International Journal of Science and Engineering(IJSE)

Home page: http://ejournal.undip.ac.id/index.php/ijse

A High Order Solution of Three Dimensional Time Dependent Nonlinear Convective-Diffusive Problem Using Modified Variational Iteration Method

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Abstract - In this paper, we have achieved high order solution of a three dimensional nonlinear diffusive-convective problem using modified variational iteration method. The efficiency of this approach has been shown by solving two examples. All computational work has been performed in MATHEMATICA.

Keywords— *Variational iteration method; Nonlinear equation; Partial differential equation; Diffusive-convective problem*

Submission: Sept 18, 2014 **Corrected : December 1, 2014** **Accepted: December 25, 2014**

Doi: 10.12777/ijse.8.1.1-5

[***How to cite this article***: Pathak, M. and Joshi, P. (2015). A High Order Solution of Three Dimensional Time Dependent Non Linear Convective-Diffusive Problem Using Modified Variational Iteration Method,*International Journal of Science and Engineering, 8(1):1-5,*
Doi: 10.12777/ijse.8.1.1-5

I. INTRODUCTION

Differential equations play an important role in several fields of science and engineering. One can easily see problems in the form of ordinary differential equations and partially differential equations in different branches of sciences and engineering. Vast applications of differential equations draw attention of engineers and scientists toward their solutions. According to the nature of differential equations their solutions always remain challenging. When a differential equation is of nonlinear type, finding its solution becomes more complicated. However, from time to time so many methods have been developed and their improved versions also came into existence. Finite difference methods, Lagrange multiplier method (Inokuti *et al.*, 1978), Backlund transformation, Darboux transformation , the inverse scattering transformation, symmetry method, the tanh method , Hirota's bilinear method, Adomian decomposition method, the, Homotopy perturbation method, Homotopy analysis method, Variational iteration method (He, 1997, 1999) and Modified variational iteration method (Abassy *et al.*, 2007, 2007a) are widely used to solve several nonlinear differential equations.

Convective – diffusive problem is another example of a nonlinear partial differential equation. It is a combination of two different equations, convection equation and diffusion equation. It models the transfer of a physical quantity inside a physical system due to convection and diffusion. A Transient nonlinear convection-diffusion equation in three dimensions (Campos *et al.*, 2014) is given by:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+u\frac{\partial u}{\partial y}+u\frac{\partial u}{\partial z}=v\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{1}$$

where $u(x,y,z,t)$ is the velocity field in the x,y,z -directions and v is the kinematic viscosity.

In this paper, modified variational iteration method(MVIM) (Abassy *et al.*, 2007) is applied successfully to obtain the highly accurate solution of the three dimensional transient nonlinear convective- diffusive problem. MVIM is developed by changing the formulation of variational iteration method(VIM) and gives better and faster results than VIM. To show the higher accuracy of the solutions obtained by modified variational iteration method, numerical results are compared with the results obtained in (Campos *et al.*, 2014).

II. FORMULATION OF THE MODIFIED VARIATIONAL ITERATION METHOD:

Materials used in this research were: In this section we will give a brief introduction about modified variational iteration method (Abassy *et al.*, 2007). Let us consider a general homogeneous unsteady nonlinear initial value problem in three dimensions of the form:

$$Lu(x,y,z,t)+Ru(x,y,z,t)+Nu(x,y,z,t)=0 \tag{2a}$$
$$u(x,y,z,0)=f(x,y,z) \tag{2b}$$

where $L = \partial/\partial t$, R is a linear operator which has partial derivatives with respect to x, y, z and $Nu(x, y, z, t)$ is a nonlinear term.

According to the variational iteration method (He, 1997, 1999, 2006) following iteration formula for equation (2) can be constructed:

$$U_{n+1}(x,y,z,t) = U_n(x,y,z,t) + \int_0^t \lambda\{LU_n + R\widetilde{U_n} + N\widetilde{U_n}\}d\tau \qquad (3)$$

where λ is called a general Lagrange multiplier (Inokuti *et al*., 1978) which can be identified optimally via variational theory (Inokuti *et al*., 1978; Finlayson, 1972), $R\widetilde{U_n}$ and $N\widetilde{U_n}$ are considered as restricted variations i.e. $\delta R\widetilde{U_n} = 0$ and $\delta N\widetilde{U_n} = 0$.

Calculating variation (He, 1997, 1999) with respect to U_n, the following stationary conditions are obtained:

$$\lambda'(\tau) = 0, \qquad (4a)$$

$$1 + \lambda(\tau)|_{\tau=t} = 0 \qquad (4b)$$

By solving equation (4), Lagrange multiplier can be identified as $\lambda = -1$. Substituting the identified multiplier into equation (3), following iteration formula can be obtained:

$$U_{n+1}(x,y,z,t) = U_n(x,y,z,t) - \int_0^t \{LU_n + RU_n + NU_n\}d\tau \qquad (5)$$

The second term on the right is called the correction term. Eq. (5) can be solved iteratively using $U_0 = u(x,y,z,0) = f(x,y,z)$ as initial approximation.

In application of iterative formula (5) to obtain approximate solution of equation (2), repeated calculations occur in every iteration which increase computational time of the method. Also, formula (5) produces non-settled terms in the approximate solution that slow convergence down.

To overcome with these problems, in 2007, Abassy *et al.* proposed the following modified variational iteration method:

$$U_{n+1}(x,y,z,t) = U_n(x,y,z,t) - \int_0^t \{R(U_n - U_{n-1}) + (G_n - G_{n-1})\}d\tau \qquad (6)$$

where $U_{-1} = 0, U_0 = f(x,y,z)$ and $G_n(x,y,z,t)$ is obtained from

$$NU_n(x,y,z,t) = G_n(x,y,z,t) + O(t^{n+1}) \qquad (7)$$

Equation (6) can be solved iteratively to obtain an approximate solution of equation (2) that takes the form

$$u(x,y,z,t) \simeq U_n(x,y,z,t) \qquad (8)$$

where n is the final iteration step.

III. NUMERICAL RESULTS

The In this section, modified variational iteration method (Abassy *et al*., 2007) is applied successfully to obtain the solutions of the three dimensional transient nonlinear diffusive-convective problems. The computational work is performed on Mathematica 9.0.

a) **Problem 1:** Taking Eq. (1) with initial condition $u(x,y,z,0) = \exp((x+y-2z)/v)$ which is directly taken from exact solution $u(x,y,z,t) = \exp((x+y-2z+6t)/v)$ (Campos *et al*., 2014).

Compare Eq. (1) and Eq. (2) to get

$$LU_n = \frac{\partial U_n}{\partial t}, RU_n = -v\left(\frac{\partial^2 U_n}{\partial x^2} + \frac{\partial^2 U_n}{\partial x^2} + \frac{\partial^2 U_n}{\partial x^2}\right), NU_n = U_n\frac{\partial U_n}{\partial t} + U_n\frac{\partial U_n}{\partial t} + U_n\frac{\partial U_n}{\partial t} \qquad (9)$$

where U_n is the nth approximation of Eq. (1). Initial conditions for Eq. (6) are given as:

$$U_{-1} = 0, U_0 = u(x,y,z,0) = \exp((x+y-2z)/v)$$

and $G_n(x,y,z,t)$ is calculated by Eq. (7) as follows:

$$U_n\frac{\partial U_n}{\partial t} + U_n\frac{\partial U_n}{\partial t} + U_n\frac{\partial U_n}{\partial t} = G_n(x,y,z,t) + O(t^{n+1}) \qquad (10)$$

Now apply modified variational iteration formula (6) to get approximate solutions as:

$$U_0 = e^{\frac{x+y-2z}{v}},$$

$$U_1 = e^{\frac{x+y-2z}{v}} + e^{\frac{x+y-2z}{v}}\left(\frac{6\mathrm{t}}{v}\right),$$

$$U_2 = e^{\frac{x+y-2z}{v}} + e^{\frac{x+y-2z}{v}}\left(\frac{6\mathrm{t}}{v}\right) + e^{\frac{x+y-2z}{v}}\frac{1}{\lfloor 2}\left(\frac{6\mathrm{t}}{v}\right)^2,$$

$$U_3 = e^{\frac{x+y-2z}{v}} + e^{\frac{x+y-2z}{v}}\frac{1}{\lfloor 2}\left(\frac{6\mathrm{t}}{v}\right)^2 + e^{\frac{x+y-2z}{v}}\frac{1}{\lfloor 3}\left(\frac{6\mathrm{t}}{v}\right)^3,$$

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$$U_n = e^{\frac{x+y-2z}{v}} + e^{\frac{x+y-2z}{v}}\frac{1}{\lfloor 2}\left(\frac{6\mathrm{t}}{v}\right)^2 + e^{\frac{x+y-2z}{v}}\frac{1}{\lfloor 3}\left(\frac{6\mathrm{t}}{v}\right)^3 + \cdots + e^{\frac{x+y-2z}{v}}\frac{1}{\lfloor \mathrm{n}}\left(\frac{6\mathrm{t}}{v}\right)^{\mathrm{n}},$$

as $n \to \infty$, U_n converges to the exact solution.

Hence, by applying modified variational iteration method, exact solution is obtained which shows superiority of the modified variational iteration method over the method in (Campos *et al*., 2014).

b) **Problem 2:** Taking Eq. (1) with initial condition $u(x,y,z,0) = z\mathrm{Cos}[2\pi y]\mathrm{Sin}[2\pi x]$ and computational domain as being an unit cube and $t = 0.1$ (Campos *et al*., 2014).

Therefore, initial conditions for Eq. (6) are given as:

$$U_{-1} = 0, U_0 = u(x,y,z,0) = z\mathrm{Cos}[2\pi y]\mathrm{Sin}[2\pi x]$$

and $G_n(x,y,z,t)$ is calculated as in Eq. (10).

Applying modified variational iteration formula (6) considering Eq. (9), first two approximate solutions can be given as:

$$U_1 = \mathrm{Cos}[2\pi y]\mathrm{Sin}[2\pi x] - \frac{1}{2}tz\mathrm{Cos}[2\pi y]\mathrm{Sin}[2\pi x](16\pi^2 v + 4\pi z\mathrm{Cos}[2\pi(x+y)] + \mathrm{Sin}[2\pi(x-y)] + \mathrm{Sin}[2\pi(x+y)])$$

$$U_2 = \mathrm{Cos}[2\pi y]\mathrm{Sin}[2\pi x] - \frac{1}{2}tz\mathrm{Cos}[2\pi y]\mathrm{Sin}[2\pi x](16\pi^2 v + 4\pi z\mathrm{Cos}[2\pi(x+y)] + \mathrm{Sin}[2\pi(x-y)] + \mathrm{Sin}[2\pi(x+y)]) + \frac{1}{2}t^2(-4\pi^2 vz\mathrm{Cos}[2\pi x]\mathrm{Cos}[2\pi y](\mathrm{Cos}[2\pi(x-y)] + \mathrm{Cos}[2\pi(x+y)] - 4\pi z\mathrm{Sin}[2\pi(x+y)]) + 4\pi v\mathrm{Cos}[2\pi y]\mathrm{Sin}[2\pi x]((-1+8\pi^2 z^2)\mathrm{Cos}[2\pi(x+y)] + 2\pi z(8\pi^2 v + \mathrm{Sin}[2\pi(x-y)] + \mathrm{Sin}[2\pi(x+y)])) + z\mathrm{Cos}[2\pi y]^2(\pi z\mathrm{Sin}[4\pi x](16\pi^2 v + 4\pi z\mathrm{Cos}[2\pi(x+y)] + \mathrm{Sin}[2\pi(x-y)] + \mathrm{Sin}[2\pi(x+y)]) + \mathrm{Sin}[2\pi x]^2(16\pi^2 v + 8\pi z\mathrm{Cos}[2\pi(x+y)] + \mathrm{Sin}[2\pi(x-y)] + \mathrm{Sin}[2\pi(x+y)] - 8\pi^2 z^2\mathrm{Sin}[2\pi(x+y)])) - \pi z\mathrm{Sin}[2\pi x](4\pi v\mathrm{Cos}[2\pi(x-y)]\mathrm{Sin}[2\pi y] + \mathrm{Cos}[2\pi(x+y)](-4\pi v\mathrm{Sin}[2\pi y] + 4\pi z^2\mathrm{Sin}[2\pi x]\mathrm{Sin}[4\pi y]) + z(16\pi^2 v\mathrm{Sin}[2\pi y]\mathrm{Sin}[2\pi(x+y)] + \mathrm{Sin}[2\pi x]\mathrm{Sin}[4\pi y](16\pi^2 v + \mathrm{Sin}[2\pi(x-y)] + \mathrm{Sin}[2\pi(x+y)])))).$$

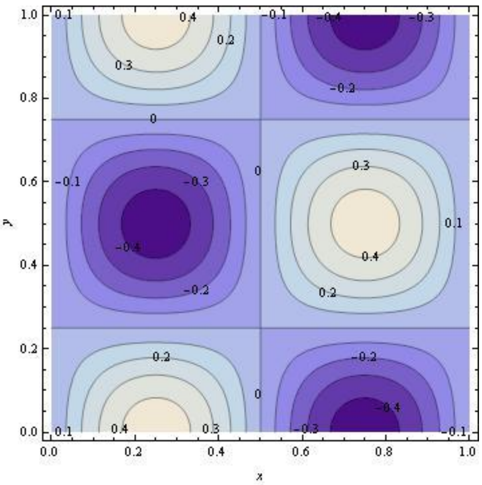

Fig. 1: Contour plot of first approximate solution U_1 at $v = 1, z = 0.5$ and $t = 0.001$ in xy-plane.

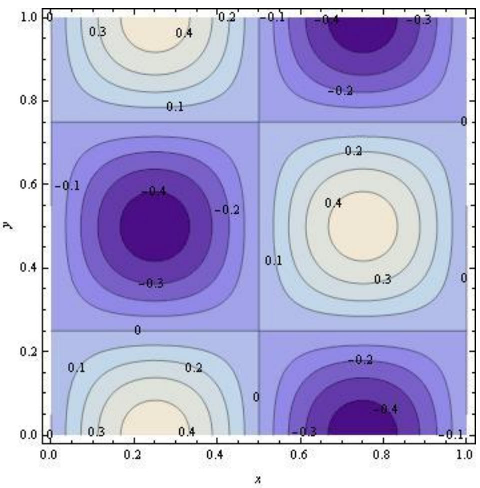

Fig. 2: Contour plot of second approximate solution U_2 at $v = 1, z = 0.5$ and $t = 0.001$ in xy-plane.

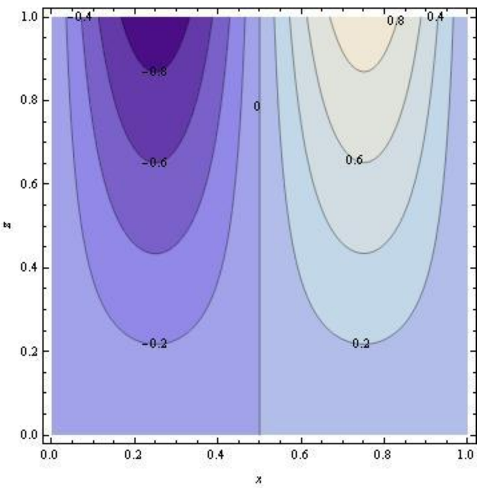

Fig. 3: Contour plot of first approximate solution U_1 at $v = 1, y = 0.5$ and $t = 0.001$ in xz-plane.

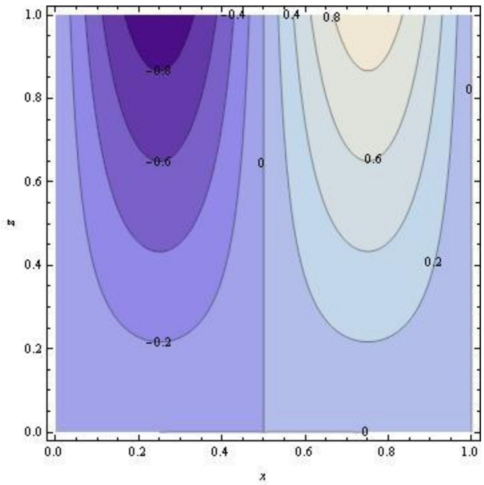

Fig. 4: Contour plot of second approximate solution U_2 at $v = 1, y = 0.5$ and $t = 0.001$ in xz-plane.

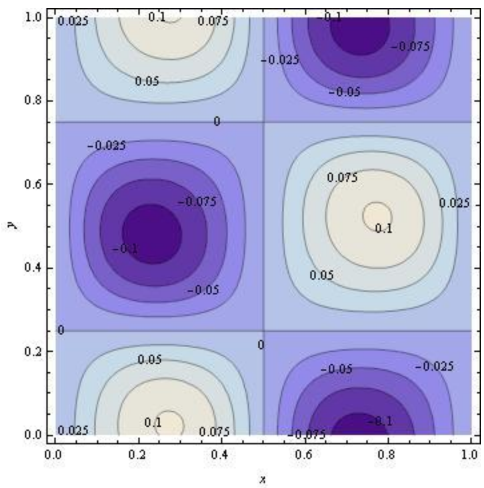

Fig. 5: Contour plot of first approximate solution U_1 at $v = 1, z = 0.5$ and $t = 0.01$ in xy-plane.

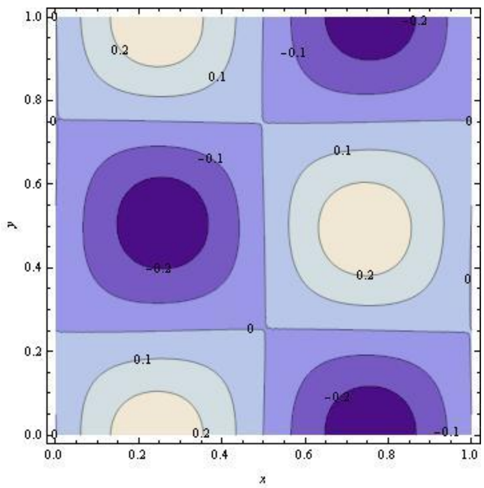

Fig. 6: Contour plot of second approximate solution U_2 at $v = 1, z = 0.5$ and $t = 0.01$ in xy-plane.

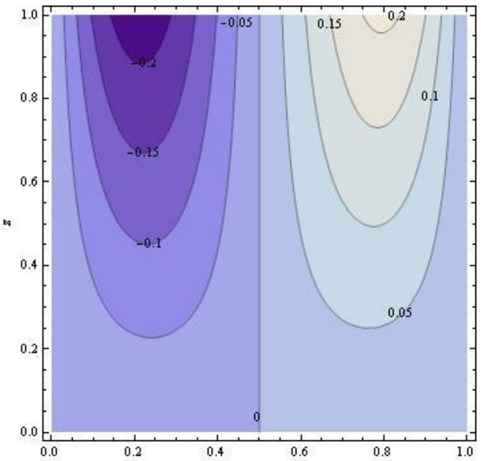

Fig. 7: Contour plot of first approximate solution U_1 at $v = 1, y = 0.5$ and $t = 0.01$ in xz-plane.

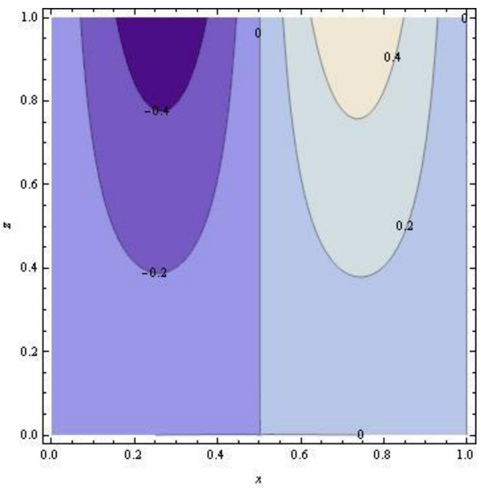

Fig. 8: Contour plot of second approximate solution U_2 at $v = 1, y = 0.5$ and $t = 0.01$ in xz-plane.

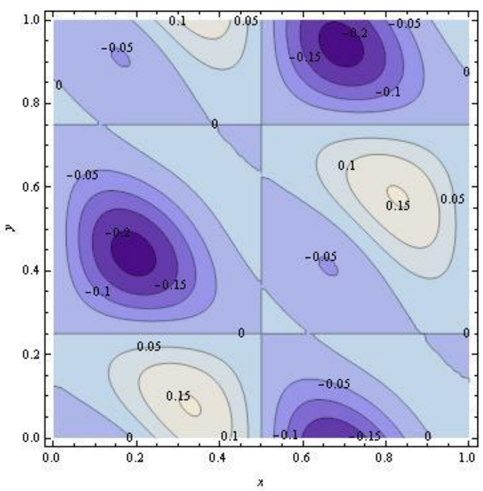

Fig. 9: Contour plot of first approximate solution U_1 at $v = 0.1, z = 0.5$ and $t = 0.1$ in xy-plane.

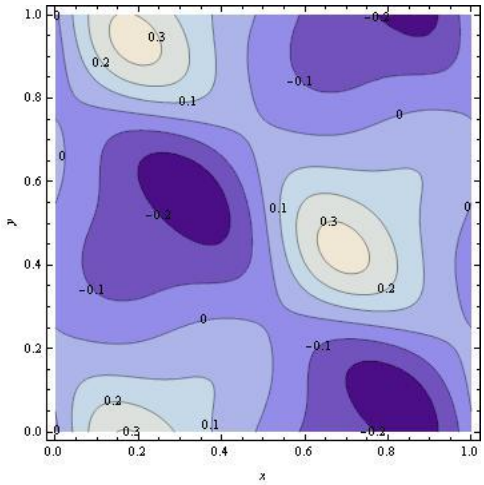

Fig. 10: Contour plot of second approximate solution U_2 at $v = 0.1, z = 0.5$ and $t = 0.1$ in xy-plane.

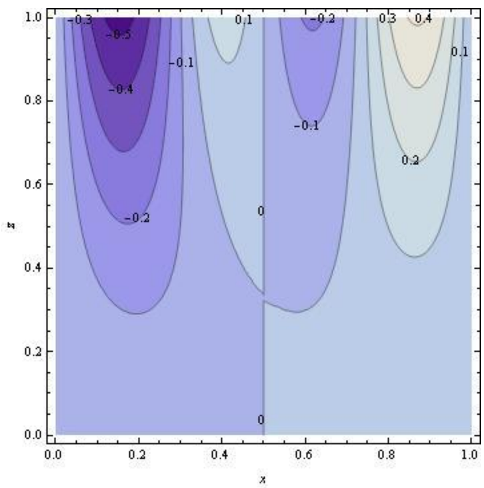

Fig. 11: Contour plot of first approximate solution U_1 at $v = 0.1, y = 0.5$ and $t = 0.1$ in xz-plane.

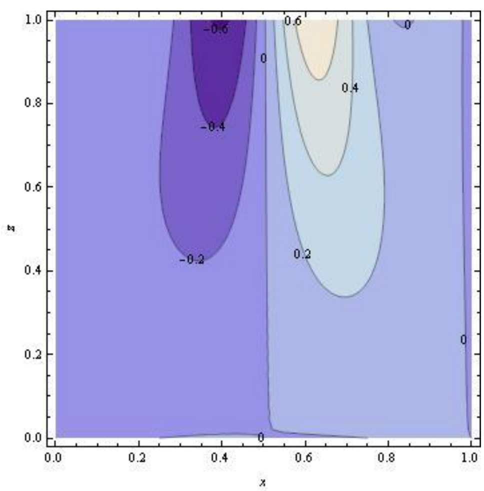

Fig. 12: Contour plot of second approximate solution U_2 at $v = 0.1, y = 0.5$ and $t = 0.1$ in xz-plane.

From Fig. 1 - Fig. 8, first and second approximations of velocity field remain smooth with respect to time in unit cube computational domain for kinematic viscosity $v = 1$, in xy and xz-plane. From Fig. 9 – Fig. 12 it is clear that second approximate solution is smoother than first approximate solution for kinematic viscosity $\upsilon = 0.1$, in xy and xz -plane. Hence second approximate solution remains smooth with respect to time and viscosity. Similarly, high order approximate solutions obtained by modified variational iteration method improve smoothness.

IV. CONCLUSIONS

In this paper, modified variational iteration method is applied successfully to solve nonlinear convection-diffusion problems in three dimensions. By comparing our results with results in (Campos *et al*., 2014), it is found that modified variational iteration method gives better results with respect to method mentioned in (Campos *et al*., 2014). Modified iteration method can be applied to other complex linear (Joshi and Kumar, 2012; Kumar and Joshi, 2012, 2012a, 2013) and nonlinear system arising in various other branches of science and engineering.

ACKNOWLEDGMENT

This paper is part of the project [No.-UCS&T/R&D/PHY SC.-10/12-13/6180] funded by Uttarakhand State Council for Science and Technology, Dehradun, Uttarakhand, India.

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