Design of Level Control in A 10 L Pure Capacitive Tank: Stability Analysis and Dynamic Simulation

Yulius Deddy Hermawan¹, Renung Reningtyas², Siti Diyar Kholisoh³, and Tutik Muji Setyoningrum⁴

¹Department of Chemical Engineering, Faculty of Industrial Technology, UPN “Veteran” Yogyakarta
Jl. SWK 104 (Lingkar Utara) Condongcatur, Yogyakarta, 55283, Indonesia
Email: ydhermawan@upnyk.ac.id

Abstract - The open loop experiment of a 10 L pure capacitive tank has been successfully done in laboratory. The square tank connected with a pump was designed for investigation in laboratory, and the water was chosen as a fluid with its input volumetric rate of \( f(t) \) [cm³/min]. The output volumetric rate of \( f_i(t) \) can be adjusted by changing the pump voltage of \( v_p(t) \) [volt]. The open loop experiment has given the steady state parameters, and it could then be used for calculating the dynamic parameters. This study has proposed the level control configuration of a 10 L pure capacitive tank system; liquid level in the tank \( h(t) \) was kept constant by manipulating the pump voltage of \( v_p(t) \) and the input water volumetric rate of \( f(t) \) was considered as a disturbance. The P-only-control was implemented to control the level. Purposes of this study are to analyze the stability of the closed loop responses and to do the closed loop dynamic simulation. The closed loop mathematical model was solved analytically with Laplace Transform, and Routh-Hurwitz criterion was chosen to analyze the stability. Since the closed loop model was found in the 2nd order system, the response depended on the value of the damping coefficient \( (Q) \), in which it was really affected by the controller gain \( (K_c) \). In order to examine the control configuration, the input water volumetric rate disturbance (with amount of ±14%) was made based on step function. Based on the stability analysis, a stable response would be achieved if the controller gain is negative \( (K_c<0) \) and the damping coefficient is positive \( (c>0) \). Based on the dynamic simulation, the controller gain was recommended in between -117.36 [volt/cm] and -1.17 [volt/cm] and the damping coefficient in between 0 and 1. This study also revealed that by tuning an appropriate controller gain, the fastest and the stable response would be achieved.

Keywords— Closed loop, Dynamic simulation, Level control, Pure capacitive, Routh-Hurwitz, Stable response

I. INTRODUCTION

The pure capacitive tank system is widely used in chemical process industries. This system is defined as a system with the transfer function equals to \( 1/s \). Thus, it is known as an integrating system (Rao et al. 2011). The pure capacitive system could be found in a tank connected with a pump for transferring the liquid out of the tank (Dias et al. 2003). Both changes of the input volumetric rate and the pump’s energy strongly affect to the level of the tank. The implementation of automatic level control in a pure capacitive tank is therefore very important. Furthermore, the level controller gain should be tuned properly in order to give fast and stable response.

Some researches in field of process control, tuning controllers, and stability analysis have been done in both of laboratory experiments and dynamic simulation with computer. Rao et al. (2011) has proposed design of Proportional Integral and Derivative (PID) controller based on internal model control (IMC), direct synthesis method (DS), stability analysis (SA) for pure integrating process with time delay. Designing PID and fuzzy controllers for liquid level process has also been done by Kala et al. (2014). Hermawan, Y.D. (2011) has implemented the process reaction curve for tuning of temperature control parameters in a 10 L stirred tank heater. Hermawan, Y.D. et al. (2012) did the open loop experiment in laboratory to study the composition dynamic in a 10 L mixing tank. Then, Hermawan, Y.D. and Haryono, G. (2012) continued the study in closed loop dynamic simulation; the composition control parameters were tuned by using the open loop method. Singh, M., and Saksea, D. (2012) presented a novel method to estimate the inverse response parameters in the first order system and a pure capacitive system. Singh, M., and Sharma, A. (2012) proposed the novel iterative method to overcome the limitation due to non-linear equation in the two opposing of first order system. Then, Singh et al. (2012) presented a novel method to compensate the inverse response of the process comprising of two opposite first order systems with a delay element.
Study on the stability analysis with Routh-Hurwitz criterion has also been done by some researchers (e.g. Sigal, R. 1990, Zahreddine, Z. 2002, Pena, J.M. 2004, and Roopamala, T.D., and Katti, S.K. 2010). Hussien et al. (2015) has successfully developed mathematical model of couple tank system using system identification; they also used Routh-Hurwitz criterion to determine the stability of the system. Hermawan, Y.D. et al. (2014) presented the process dynamic of pure capacitive system of 2 tanks in series; their study has been experimentally done in laboratory. Then, Hermawan, Y.D. (2014) continued the study in dynamic simulation and liquid level control in pure capacitive system of 2 tanks in series, where its level control parameters were tuned by trial and error.

In this work, we propose the liquid level control system in a 10 L pure capacitive tank, and analyze its closed loop stability. Steady state parameters of a 10 L pure capacitive tank system has been resulted by Hermawan, Y.D. et al. (2014) experimentally. The level control parameter would be tuned based on the Routh-Hurwitz stability criterion. The closed loop responses of the tank’s level to a change in the input volumetric rate will also be explored.

II. MATERIAL AND METHOD

Figure 1 shows the experimental apparatus setup. As can be seen in Figure 1, No.1 is a main tank that represents a pure capacitive tank. The tank No. 1 was designed in square shape and made of glass, thus the liquid level in the tank can be seen clearly. A ruler was attached on the tank’s wall (No. 5 in Figure 1) to indicate the liquid level in the tank. The tank No. 1 has an input stream that comes from the feeding tank No.7. In this work, water was used as an input stream with its volumetric rate \( f_i \) [cm³/minute]. The input volumetric rate can be adjusted by valve No. 6a. The water in tank No. 1 was then be pumped out of the tank (by pump No. 2a) with its volumetric rate \( f_o \) [cm³/minute]. The output volumetric rate \( f_o \) can be adjusted by means of pump volt regulator (No. 3 in Figure 1).

II.1. RESEARCH PROCEDURES

This work was done through some procedures as follows:

1) Open loop experiment in laboratory: was done to determine the steady-state parameters as follows volumetric rates: \( f_i \) and \( f_o \), liquid level in the tank: \( h \), pump voltage: \( v_{pu} \), pump time constant: \( \tau_{pu} \), and pump gain: \( K_{pu} \). Those steady-state parameters were then used to calculate the dynamic parameters of the proposed level control system. The level control system in a pure capacitive tank is shown in Figure 2.

2) Stability Analysis by using Routh-Hurwitz criterion: was done to tune the controller gain that gives a stable response. The Proportional (P) feedback control would be implemented to control the liquid level constant (Figure 2).

3) Closed loop simulation by means of computer: was done to explore the response of liquid level in the tank to a change in the input volumetric rate. The input volumetric rate was made based on step function with amount of ±14%. The closed loop model was solved analytically with Laplace Transform, and the

II.2. OPEN LOOP MODEL

The level control parameter would be tuned based on the Hurwitz stability criterion. The closed loop responses of the system of 2 tanks in series, where its level control parameters were tuned by trial and error.

Figure 1. Experimental apparatus setup.

Figure 2. The proposed level control system in a 10 L pure capacitive tank.
In this study, the water density ($\rho$) is assumed constant, the un-steady state mass balance of the pure capacitive tank can be written as follows:

$$f_i(t) - f_o(t) = \tau \frac{df_o}{dt}$$  

where $A$ is the cross-sectional area of tank [cm$^2$].

The steady-state material balance on the contents of the tank at the initial condition is:

$$f_i(0) - f_o(0) = \tau \frac{dH(0)}{dt}$$  

Subtracting equation (2) from equation (1) yields:

$$K_{pu}F_i(t) - K_{pu}F_o(t) = \frac{dH(t)}{dt}$$  

with the following deviation variables as follows:

$$F_i(t) = f_i(t) - f_i(0)$$

$$F_o(t) = f_o(t) - f_o(0)$$

$$H(t) = h(t) - h(0)$$

and process gain is:

$$K_{pu} = \frac{1}{A}$$

The relation between the output volumetric rate ($f_o$) and the pump voltage ($V_{pu}$) is given by Smith, C.A. and Corripio, A.B. (1997):

$$\tau_{pu} \frac{df_o}{dt} + f_o(t) = K_{pu} V_{pu}(t)$$

where $\tau_{pu}$ is pump time constant [minute], and $K_{pu}$ is pump gain [cm$^3$/minute/volt).

At the initial condition, equation (8) becomes:

$$\tau_{pu} \frac{df_o(0)}{dt} + f_o(0) = K_{pu} V_{pu}(0)$$

Subtracting equation (9) from equation (8) yields:

$$\tau_{pu} \frac{dF_i(t)}{dt} + F_o(t) = K_{pu} V_{pu}(t)$$

with the following deviation pump voltage:

$$V_{pu}(t) = v_{pu}(t) - v_{pu}(0)$$

Laplace transform of linear equations (3) and (10) yields:

$$H(s) = \frac{K_{pr} F_i(s) - K_{pr} F_o(s)}{s}$$

Substitution of equation (13) into equation (12) results:

$$H(s) = \frac{K_{pr} F_i(s) - K_{pr} F_o(s)}{\tau_{pu} s + 1}$$

Equation (14) shows that system is pure integrator, and the liquid level in the tank is affected by the input volumetric rate and the pump voltage.

II.3. CLOSED LOOP MODEL

In this work, the liquid level will be kept constant at its set point by manipulating the pump voltage. The transfer function of measurement process and final control element are assumed equal to 1, and the transfer function for a Proportional controller is:

$$G_c(s) = K_c$$

where $K_c$ is controller gain [volt/cm]. The closed loop block diagram of the proposed level control is shown in Figure 3.

![Figure 3. The closed loop diagram block of the proposed level control](image-url)

Since the input volumetric rate is considered as a disturbance, the set point of liquid level remains the same, i.e. $R(s)=0$, the closed loop model is written as follows:

$$H(s) = \frac{K}{s + \tau^2 s^2 + 2\zeta \tau s + 1} F_i(s)$$

where overall gain ($K$), overall time constant ($\tau$), and damping coefficient ($\zeta$) are as follows:

$$K = \frac{1}{K_{pu}(1 - K_c)}$$

$$\tau = \frac{\tau_{pu}}{K_{pr} K_{pu}(1 - K_c)}$$

$$\zeta = \frac{1}{2\sqrt{\tau_{pu} K_{pr} K_{pu}(1 - K_c)}}$$

In this work, the input disturbance would be made based on the step function with the magnitude of $\pm \Delta F_i$ [cm$^3$/minute]. The Laplace equation of input disturbance is then given by:

$$F_i(s) = \frac{\pm \Delta F_i}{s}$$

© IJSE - ISSN: 2086-5023, 15*January, 2016, All rights reserved
Substitution of equation (20) into equation (16) yields the 2nd order closed loop response of liquid level to a change in the input volumetric rate as follows:

$$H(s) = \frac{K(\pm \Delta f_i)}{\tau^2 s^2 + 2\zeta \tau s + 1}$$

(21)

The characteristic equation of the closed loop model is in 2nd order type (Luyben, W.L. 1996):

$$\tau^2 s^2 + 2\zeta \tau s + 1 = 0$$

(22)

II.4. STABILITY ANALYSIS WITH ROUTH-HURWITZ CRITERION

Routh-Hurwitz criterion was chosen to analyze the stability of the closed loop responses (Stephanopoulos, G., 1984). The stability of the closed loop response is really affected by its damping coefficient which is depended on the controller gain. The Routh array for stability analysis of closed loop response is given as follow:

Row 1
$$\begin{bmatrix} \tau_{pu} \\ K_{pu} K_{pu} (-K_c) \end{bmatrix}$$
1 0

2
$$\sqrt{\tau_{pu} K_{pu} K_{pu} (-K_c)}$$
0 0

3
$$\begin{bmatrix} \tau \\ \tau_{pu} K_{pu} (-K_c) \end{bmatrix}$$
1 0 0

(23)

The process system will give stable response if all members in the 1st column of Routh array are positive. According to equation (23), the values of 1st and 2nd members of 1st column depend on the controller gain; In order to allow them be positive, the controller gain must be negative, i.e. \(K_c < 0\), as a result that the control system will give a stable response.

II.5. CLOSED LOOP RESPONSES

The closed loop model of equation (21) should be inverted to obtain the closed loop response. Since the closed loop model is the 2nd order system, the solution depends on the value of damping coefficient (Marlin, T.E. 2000). On the other hand the damping coefficient (\(\zeta\)) is really affected by the controller gain (\(K_c\)).

When \(\zeta > 1\), the control system produces the overdamped response as follows:

$$h(t) = h(0) \pm K\Delta f_i \left[1 - \frac{1}{2} \left(1 + \frac{1}{\tau} e^{-\tau_{el}/\tau} \right) \right]$$

(24)

where the effective time constants (\(\tau_{el}\)) are as follows:

$$\tau_{el} = \frac{\tau}{\zeta - \sqrt{\zeta^2 - 1}}$$

(25)

When \(\zeta = 1\), the critically damped response of the liquid level would be produced as follows:

$$h(t) = h(0) \pm K\Delta f_i \left[1 - \frac{1}{\tau} e^{-\tau_{el}/\tau} \right]$$

(26)

When the damping coefficient is between 0 and 1 (\(0 < \zeta < 1\)), the under damped response of the liquid level would be obtained as follows:

$$h(t) = h(0) \pm K\Delta f_i \left[1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta \pi/\tau} \sin(\sqrt{\zeta^2 - 1}) \right]$$

(27)

$$\psi = \frac{1}{\tau} \sqrt{1 - \zeta^2}, \text{ [radian/time or hertz]}$$

(28)

$$\phi = \tan^{-1} \left(\frac{1 - \zeta^2}{\zeta}\right), \text{ [radian]}$$

(29)

Dynamic performance of the level control system will be formulated from the complete closed loop response, from time \(t = 0\) until steady state has been reached. Integral of the absolute value of the error (IAE) for level controller would be used for the formulation of the level dynamic performance. The IAE can be calculated as bellows:

$$IAE = \int_{0}^{\infty} |e(t)| dt$$

(30)

where \(e(t)\) is the deviation (error) of the level response from its desired set point (\(h_{sp}\)), and written as follows:

$$e(t) = h_{sp} - h(t)$$

(31)

The desired level set point (\(h_{sp}\)) is the level at the initial condition:

$$h_{sp} = h(0)$$

(32)

III. RESULT AND DISCUSSION

Steady state parameters of a 10 L pure capacitive tank are shown in Table 1. The pump time constant is found to be 0.38 minute. Therefore the pump is considered quite sensitive to the liquid level and suitable to be manipulated to control the liquid level. The steady state parameters would then be used as the initial conditions for dynamic simulation. The initial liquid level in the tank is 10 cm; this is the desired set point. The initial pump voltage is 63 volt; this is then manipulated to maintain the liquid level at its set point.

<table>
<thead>
<tr>
<th>No</th>
<th>Variable</th>
<th>Steady State</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The input volumetric rate, (f_i) [cm³/minute]</td>
<td>14,125.76</td>
</tr>
<tr>
<td>2</td>
<td>The output volumetric rate, (f_o) [cm³/minute]</td>
<td>14,125.76</td>
</tr>
<tr>
<td>3</td>
<td>Pump voltage, (v_{pu}) [volt]</td>
<td>63</td>
</tr>
<tr>
<td>4</td>
<td>Liquid level, (h) [cm]</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>Pump gain, (K_{pu}) [cm³/(minutevolt)]</td>
<td>224.22</td>
</tr>
<tr>
<td>6</td>
<td>Pump time constant, (\tau_{pu}) [minute]</td>
<td>0.38</td>
</tr>
<tr>
<td>7</td>
<td>The cross sectional area of the tank, (A) [cm²]</td>
<td>400</td>
</tr>
<tr>
<td>8</td>
<td>Tank height, (h) [cm]</td>
<td>25</td>
</tr>
</tbody>
</table>

III.1. ROUTH-HURWITZ STABILITY ANALYSIS FOR TUNING OF LEVEL CONTROL PARAMETER

Based on the Routh-Hurwitz analysis, the closed loop response would be stable if the controller gain is less than zero.
(\(K_c < 0\)). The effect of the controller gain to the dynamic parameters, such as damping coefficient (\(\zeta\)), overall gain (\(K\)), and overall time constant (\(\tau\)) are listed in Table 2. The greater controller gain that be tuned, i.e. close to zero, the greater damping coefficient would be obtained; that means the closed loop response is slower, monotonic stable, and called overdamped response. And vice-versa, the less controller gain, i.e. more negative, the less damping coefficient; that means the closed loop response is faster and more oscillatory, and called underdamped response.

### Table 2: Dynamic Parameters

<table>
<thead>
<tr>
<th>No</th>
<th>(K_c) Controller Gain [volt/cm]</th>
<th>(\zeta) Damping Coefficient</th>
<th>(K) Overall Gain [minute/cm(^2)]</th>
<th>(\tau) Overall Time Constant [minute]</th>
<th>(\text{IAE}) Integral Absolute Error</th>
<th>Type of Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.46</td>
<td>1.6</td>
<td>972.8 x 10(^{-5})</td>
<td>1.22</td>
<td>123.23</td>
<td>Overdamped</td>
</tr>
<tr>
<td>2</td>
<td>-0.69</td>
<td>1.3</td>
<td>642.2 x 10(^{-5})</td>
<td>0.99</td>
<td>95.81</td>
<td>Overdamped</td>
</tr>
<tr>
<td>3</td>
<td>-1.17</td>
<td>1.0</td>
<td>380 x 10(^{-5})</td>
<td>0.76</td>
<td>64.49</td>
<td>Critically damped</td>
</tr>
<tr>
<td>4</td>
<td>-4.69</td>
<td>0.5</td>
<td>95 x 10(^{-5})</td>
<td>0.38</td>
<td>18.29</td>
<td>Underdamped</td>
</tr>
<tr>
<td>5</td>
<td>-117.36</td>
<td>0.1</td>
<td>3.8 x 10(^{-5})</td>
<td>0.08</td>
<td>0.76</td>
<td>Underdamped</td>
</tr>
</tbody>
</table>

### III.2. Dynamic Simulation of Level Control System

Closed loop responses to a change in the input volumetric rate \(f_i\) are illustrated in Figure 4 and Figure 5. The disturbances were made by following both functions of step increase and step decrease.

**Step increase change in the input volumetric rate:** For step increase’s disturbance, the input volumetric rate \(f_i\) is increased by an amount of 2000 cm\(^3\)/minute. The solid line in both of Figure 4 and Figure 5 represents the closed loop responses to a step increase change in the input volumetric rate.

When the controller gain is greater than \(-1.17\) volt/cm, the damping coefficient is greater than unity; as a result the dynamic behaviour of the closed loop response is sluggish monotonic stable as shown in Figure 4.

![Figure 4](image)

**Figure 4.** The closed loop overdamped responses to a step change in the input volumetric rate:

(a). liquid level, (b). pump voltage

The liquid level in the tank increases and could not be backed to its set point of 10 cm, even though the pump voltage rises to the new steady state value of 72 volt (Figure 4.b). The level control system with the controller gain of \(-0.46\) volt/cm produces the damping coefficient of 1.6, thus the liquid level response is very slow; this is called overdamped response;
finally the liquid outs of the tank at time of about 6 minutes (Figure 4.a).

When the controller gain is equal to –1.17 volt/cm, the damping coefficient is equal to unity, the closed loop response is critically damped; it is almost oscillatory. Finally at time of about 5 minutes, the liquid level achieves the new steady state value of 15 cm (Figure 4.a), and the pump voltage constants at the value of 72 volt (Figure 4.b).

When the controller gain is less than –1.17 volt/cm, the damping coefficient is in between 0 and 1; consequently the dynamic behaviour of the closed loop response is faster but more oscillatory as shown in Figure 5. This oscillatory response is called underdamped response (Luyben, W.L. 1996).

The closed loop responses become more oscillatory as the damping coefficient decreases. As can be seen from Figure 5, the closed loop responses of level control system with the controller gain of –117.36 volt/cm (gives damping coefficient of 0.1) is faster and more oscillatory than those with the controller gain of –4.69 volt/cm (gives damping coefficient of 0.5). The level control system with the controller gain of –4.69 volt/cm still could not back the liquid level to its set point of 10 cm; the new steady state level of 11.9 cm is achieved at time of about 3.5 minutes. The liquid level could be returned to its set point of 10 cm for the controller gain of –117.36 volt/cm.

**Step decrease change in the input volumetric rate:**

For step decrease’s disturbance, the input volumetric rate \( f_i \) is decreased by an amount of 2000 cm\(^3\)/minute. The dashed line in both of Figure 4 and Figure 5 represents the closed loop responses to a step decrease change in the input volumetric rate.

The liquid level drops as the input volumetric rate jumps down. As shown in Figure 4, the liquid level drops to the value of zero for the level control system with its controller gain is greater than –1.17 volt/cm (the damping coefficient is greater than unity). For the controller gain of –0.46 volt/cm (damping coefficient of 1.6), the liquid level in the tank drops to the value of zero at the time equal to 3 minutes (Figure 4.a), even though the pump voltage drops to the new steady state value of 54 volt (Figure 4.b).

The liquid in the tank could be maintained if the controller gain is tuned at the value of less than –1.17 volt/cm, i.e. the damping coefficient is in between 0 and 1. As shown in Figure 5, the liquid in the tank still remain. The liquid level in the tank could be returned to its set point of 10 cm (Figure 5.a) for the controller gain of –117.36 volt/cm (damping coefficient of 0.1); the pump voltage finally constant at 54 volt.

![Figure 5. The closed loop underdamped responses to a step change in the input volumetric rate:](image-url)
IV. CONCLUSIONS

This paper has discussed stability analysis of closed loop responses and closed loop dynamic simulation in a 10 L pure capacitive tank. The level control system in a 10 L pure capacitive tank has been proposed. Closed loop dynamic behaviors of the proposed control system have also been explored. According to the stability analysis and the dynamic simulation, the controller gain was recommended to be tuned in between -117.36 and -1.17 [volt/cm]. This gives the fast and stable response.

ACKNOWLEDGMENT

I appreciate the technical support on the use of free software SCILAB. I also thank Hamdani, A.F. and Puspita, D.D. for helping research in laboratory.

REFERENCES


http://ejournal.undip.ac.id/index.php/reaktor/article/view/4802/4352


http://journal.uny.ac.id/index.php/kejuangan/article/view/519


http://www.davidspublishing.com/Download/?id=1573


http://www.ijitee.org/attachments/Files/v4i12/2L20608541215.pdf


doi:10.1016/0895-7177(90)90072-U


http://www.ccsjournals.com/IITKM/PDF%206-1/Article_5.pdf

