Running-in as an Engineering Optimization

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Abstract

Running-in is a process which can be found in daily lives. This phenomenon occurs after the start of the contact between fresh solid surfaces, resulting in changes in the surface topography, friction and wear. Before the contacting engineering solid surfaces reach a steady-state operation situation this running-n enhances the contact performance. Running-in is very complex and is a vast problem area. A lot of variable occurs in the running-in process, physically, mechanically or chemically. These transient phenomena should be optimized so that it is beneficial. In this paper the global analysis of running-in in term of engineering optimization is presented. Literature that reports of what have been published about knowledge and ideas, on the running-in topic by accredited scholars and researchers, are reviewed. The running-in model which can predict the real engineering surfaces in its operation is proposed.

Key words: *optimization, running-in, elastic-plastic contact, friction, wear,*

I. INTRODUCTION

Some application of contact mechanics can be meet in almost every aspect of our daily lives. Gripping, holding, sliding, brushing, machinery works, friction between skin and clothes, et cetera all demonstrate the impact of contact mechanics. Running-in is one of the manifestations of contact mechanics. The nature and consequence of the interactions that take place at the interface control friction and wear of the contacting bodies. During the interactions, forces are transmitted, mechanical energy is converted, physical and chemical natures of the interacting materials are altered.

Many literatures have defined running-in in their own way, however, it has been agreed that there is a "change" during running-in process. GOST (former USSR) Standard defines running-in as: "The change in the geometry of the sliding surfaces and in the physicomechanical properties of the surface layers of the material during the initial sliding period, which generally manifests itself, assuming constant external conditions, in a decrease in the frictional work, the temperature, and the wear rate" [1]. Summer-smith [2] defines running-in as: "The removal of high spots in the contacting surfaces by wear or plastic deformation under controlled conditions of running giving improved conformability and reduced risk of film breakdown during normal operation".

This paper presents the study of running-in in term of engineering optimization. Literature that reports of what have been published (knowledge and ideas) on the running-in topic, by accredited scholars and researchers, are explored. A new model to simulate the characteristics of a running-in process is proposed.

II. RUNNING-IN PHENOMENON

Changes in the condition of both surfaces generally occur when two surfaces are loaded for the first time and moved relatively to one another, These changes are usually a combination of many things, such as the alignment of axes, shape, surface roughness, and the equalizing of various mechanical and chemical properties between the moving surfaces (the micro-hardness, which is produced by selective work hardening or the formation of oxide layers and other boundary layers). All these changes are adjustments to minimize energy flow, whether mechanical or chemical, between the moving surfaces [3]. The changes which occur between start-up and steady state are associated with running-in (also called breaking-in or wearing-in, [4]).

Running-in occurs in the first period in the life-time of a rolling or sliding contact of a lubricated system, which is schematically shown in Fig. 1. Prior to running-in, the various pairs of contacting surfaces in, for instance, a new engine are not 'mated together'. There may be a slight initial misalignment and there will certainly be 'high spots' on all surfaces. Initially the clearances will be small and therefore the cooling

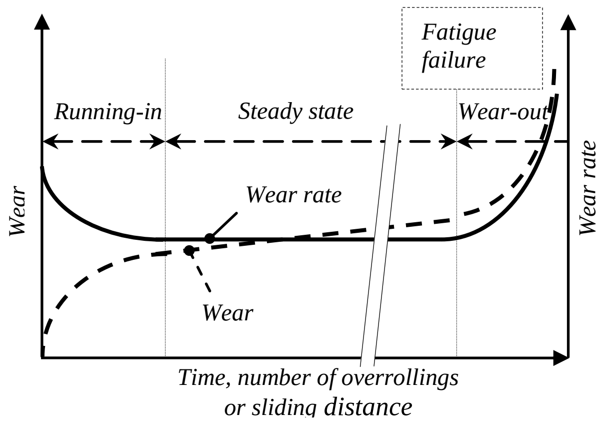

Figure 1.
Schematic representation of the wear behavior as a function of time, number of overrollings or sliding distance of a contact under constant operating conditions.

flow or oil is low and this, together with the initial higher friction, leads to operating temperatures higher than normal. During the running-in period, the high spots left from the final machining process are reduced by plastic flow, voids are filled and overall shapes are matched. The higher temperatures usually cause higher wear rates, but as the surfaces become smoother and the more prominent asperities are flattened, the wear rate falls to a steady state. There are two dominant mechanisms in the running-in period; plastic deformation and mild wear [5]. The plastic deformation mechanism is similar to roller burnishing; the asperities literally get squashed down. The change of the surface topography can be the amplitude and/or the texture depending on the load and moving direction. The higher asperities are rubbed off. This mechanism is also called truncating or censoring the height distribution. Frictional losses usually decrease during this period and contact clearances increase, thus reducing the surface temperatures. The wear rate decreases until it reaches the normal steady-state wear rate for the design contact pairs. The wear rate during running-in, even when misalignments are minimal, is higher than during normal running.

After the running-in period, of which duration is invariably depending on the tribo-system, the full service conditions can be applied without any sudden increase in wear rate. The load carrying capacity reaches to its operating design. The steady low wear rate regime is maintained for the designed operational life. The term steady state is defined as the condition of a given tribo-system in which the average dynamic coefficient of friction, wear rate, and other specific parameters have reached and maintained a relatively constant level [6].

The wear rate may rise again once the operating time becomes sufficiently long for a fatigue process to occur in the upper layers of the loaded surface. A significant contribution to material loss driven by cyclic loading is started. The particles from such a fatigue wear process are characteristically much larger than the small fragments associated with adhesive or abrasive wear [7]. This form of wear generates a 'pitted' surface (pitting failure). Once the wear particles due to fatigue wear accumulate the surface, it will wear-out i.e. total failure occurs.

Although the running-in subject is somewhat vague, numerous investigations have been conducted to study running-in. The first study of running-in is probably the experimental work by Hirn in 1854 [8]. The effect of running-in upon bearing friction was discovered and it pointed out that lubricated bearing must be run continuously for a certain time before a steady value of friction is attained. Running-in process is a complicated phenomenon. Jamari and Schipper [9] have reviewed the study of running-in. They found that most studies are based on experiments [10-42] in order to get an impression of the running-in behavior. Furthermore, the initial surface topography shows the most influencing factor with respect to running-in. This variable is the important issue to be used for the optimization process.

III. MAIN ASPECTS OF RUNNING-IN

There are two phases during the running-in period, i.e. Phase I and Phase II. In Phase I, the coefficient of friction strongly decreases and the change in surface topography shows similarities with the decrease of the center line average roughness, R_a, value. In Phase II, there is only a slight decrease in the coefficient of friction as well as in the reduction of R_a for quite some time. In this phase mild wear is considered due to the removal of boundary layers formed by a reaction of the additives and oxygen in the lubricant and the contacting metal surfaces.

Schipper [43] studied the running-in effect on the frictional behavior of lubricated concentrated contacts, which can be represented in generalized Stribeck curves. The coefficient of friction, μ, is plotted as a function of the lubrication number, $(\eta V_+)/p$ or H, in a logarithmic scale. η is the lubricant inlet viscosity, V_+ is the sum velocity and p is the mean contact pressure. During the running-in period, the decrease in the micro-geometry increases the hydrodynamic action. The succeeded running-in increases the load carrying capacity, i.e. increases the hydrodynamic action hence decreases the friction at constant operational conditions. For the low pressure situation, running-in manifests itself by shifting the mixed lubrication (ML) regime to lower values of the lubrication number, H and by decreasing the coefficient of friction. The change in micro-geometry affects the coefficient of friction in the boundary lubrication (BL) regime to lower values. The same shifts are found for the high pressure situation, except that the minimum coefficient of friction, at the transition from mixed lubrication to elasto-hydrodynamic lubrication (E(HL)) regime, shifts to higher values.

IV. MODELING RUNNING-IN

Blau [44] collected from the published work numerous examples of running-in experiments, which resulted in sliding coefficient of friction versus time behavior graphs, and own laboratory experiments in order to be able to develop a physical realistic and useful running-in model [45]. Based on a survey of literature eight common forms of friction versus sliding time curves are revealed, see Fig. 2.

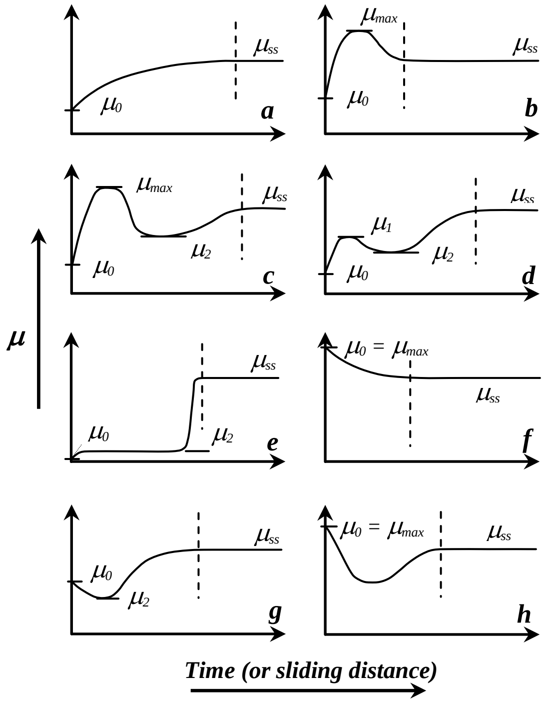

Figure 2.
Eight commonly observed forms of initial frictional behaviour as a function of time or sliding distance, after Blau [45].

The basic shape of the eight transition curves is a starting point for developing a semi-empirical running-in model of Blau. The model can be represented in its simple form as a product of two factors:

$$\mu(t) = L(t)S(t) \qquad (1)$$

where $\mu(t)$ is the time-dependent coefficient of friction, $L(t)$ is the time-dependent lubrication factor, and $S(t)$ is the time-dependent contribution of the solid materials in contact. Each factor in the model is further broken down into a form which permits the magnitude and rate of change in the different frictional contributions to be incorporated.

Several kinds of frictional behavior can be represented by summing up contributions of the various terms and factors after the proper time scales and magnitudes of contributory processes have been determined. By using various combinations of L, D and T terms, all the eight transition curve shapes can be produced. It can be summarized that a simple generalized model of Blau is able to generate the various types of frictional transitions including the running-in friction. However, the model may be applied to a frictional system behavior globally or phenomenological rather than to study the local micro-geometry changes which affect the global frictional behavior deterministically.

Another approach in modeling the running-in process is statistically. In static contact situations in which the contact pressure is lower than the elastic limit or yield stress, a material element will return to its original geometry once the load has been removed. If the contact pressure is larger than the elastic limit then some material will undergo plastic flow. There are two significant consequences with respect to this situation; residual stresses will develop and the material may strain-harden so increase its effective yield stress [46].

In repeated contact situations, the developed residual stresses will increase the yield stress for the subsequent loading. These residual stresses are essentially protective, together with any effects of strain-hardening and geometric changes which may ensure that the repeated contact is still in the elastic regime. This process is referred to as shakedown. Shakedown is the process in which a cyclically loaded structure or material element deforms plastically at the first loading and finally achieves a steady state in which the response is perfectly elastic [47]. The influence of residual stresses in promoting shakedown was governed by the Melan's theorem [48] which states: "If any system of self-equilibrating residual stresses can be found which, in combination with the stresses due to the repeated load, do not exceed yield at any time, then elastic shakedown will take place".

The response of the structure is entirely elastic for loading up to the elastic limit. The plastic flow is encountered when applying the load above the elastic limit. The shakedown process takes place and the structure responses to the elastic steady-state. The upper limit for this behavior is shown as elastic shakedown limit. When increasing the load, the plastic flow is encountered, even in the steady-state condition. If the load is below the plastic shakedown limit a closed cycle of plastic deformation occurs. In this stage plastic flow occurs at two instances in each load cycle but there is no net accumulation of deformation. However, if the load lies above the plastic shakedown limit, then an open cycle of plastic deformation occurs and the material accumulates small increments of plastic deformation in each loading cycle or ratcheting.

Kapoor & Johnson [49] consider running-in as a shakedown process. They hypothesize that due to plastic flow in early passages of a body, the shape and height of asperities at the surface will be modified as such that, in steady state, the load will be carried purely elastically in order to model running-in. This approach has been used extensively to different applications. Based on the shakedown hypothesis of [50] and the statistical approach, the new distribution of asperity heights is completely defined which depend only on the surface separation and the group of rough surfaces. The asperity heights follow a Gaussian distribution, but their radii remain constant. The non-dimensional nominal pressure at shakedown for point contact is derived as:

$$\overline{P}_s \equiv \frac{\left(P_s / p_0{}^s\right)}{N R_1 \sigma_1} = \left(\frac{9\pi^{5/2}}{16\sqrt{2}}\right)\frac{1}{\psi_s{}^2} \times \int_{d/\sigma_1}^{h/\sigma_1} \left\{1+\left(\frac{4}{3\pi}\right)^2 \psi_s{}^2 \left(\frac{z}{\sigma_1}-\frac{h}{\sigma_1}\right)\right\}^{3/2} e^{-z^2/2\sigma_1} d\left(\frac{z}{\sigma_1}\right) + \left(\frac{9\pi^3}{32}\right)\frac{1}{\psi_s{}^2}\left[1-erf\left(\frac{h}{\sqrt{2}\sigma_1}\right)\right] \qquad (2)$$

where

$$\frac{1}{\psi_s{}^2} = \left(\frac{p_0{}^s}{E}\right)^2 \frac{R_1}{\sigma_1} \qquad (3)$$

P_s is the nominal shakedown pressure, $p_0{}^s$ is the asperity shakedown pressure, N is the number of asperities per unit area, R_1 is the radius of hard asperities, ψ_s is the plasticity index, h is the cut-off height of hard asperities and σ_1 is the r.m.s. roughness of hard asperities. Eq. (2) has been evaluated numerically and the resulting values of the nominal shakedown pressure are plotted against the value of ψ_s in Fig. 3 for various values of h/σ_1. The process of running-in can be interpreted by referring to Fig. 3. Initially, the softer surface has asperities with radius R_2 and r.m.s. height σ_2.

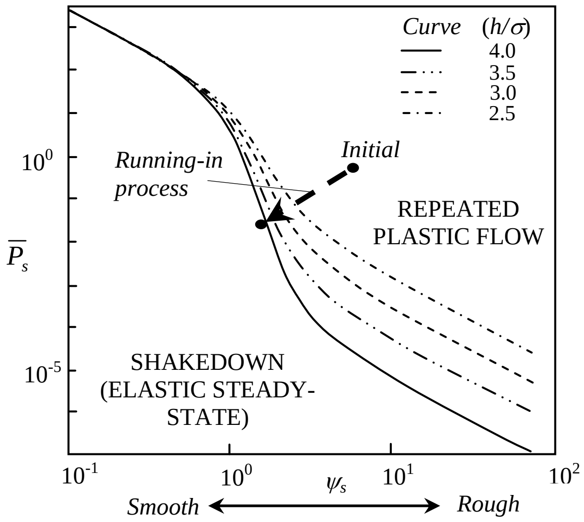

Figure 3.
Shakedown map for rough in sliding contact.

In the first sliding pass, the system can be represented by a point somewhere in Fig. 3 with coordinates ψ_s* and P_s. If the point lies below the shakedown curve, then the load is carried purely elastically without any change in the softer surface topography. However, if the point lies above the shakedown curves, plastic flow will occur during sliding and the softer asperities will deform. R_2 will increase and σ_2 will decrease such that ψ_2* will reduce. The run-in (shakedown) state will be reached if, and only if, the curve in Fig. 3 is crossed with R_2 less than infinity and σ_2 greater than 0. If $R_2 = \infty$ and $\sigma_2 = 0$ (i.e. when the soft surface has become flat and thus is capable of carrying its maximum load) the point still lies to the right of the shakedown curves, then running-in will not lead to the conditions of elastic sliding and the steady state will be one of repeated plastic deformation.

This statistical model is promising with respect to running-in, however, there is a fundamental shortcoming, the radius of the asperities is assumed to be equal, and that should be taken into consideration for the improvement of the model for real rough surfaces. Due to the fact that the change of the micro-geometry is dominant, many efforts have been made in order to study the behavior of the micro-geometry changes by applying a contact model. From the literature it can be concluded that there is no model which predicts the surface topography changes during running-in at roughness level deterministically. Recently, Jamari and his co-worker [51-56] have studied a lot the behaviour of the running-in process based on the contact mechanics (an elastic-plastic deterministic contact model). In their work, a model which predicts the process roughness of real surfaces as running-in proceeds is developed. From the above discussion it is clear that the contact model is crucial in developing such a model and therefore the asperity contact model as well as the asperity deformation model is formulated.

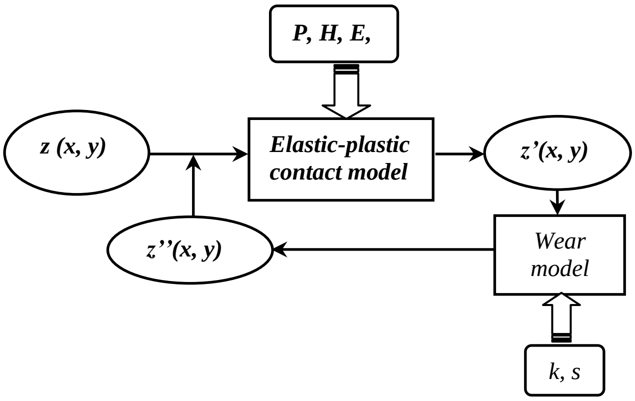

Figure 4.
Schematic illustration of the proposed running-in model as an optimization process.

Figure 4 shows schematically the proposed running-in model. In this model the initial (measured) surface geometry, *z*(*x*,*y*), will be used as an input for the elastic-plastic contact model and *z'*(*x*,*y*) and *z''*(*x*,*y*) will be the output of surface geometries after applying the elastic-plastic contact model and wear model respectively. For calculating with the elastic-plastic contact model the applied load, *P*, the material hardness, *H*, the elasticity modulus, *E*, et cetera are needed. The wear coefficient, *k*, and sliding distance, *s*, are necessary to calculate *z''*(*x*,*y*) with the wear model. *z''*(*x*,*y*) is now used as input for the elastic-plastic contact model until a near steady-state or process roughness is obtained. The surface topography changes during the running-in process for a certain number of cycles or sliding distances may then be predicted by using this model. These processes are continued and iterated until the optimum roughness of the surface is developed. Once the engineering surface is optimized the operational performance of the mechanical component will be maximized.

V. SUMMARY

Running-in is the best and effective way of matching or conforming two contacting mechanical components in a functional situation of a contact system. There are many parameter changes during this process, chemically or mechanically. However, the change of the micro-geometry at the asperity level due to plastic deformation is dominant. This process should be optimized so that it is beneficial in many aspects.

Studies on running-in have been performed for many years; however, due to the complexity of its phenomena many problems encountered and have not been solved yet. Theoretical published works with respect to running-in has been reviewed. Running-in is modeled globally or phenomenologically due to its high complexity. In this model the running-in process is qualitatively explained. A more quantitative approach is the statistical approach. This model is used widely for modeling running-in and the Gaussian distribution of the surface is always assumed. Since the most change during the running-in process is the surface topography, modeling running-in based on the contact model is the best approach. A new running-in model is proposed based on the elastic-plastic micro-contact model and the wear model. This running-in model is able to predict the change of the surface topography during running-in locally or deterministically, therefore, the running-in process as a system can be optimized.

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