrete is selected to be used in RBDO model for concrete mix parameters.

Prediction models for cost of concrete

In this study, no cost is associated with the water content and as such, cost of concrete is a linear function of fine aggregate content, coarse aggregate content and cement content. Firstly, it can be noted from Table 4 that pair wise correlations between predictor variables lie between 0.026 and 0.375 numerically. Also, VIF values for predictor variables are 1.016, 1.181 and 1.169. All the three VIF values are very near to one. The above observations suggest that multicollinearity does not affect the linear OLSR model for cost of concrete. So, linear OLSR model is developed for cost of concrete. Regression coefficients for linear OLSR model for cost of concrete are given in Table 8.

Table 8. OLSR models for cost of concrete

Parameter	$cost$ $(Rs.)$
fa $((kg/m^3)$	0.629
ca (kg/m^3)	0.333
c (kg/m^3)	4.892
$Intercept$	236.461

V. OPTIMIZATION MODEL FORMULATION

Formulation of RBDO models for concrete mix parameters

The RBDO model for concrete mix parameters is formulated with cost of concrete as objective function. This objective is minimized satisfying a ratio constraint, an absolute volume constraint, boundary constraints on input variables and a reliability constraint on 28 days compressive strength requirement. All the four input variables, namely, w, fa, ca and c have been considered as random variables that follow normal distribution with their respective means and standard deviations listed in Table 3. So, the design variables for the proposed multi-objective RBDO problem have been taken as the mean values of water content, fine aggregate content, coarse aggregate content and cement content denoted as $\mu_w, \mu_{fa}, \mu_{ca}$ and μ_c, respectively.

The RBDO problem formulated for concrete mix cost optimization is given below:

$$\left.\begin{array}{ll} & \underset{\mu_w,\mu_{fa},\mu_{ca},\mu_c}{Minimize} \quad cost\ (fa, ca, c) \\ Subject\ to: & \text{Prob}(st28(w, fa, ca, c) \geq f_c) \geq R \\ & 0.42 \leq {}^{w}/_{c} \leq 0.55 \\ & \left(w + \frac{fa}{\text{G}_{fa}} + \frac{ca}{\text{G}_{ca}} + \frac{c}{\text{G}_c}\right) * 0.001 + 0.02 = 1.00 \\ & w_l \leq w \leq w_u \\ & fa_l \leq fa \leq fa_u \\ & ca_l \leq ca \leq ca_u \\ & c_l \leq c \leq c_u \end{array}\right\} \quad (8)$$

where,

$cost$ - Cost of concrete

$st28$ - 28 days compressive strength of concrete

f_c - Target 28 days compressive strength of concrete

Prob(.) - Probability of constraint satisfaction

R - Target reliability level

w/c - Water-cement content ratio

G_{fa}, G_{ca} and G_c – specific gravities of fine aggregate, coarse aggregate and cement content, respectively

w_l, fa_l, ca_l, and c_l - Lower bounds for water, fine aggregate, coarse aggregate and cement content, respectively

w_u , fa_u , ca_u , and c_u - Upper bounds for water, fine aggregate, coarse aggregate and cement content, respectively

First constraint in (8) is the reliability constraint on compressive strength of concrete which ensures that $st28$ is more than a specified value of compressive strength f_c with reliability R . The second constraint is a deterministic constraint which shows that w/c ratio lies between 0.42 and 0.55. Third constraint is a condition that ensures that total volume of components of concrete should be equal to 1 m^3. In this constraint, 0.02 signifies the percentage of air content in concrete mix. Last four constraints are boundary constraints for design variables. The specific gravities, lower bounds and upper bounds for the design variables are given in Table 3.

Using SORA method, RBDO problem formulated in Eq. (8) is replaced by deterministic optimization problem as given below:

$$\left.\begin{array}{c} \underset{\mu_w,\mu_{fa},\mu_{ca},\mu_c}{Minimize}\ cost\,(fa,ca,c) \\ Subject\ to\!:\ st28\big(\mu_w - s_w^{(k)}, \mu_{fa} - s_{fa}^{(k)}, \mu_{ca} - s_{ca}^{(k)}, \mu_c - s_c^{(k)}\big) \ge f_c \\ \left(w + \dfrac{fa}{G_{fa}} + \dfrac{ca}{G_{ca}} + \dfrac{c}{G_c}\right) * 0.001 + 0.02 = 1.00 \\ 0.42 \le {}^{w}/_{c} \le 0.55 \\ w_l \le w \le w_u \\ fa_l \le fa \le fa_u \\ ca_l \le ca \le ca_u \\ c_l \le c \le c_u \end{array}\right\} \quad (9)$$

where k denotes the cycle number and s_w, s_{fa}, s_{ca}, s_c are called shift factors defined as:

$$\begin{bmatrix} s_w^{(k)} \\ s_{fa}^{(k)} \\ s_{ca}^{(k)} \\ s_c^{(k)} \end{bmatrix} = \begin{cases} \mathbf{0}, & k = 1 \\ \begin{bmatrix} \mu_w^{(k-1)} - w_{\mathrm{MPP}}^{(k-1)} \\ \mu_{fa}^{(k-1)} - fa_{\mathrm{MPP}}^{(k-1)} \\ \mu_{ca}^{(k-1)} - ca_{\mathrm{MPP}}^{(k-1)} \\ \mu_c^{(k-1)} - c_{\mathrm{MPP}}^{(k-1)} \end{bmatrix}, & k \ge 2 \end{cases} \quad (10)$$

Here, $(w_{\mathrm{MPP}}, fa_{\mathrm{MPP}}, ca_{\mathrm{MPP}}, c_{\mathrm{MPP}})$ denotes the inverse Most Probable Point (MPP) of failure of compressive strength constraint corresponding to the given reliability level R at deterministic optimal solution $(\mu_w, \mu_{fa}, \mu_{ca}, \mu_c)$. The deterministic optimization problem in each cycle is updated on the basis of MPP information obtained from the previous cycle. The above procedure is repeated cycle by cycle until the $cost$ function converges and the reliability requirement for compressive strength constraint is achieved.

Formulation of Safety factor based DDO model

In deterministic design procedures, safety margin is set before the optimization process. As per *IS 10262* (2009), for a specified target compressive strength of f_c MPa, the concrete mix should be proportioned for an average compressive strength not less than $(f_c + 1.65s)$ MPa, so that, no more than 5% of the results will fall below f_c MPa. Here, s denotes the assumed standard deviation of the compressive strength data. So, in DDO problem based on safety factor approach, probabilistic constraint given in Eq. (8) is replaced by the deterministic constraint given below:

$$st28 \ge f_c + 1.65s \quad (11)$$

Also, the design variables w , fa , ca and c are taken as deterministic design variables.

VI. RESULTS AND DISCUSSIONS

RBDO models based on full quadratic GRR model are solved by SORA method. The influence of reliability level on optimization results is also investigated in the present study. The RBDO problem formulated in preceding section is solved for three target reliability levels of 0.90, 0.95 and 0.99. The optimal mix proportions obtained are presented in Table 9. For finding optimal proportions of concrete mix, the minimum target compressive strength is taken as 27 MPa and is increased in steps of 3 MPa. The results are reported up to the maximum target compressive strength for which SORA optimizer converged for a given reliability.

Safety factor based DDO model is solved using sequential quadratic programming method. As per IS 10262 (2009), assumed standard deviations for different grades of concrete are listed in Table 10. It can be noted from Table 10 that the value of s is 4.0 MPa for target compressive strength of 27 MPa and this is 5 MPa for target compressive strength $\ge$ 30 MPa. As such, the safety margin is taken 6.60 MPa for target compressive strength of 27 MPa and 8.25 MPa in all the remaining cases.

The results of safety factor based deterministic design optimization are summarized in Table 11. Reliability analysis using mean value approximation method is carried out at DDO optimal designs and computed reliabilities are also reported in Table 11. Fig. 2 shows the variation of optimal cost with target compressive strength for different reliability levels and for safety factor based DDO approach.

Effect of reliability level on optimal cost

Fig. 2 shows variation of optimal cost with target compressive strength and reliability level. It can be seen from this figure that difference between heights of the curves for reliability levels of 0.95 and 0.99 is higher than that of the curves for reliability levels of 0.90 and 0.95. It indicates that raising the reliability level from 0.95 to 0.99 is costlier than raising the reliability level from 0.90 to 0.95, for a given target compressive strength. The rise in optimal cost lies between 2.3% to 4.0% when reliability level is increased from 0.90 to 0.95. However, to raise the reliability level from 0.95 to 0.99, the rise in optimal cost lies between 4.28% to 5.52%.

Effect of reliability level on design variables

Figures 3 and 4 show variation of design variables and water-cement content ratio with target compressive strength and reliability level. It can be noted from Fig. 3(a) that water content w shows some variations for higher compressive strengths for a given reliability. Figures 3(b) and 3(c) reveal that fa and ca contents exhibit wide fluctuations as target strength and reliability level varies. Also, it can be noted from

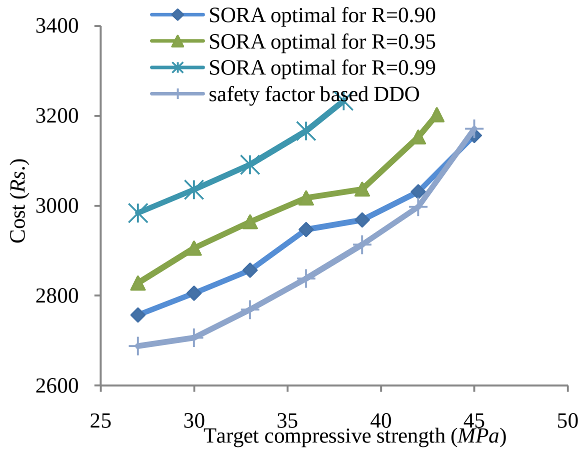


Figure 2. Variation of optimal cost with target compressive strength

Table 9. Reliability based design optimization results

Target Reliability R	Target $st28$ (MPa)	$cost$ $(Rs.)$	Predicted $st28$ (MPa)	w (kg/m^3)	fa (kg/m^3)	ca (kg/m^3)	c (kg/m^3)	w/c
0.90	27	2756.46	40.20	180.00	577.31	1208.50	358.63	0.502
	30	2805.50	42.60	180.00	548.87	1227.58	371.02	0.485
	33	2856.27	45.02	180.00	514.80	1251.91	384.12	0.469
	36	2947.31	45.98	180.00	617.76	1133.53	397.55	0.453
	39	2968.33	49.26	180.00	493.83	1252.05	409.71	0.439
	42	3030.93	51.31	180.00	484.71	1249.56	423.85	0.425
	45	3156.65	53.06	191.85	429.28	1248.03	456.78	0.420
0.95	27	2827.77	43.39	180.00	553.92	1218.51	375.54	0.479
	30	2905.33	44.13	180.00	637.13	1122.11	387.25	0.465
	33	2964.37	46.08	180.00	631.09	1117.24	400.43	0.450
	36	3017.46	48.18	180.00	610.09	1128.23	413.24	0.436
	39	3037.03	51.52	180.00	482.00	1251.11	425.34	0.423
	42	3153.04	53.04	191.23	432.65	1247.10	455.67	0.420
	43	3202.52	52.99	195.44	463.03	1196.21	465.35	0.420
0.99	27	2983.80	46.65	180.00	630.20	1114.54	404.70	0.444
	30	3035.78	48.33	180.00	620.17	1114.38	416.56	0.432
	33	3091.31	49.97	180.37	608.17	1115.55	429.44	0.420
	36	3166.28	52.92	192.18	451.49	1223.40	457.57	0.420
	38	3233.30	53.49	199.34	432.73	1209.56	474.63	0.420

Table 10. Assumed standard deviation

Grade of concrete	Assumed standard deviation (MPa)
M10 M15	3.5
M20 M25	4.0
M30 M35 M40 M45 M50 M55	5.0

Table 11. Safety factor based approach optimization results

Target $st28$ (MPa)	Safety margin (MPa)	$cost$ $(Rs.)$	Predicted $st28$ (MPa)	w (kg/m^3)	fa (kg/m^3)	ca (kg/m^3)	c (kg/m^3)	w/c	Reliability
27	6.60	2687.79	33.60	191.93	512.30	1251.90	350.00	0.548	0.730940
30	8.25	2706.22	38.25	180.44	541.66	1251.68	350.00	0.516	0.801893
33	8.25	2768.64	41.25	180.07	531.32	1251.47	364.11	0.495	0.823800
36	8.25	2838.22	44.25	180.08	518.49	1251.27	380.00	0.474	0.846715
39	8.25	2913.36	47.25	180.01	504.45	1251.63	397.14	0.453	0.872969
42	8.25	2997.75	50.25	180.06	488.45	1251.81	416.44	0.432	0.903361
45	8.25	3171.25	53.25	193.63	417.40	1252.05	461.02	0.420	0.951700

graphs that fa and ca contents show opposite variations, i.e., as fa content rises, ca content falls and vice-versa, for a given reliability curve. Fig. 3(d) depicts that cement content c increases according to target strength and reliability level. Fig. 4 shows that water-cement content ratio falls as target strength increases for a given reliability. The above two observations are as per already proven trends for concrete mix design. It further strengthens the idea that the formulated RBDO model behaves in accordance with the existing standards and hence is of great practical importance.

Comparison of RBDO approach with safety factor based DDO approach

It can be noted from Fig. 2 that optimal costs for safety factor based DDO designs are less than that for RBDO designs in each corresponding case and for each reliability level except for f_c = 45 MPa. Safety factor based DDO curve lies above

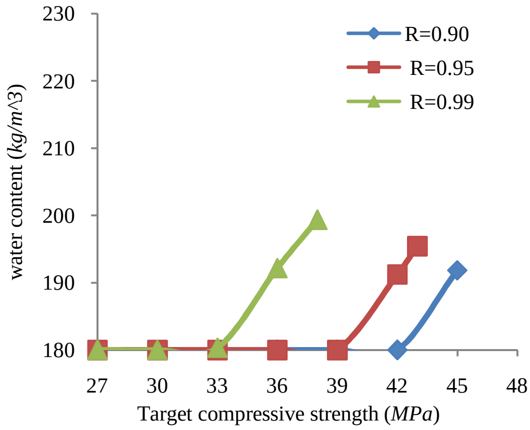


Fig. 3(a)

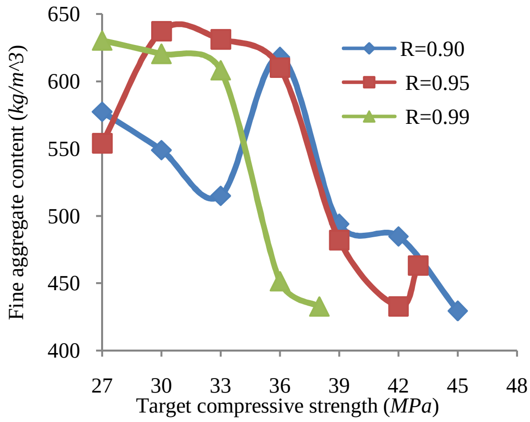


Fig. 3(b)

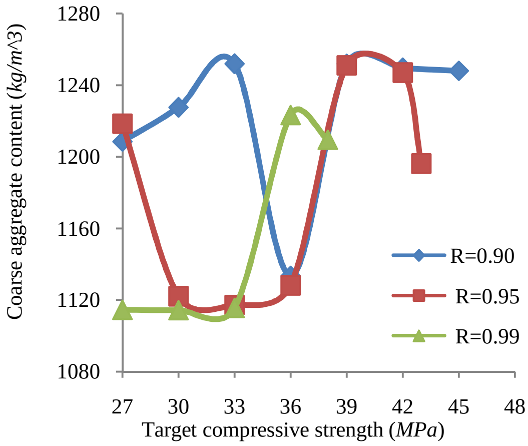


Fig. 3(c)

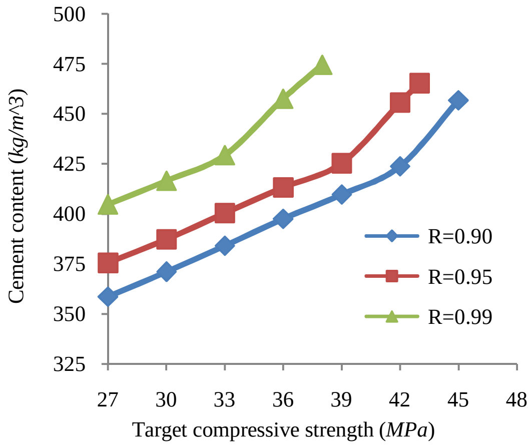


Fig. 3(d)

Figure 3(a). Variation of water content with target compressive strength; Figure 3(b). Variation of Fine aggregate content with target compressive strength; Figure 3(c). Variation of Coarse aggregate content with target compressive strength; Figure 3(d). Variation of cement content with target compressive strength the RBDO curve with R = 0.90 for this value of target strength. This is justified as Table 11 reveals that reliability of safety factor based optimal design for this strength is 0.9517, that is greater than 0.90.

In particular, for reliability of 0.95 for which safety factor based DDO designs are obtained, comparison shows that optimal costs achieved for RBDO designs are 5.18% to 7.36% more than optimal costs for safety factor based designs.

Also, Table 11 reveals that in safety factor based DDO results, optimal designs do not meet the reliability requirement of 0.95 in all cases except for f_c = 45 MPa, although reliability improves as target compressive strength increases.

Thus using DDO approach, required reliability levels cannot be maintained. The disadvantage of safety factor based DDO approach is that recommended safety margins are not always suitable for the given system. But, as already mentioned, RBDO process modulates the safety margins within the optimization process taking into account uncertainty effect for each variable. Hence, the resulting designs obtained by RBDO approach are the best solution relative to the designs obtained by safety factor based DDO approach as the objective is to provide best compromise between cost and safety.

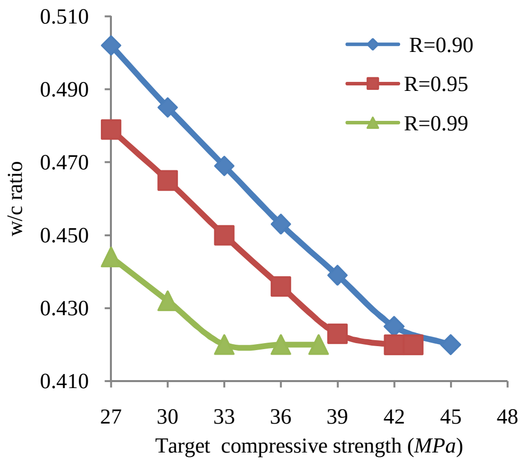


Figure 4. Variation of water-cement content ratio with target compressive strength

VII. CONCLUSIONS

In this paper, optimal concrete mix proportions are determined that are less sensitive to uncertainties involved in concrete mix design process. The first part of the study focuses on development of prediction models for cost and compressive strength of concrete. OLSR, TRR and GRR techniques are used to develop compressive strength models. DE is used to find optimal ridge parameters. It has been seen that full quadratic GRR model performs best with respect to prediction accuracy of the model. Linear OLSR model is developed for cost of concrete. RBDO model to minimize the cost of concrete with a reliability constraint on compressive strength of concrete is formulated and solved in second part of the study. RBDO model is solved using SORA method. Also, optimal concrete mix proportions are found using safety factor based DDO approach. Following conclusions have been drawn from this study:

i. Safety factor based DDO designs are cost effective but, these lead to low reliability levels. However, RBDO results respect the required reliability level.
ii. Cement content is the most significant parameter in reliability based concrete mix design process.
iii. The proposed RBDO model is applicable for concrete mix proportioning and can be used for finding the proportions of constituents for desired compressive strength and reliability.

REFERENCES

Aggarwal, H. 2004. Reliability based design optimization: formulations and methodologies. PhD thesis, University of Notre Dame, Indiana.

Baykasoğlu, A., Öztas, A., and Özbay, E. 2009. Prediction and multi-objective optimization of high strength concrete parameters via soft computing approaches. Expert System with Applications, 36: 6144 – 6155; DOI 10.1016/j.eswa.2008.07.017

Chateauneuf, A. 2008. Principles of reliability-based design optimization. Structural design optimization considering uncertainties (eds Y. Tsompanakis, N. D. Lagaros & M. Papadrakakis), pp. 3-30. Taylor & Francis Group, London, UK.

Chen, X., Hasselman, T. K., and D. Neill, 1997. Reliability based structural design optimization for practical applications. Proceedings of 38th AIAA Conference on Structures, Structural Dynamics and Materials, April 7-10, Kissimmee, Florida, pp: 2724-2732.

Du, X., and Chen, W. 2004. Sequential optimization and reliability assessment method for efficient probabilistic design. ASME Journal of Mechanical Design, 126(2): 225-233; DOI 10.1115/1.1649968

Enevoldsen, I., and Sorensen, J. D. 1993. Reliability-based optimization of series systems of parallel systems. ASCE Journal of Structural Engineering, 119(4): 1069-1084; DOI 10.1061/(ASCE)0733-9445(1993)119:4(1069)

Gambhir, M. L., 1995. Concrete Technology. 4th Ed. Tata McGraw Hill Publishing Company Limited, India. ISBN: 0070151369, pp: 716.

Hoerl, A. E., and Kennard, R. W. 1970. Ridge regression: Biased estimation for nonorthogonal problems. Technometrics, 12: 55-67.

IS 10262. 2009. (First Revision), Indian standard concrete mix proportioning- Guidelines. BIS, New Delhi, India.

IS 516 .1959. Indian standard methods for tests for strength of concrete. BIS, New Delhi, India.

Jayaram, M. A., Natraja, M. C., and Ravikumar, C. N. 2009. Elitist genetic algorithm models: optimization of high performance concrete mixes. Materials and Manufacturing Processes, 24: 225-229; DOI 10.1080/10426910802612387

Karihaloo, B. L., and Kornbak, D. L. 2001. Optimization techniques for the design of high performance fibre-reinforced concrete. Structural and Multidisciplinary Optimization, 21 (1): 32-39; DOI 10.1007/s001580050165

Kumar, M. 2002. Reliability based design of structural elements. PhD thesis, Thapar University, Patiala, India.

Lagaros, N. D., Tsompanakis, Y., Fragiadakis, M., Plevris, V., and Papadrakakis, M. 2008. Metamodel-based computational techniques for solving structural optimization problems considering uncertainties. Structural design optimization considering uncertainties (eds Y. Tsompanakis, N. D. Lagaros & M. Papadrakakis), pp. 567-597. Taylor & Francis Group, London, UK.

Lee, B. Y., Kim, J. H., and Kim, J. K. 2009. Optimum concrete mixture proportion based on a database considering regional characteristics. Journal of Computing in Civil Engineering, 23: 258-265; DOI 10.1061/(ASCE)0887-3801(2009)23:5(258)

Lim, C. H., Yoon, Y. S., and Kim J .H. 2004. Genetic algorithm in mix proportioning of high–performance concrete. Cement and Concrete Research, 34: 409-420; DOI 10.1016/j.cemconres.2003.08.018

Luo, X. and R. V. Grandhi, 1995. ASTROS for reliability based multidisciplinary structural analysis and optimization. Proceedings of 36th AIAA/ASME/ASCE/AHS/ASC Conference on Structures, Structural Dynamics and Materials, April 10-13, New Orleans, LA, pp: 93-102.

Ngo, S. H., Kemény, S. and Deák, A. 2004. Application of ridge regression when the model is inherently imperfect: a case study of phase equilibrium. Chemometrics and Intelligent Laboratory systems, 72: 185-194; DOI 10.1016/j.chemolab.2004.01.015

Özbay, E., A. Baykasoğlu, A. Öztas, and H. Özbebek, 2006. An experimental comparison of optimum mix proportions of high strength concrete proposed by Taguchi method and genetic algorithm. Proceedings of 5th International Symposium on Intelligent Manufacturing Systems, May 29-31, 2006, Sakarya, Türkiye, pp: 1062-1070.

Price, K. V., Storn, R. M., and Lampinen, J. A., 2005. Differential evolution: A practical approach to global optimization. Springer,Germany.

Rawling, J. O., Pantula, S. G., and Dickey, D. A., 1998. Applied Regression Analysis: A Research Tool. Springer, New York.

Royset, J. O., Kiureghian, A. D., and Polak, E. 2001. Reliability-based optimal structural design by decoupling approach. Reliability Engineering & System Safety, 73: 213-221; DOI 10.1016/S0951-8320(01)00048-5

Ryan, T. P. 1996. Modern Regression Methods. 2nd Ed. John Wiley & Sons, New York.

Sorenson, J. D. 2004. Notes in structural reliability theory and risk analysis, Aalborg, Denmark.

Thanedar, P. B., and Kodiyalam, S. 1992. Structural optimization using probabilistic constraints. Structural Optimization, 4: 236-240; DOI 10.1007/BF01742750

Valdebenito, M. A., and Schuëller, G. I. 2010. A survey on approaches for reliability-based optimization. Structural and Multidisciplinary Optimization, 42: 645-663; DOI 10.1007/s00158-010-0518-6

Wang, L., and Grandhi, R. 1995. Structural reliability optimization using an efficient safety index calculation procedure. International Journal of Numerical Methods in Engineering, 38: 1721-1738; DOI 10.1002/nme.1620381008

Yan, X. 2008. Modified nonlinear generalized ridge regression and its application to develop naphtha cut point soft sensor. Computers and Chemical Engineering, 32: 608-621; DOI: 10.1016/j.compchemeng.2007.04.011

Yeh, I. C. 1999. Design of high performance concrete mixture using neural networks and non linear programming. Journal of Computing in Civil Engineering, 13 (1): 36-42; DOI 10.1061/(ASCE)0887-3801(1999)13:1(36)

Yeh, I. C. 2003. A mix proportioning methodology for fly ash and slag concrete using artificial neural networks. Chung Hua Journal of Science and Engineering, 1 (1): 77-84.

Yeh, I. C. 2007. Computer aided design of optimum concrete mixtures, Cement and Concrete Composites, 29(3):193-202; DOI 10.1016/j.cemconcomp.2006.11.001

Yeh, I. C. 2009. Optimization of concrete mix proportioning using a flattened simplex-centroid mixture design and neural networks. Engineering with Computers, 25 (2): 179-190; DOI 10.1007/s00366-008-0113-2

Zou, T., and Mahadevan, S.A. 2006. Direct decoupling approach for efficient reliability-based design optimization, Structural and Multidisciplinary Optimization, 31: 190-200; DOI 10.1007/s00158-005-0572-7