Hydrodynamic Forces on Submerged Floating Tube: The Effect of Curvature Radius and Depth Level

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1. Introduction

Tubular structures are commonly used in maritime structures, such as: structural support poles [1], submarine cable lines [2], oil and gas pipelines [3, 4], mooring cables for Tension Leg Platform/TLP [5], and so on. Today, the use of this structure has developed into various fields and requires increasingly complex studies. For example, the use of this structure is being studied for application as a submerged floating tunnel bridge/SFTB [6, 7]. One important issue that needs to be discussed is the hydrodynamic force. This force occurs due to the interaction between the fluid and the tubular structure through the waves. The analysis is developed based on theoretical approaches to defining the various type of waves, including small-amplitude wave theory and finite wave theory [8, 9].

The hydrodynamic forces are affected by the KC number and the drag coefficient, \( C_d \) [10]. The KC number is the ratio of the wave motion to the cylinder diameter, while the \( C_d \) value depends on the geometry and Reynolds number. This theory has been proved analytically and numerically [11, 12]. One example of the application of the KC number is to calculate the hydrodynamic force on a pipeline on the seabed using the Wake II Model, which was adopted for the analysis of lift force, drag force, inertia force and the total hydrodynamic force. This equation can predict the hydrodynamic force accurately [13].

The hydrodynamic forces consist of two components; the drag and the inertial force, each affected by the velocity and acceleration of the water particles, as proposed by Morison, well known as Morison Equation [14, 15]. Morison Equation is effectively used to predict the hydrodynamic forces on a vertically or horizontally installed tube [16]. The numerical study of the hydrodynamic forces on a floating tube subjected to internal solitary waves also proves the accuracy of this equation [17, 18].

In certain constructions, a curved tubular structure is required for several technical reasons, such as the Submerged Floating Tunnel Bridge/SFTB [19]. One of the reasons for using a curved tubular structure is to gain flexibility during operation, especially anticipating changes in length caused by temperature differences [20]. Flexibility is needed to reduce cyclic loads that can cause fatigue failure in structures [21, 22]. The curved cylindrical construction also improves stability by increasing stiffness to reduce lateral movement. However, there is not much research on hydrodynamic forces on the curved tubular structure yet. For this reason, this article discusses the effect of the degree of curvature on the hydrodynamic forces that occur. The study was conducted to obtain a correction factor \( C_d \) to calculate the hydrodynamic force on a curved tube based on the Morison equation.
2. Methods

The research was conducted on an experimental pool, which is a modified model from our previous research [23], as shown in Fig. 1. Specimens are made of tubes with varying degrees of curvature, being assembled on a holder frame. At both ends of the specimen, the supports are equipped with load cells to measure the force received by the waves. The signal from the loadcell is read by a set of data acquisition system tools. Fig. 2 shows the design of the specimens with variations in the degree of curvature (R/L) and their sizes are shown in Table 1. The specimens are placed with varying degrees of depth (z/d).

![Figure 1. Experimental pool set-up [23]](image)

<table>
<thead>
<tr>
<th>Radius curvature, R (mm)</th>
<th>Tube length, L (mm)</th>
<th>Ratio, R/L</th>
<th>Tube Diameter, D (inch)</th>
<th>Total depth (mm)</th>
<th>Depth level, z/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>(straight)</td>
<td></td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>500</td>
<td>0.8</td>
<td>1.5 (38.1)</td>
<td>600</td>
<td>1/6</td>
</tr>
<tr>
<td>600</td>
<td></td>
<td>1.2</td>
<td>2 (50.8)</td>
<td></td>
<td>2/6</td>
</tr>
<tr>
<td>800</td>
<td></td>
<td>1.6</td>
<td>2.5 (63.5)</td>
<td>600</td>
<td>3/6</td>
</tr>
<tr>
<td>1000</td>
<td></td>
<td>2</td>
<td>3 (76.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1200</td>
<td></td>
<td>2.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Figure 2. Specimens (tubes) with variations in the degree of curvature (R/L)](image)

2.1. Kinematics of Water Particles and Hydrodynamic Forces

In this research, wave characteristics were developed based on Airy’s theory. This theory is considered the most relevant because the waves that occur are relatively small and conform to Eq. (1), where \( \eta_0 \) is the wave amplitude and \( H \) is the total wave height, as shown in Fig. 3. The motion of water particles can be determined from the potential velocity using the Laplace equation, as shown in Eq. (2) [24]. Here, the x-axis represents a horizontal direction, while the z-axis represents a vertical direction.

\[
\frac{\eta_0}{0.5} = H
\]

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0
\]
Setting the boundary conditions on the seabed (at \( z/d=1 \), \( \partial \phi/\partial z=0 \), the solution for potential velocity can be solved by the variable separation method, as shown in Eq. (3). Here, \( k \) is the wavenumber, as shown in Eq. (4).

\[
\phi = \frac{\pi H}{kT} \frac{\cosh[k(z + d)]}{\sinh(kd)} \sin(kx - \omega t) \tag{3}
\]

\[
k = \frac{2\pi}{L} \tag{4}
\]

The velocity and acceleration of water particles for various depth levels based on Airy’s theory can be seen in Eq. (5) and Eq. (6), respectively [25].

\[
u = \frac{\pi H}{T} \frac{\cosh[k(z + d)]}{\sinh(kd)} \sin \theta \tag{5}
\]

\[
u = \frac{\pi H}{T^2} \frac{\cosh[k(z + d)]}{\sinh(kd)} \cos \theta \tag{6}
\]

The hydrodynamic force is obtained from the Morison Equation, as shown in Eq. (7) where \( C_D \) is the coefficient of drag, \( C_m \) is the coefficient of inertia, \( \rho \) is water density, \( D \) is tube diameter, \( u \) and \( \ddot{u} \) is velocity and acceleration, respectively, as shown in Eq. (5) and Eq. (6).

\[
f(t) = \frac{1}{2} C_D \rho u |u| + C_m \frac{\rho \pi D^2}{4} \ddot{u} \tag{7}
\]

### 2.2. Numerical Analysis

The parameters used in the numerical analysis are the same as the specifications on the experimental equipment. The analysis stages consist of preprocessing, solution, and postprocessing. In the preprocessing stage, geometry and meshing are made. In the solution stage, boundary conditions and mechanical properties of the material are defined. The results of the analysis are presented in the Postprocessing stage.

The numerical analysis is modeled as Volume of Fluid (VoF)-Open Channel Wave BC. The meshing element size is 30 mm with a Quadrilateral/Hexahedron shape. There are four boundary conditions; Cylinder Wall (Wall), Channel Wall (Wall), Inlet (Velocity Inlet), and Outlet (Pressure Outlet), as shown in Fig. 4.
2.3. Experimental set up
The experiment was carried out in an experimental pool equipped with a set of wave generators, as shown in Fig. 5. The dimensions of the pool and the characteristics of the waves are shown in Table 2.

![Figure 5. Test equipment settings](image)

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td></td>
<td>mm</td>
<td>2000</td>
</tr>
<tr>
<td>Wide</td>
<td>d</td>
<td>mm</td>
<td>600</td>
</tr>
<tr>
<td>Depth</td>
<td>F(t)</td>
<td>mm</td>
<td>600</td>
</tr>
<tr>
<td>Wave direction</td>
<td>ω</td>
<td>Hz</td>
<td>1,55</td>
</tr>
<tr>
<td>Excitation frequency</td>
<td></td>
<td>Hz (rad/s)</td>
<td>(9,77)</td>
</tr>
<tr>
<td>Wave Height</td>
<td>H</td>
<td>mm</td>
<td>23</td>
</tr>
<tr>
<td>Wavelength</td>
<td>L</td>
<td>mm</td>
<td>558</td>
</tr>
</tbody>
</table>

3. Results And Discussion
In this section, research results will be presented and discussed, including; velocity and acceleration of water particles and their effect on hydrodynamic forces. Then discussed the effect of curvature, the effect of depth level and the effect of tube diameter on the hydrodynamic force.

3.1. Profile graph of velocity and acceleration of water particles
At the surface of the water, the velocity and acceleration of the water particles are relatively larger than those in deeper positions. These profiles are obtained from solving Equations (2) and (3), respectively. Based on this graph, at depths z=0.2 to z=-0.6 (z/d=1/3 to z/d=1), the rate of reduction of the hydrodynamic force does not change significantly. Thus, it can be recommended that the optimum placement for SFTB is at the level of z/d=1/3, as shown in Fig. 6(a) and (b).

![Figure 6. Fluid particle kinematics (a) Velocity and (b) Acceleration](image)

3.2. Verification of hydrodynamic force on straight tube
Fig. 7 shows the hydrodynamic force on a straight tube obtained experimentally and numerically. The analytical solution of the Morison Equation is also shown for comparison. Visually, all the graphs tend to have the same trend, although there are slight inaccuracies. The numerical graph is relatively more precise than the experimental one, compared to Morison’s graph. The experimental graph is relatively higher than Morison’s graph near the water surface (z/d=1/6). Otherwise, it is
lower at the deeper position (z/d=3/6). It is due to the wave generator’s closer placement to the surface and causes it unable to reach the pool’s bottom. In future research, the difference of the domain between experimental and numerical can be solved by improving the design of the wave generator to get more accurate results.

Figure 7. Hydrodynamic forces: (a) depth level z/d=1/6, (b) depth level z/d=2/6, and (c) depth level z/d=3/6

3.3. Effect of Curvature radius on Hydrodynamic Force

The hydrodynamic forces on the specimen are corrected by the curvature radius R/L, where the smaller R/L ratio given will results in the smaller amount of forces received, as shown in Fig. 8. If the Morison Equation applied to a straight tube is considered a reference, multiplying it by the correction factor (C) will make it applicable for a curved tube. The correction
factor (C) in each variation of curvature radius for the most recommended depth level (z/d=1/3) based on Fig. 6 is served in Table 3. This correction factor is calculated numerically, which is considered relatively more accurate. The value of C is calculated based on the ratio of the hydrodynamic force on the curved tube to the hydrodynamic force of the straight tube. Thus, Equation (7) can be modified to calculate the Morison force on a curved tube, as shown in Eq. (8).

\[
f(R/L) = C \left( \frac{1}{2} C_D \rho \nu |u| + C_m \frac{\rho \pi D^4}{4} \ddot{u} \right)
\]

Tabel 3. The correction factor for variations in the curvature radius at optimal placement (z/d = 1/3)

<table>
<thead>
<tr>
<th>R/L</th>
<th>Hydrodynamic Force (N)</th>
<th>Correction Factor, C</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Straight)</td>
<td>0.296</td>
<td>1</td>
</tr>
<tr>
<td>0.8</td>
<td>0.284</td>
<td>0.96</td>
</tr>
<tr>
<td>1.2</td>
<td>0.279</td>
<td>0.94</td>
</tr>
<tr>
<td>1.6</td>
<td>0.275</td>
<td>0.92</td>
</tr>
<tr>
<td>2</td>
<td>0.272</td>
<td>0.91</td>
</tr>
<tr>
<td>2.4</td>
<td>0.272</td>
<td>0.91</td>
</tr>
</tbody>
</table>

3.4. Effect of depth level on hydrodynamic forces

Fig. 9 shows the depth level’s effect on the hydrodynamic force amplitude, both experimentally and numerically. Placing the specimen at a high depth level further reduces the amplitude of the hydrodynamic force. This result is in good agreement with the Morison force equation (Equation 4), where the hydrodynamic forces are directly proportional to the velocity and acceleration of the water particles. In the kinematics equation of water particles as stated in Equation (2) and Equation (3), it can be seen that at deeper positions, the velocity and acceleration of water particles become smaller. The ratio of curvature (R/L) affects the drag coefficient (C_D) in the Morison Equation. Smaller R/L value will produce smaller C_D coefficient, thus reducing the hydrodynamic force.

![Figure 9. Effect of depth level on hydrodynamic forces](image)

3.5. Effect of tube diameter on hydrodynamic forces

Fig. 10 shows the tube diameter’s effect on the hydrodynamic forces’ amplitude, numerically. The diameter of the tube is directly proportional to the hydrodynamic forces it receives. This corresponds with the Morison Force Equation (Equation 7): the larger the diameter, the greater the drag and inertia forces received. In addition, the curvature ratio (R/L) affects the hydrodynamic forces, in agreement with those discussed in section 3.4.

![Figure 10. Effect of tube diameter on hydrodynamic forces](image)
4. Conclusion

The hydrodynamic forces are influenced by depth level, curvature radius, and tube diameter. The hydrodynamic forces are the greatest near the water surface (z/d=0) and will gradually decrease at a deeper position until the bottom of the pool (z/d=1). The rate of reduction of the hydrodynamic forces is not linear but satisfies a hyperbolic function. At the depth level z/d=1/3 to z/d=1, the hydrodynamic forces are not significantly reduced. Thus, we recommend z/d=1/3 is optimal for SFTB placement. In addition, the smaller the curvature radius (R/L), the smaller the hydrodynamic force. A correction factor, C, has been determined to calculate the hydrodynamic force on a curved tube based on the Morison equation which applies to a straight tube.

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References


