APPLICATION OF MODEL PREDICTIVE CONTROL TO REDUCE THE VIBRATION OF TWO CONNECTED BUILDINGS

Sutrisno[1] and Dita Anies M.[2]
Department of Mathematics, Diponegoro University,
[1]tresno.math@undip.ac.id, tresnowijoyo@gmail.com, [2]dita.anies.m@gmail.com

Abstract. In this paper, model predictive control (MPC) will be applied to reduce the vibration of two connected buildings (TCB) that can be caused by, for example, seismic activities. Mathematical model of TCB can be formulated as a linear time invariant state space equation. The state of this plant is the displacement of each building measured from the origin. We will regulate or bring the state to the origin as fast as possible with optimal control input. This optimal control input will be determined using MPC. We give a numerical simulation to observe how this control method is working. From the simulation result, MPC reduces the vibration of TCB and bring the displacement of each building to the origin faster than uncontrolled system.

Key words and Phrases: model predictive control, vibration control, two connected buildings.

1. INTRODUCTION

Buildings are susceptible to be damaged if some seismic activities attack them. To prevent it, we can apply some controller to reduce the vibration of the buildings by regulating or bringing the displacement of each building to the origin. Some building structures have two buildings connected by a bridge, said two connected buildings (TCB). The mathematical model representing the vibration (displacement) of TCB can be written as a linear time invariant state space equation appeared in [1].

Several control methods have been developed such as linear quadratic regulator, H_2 and H_∞ based on linear matrix inequality (LMI) that were applied to design an active vibration controller that were given by [1, 2, 3, 4, 5, 6]. One of several optimal control method alternatives is model predictive control (MPC) which is famous for practitioners or researchers. Model predictive control (MPC) is an optimization based control method and it is applicable for regulating and tracking problems as mentioned by [7]. MPC predicts the state and input variables along horizon prediction, substituting these predictions into some objective function and solving it using optimization method to determine the optimal control action. MPC control method can be used to control a discrete state space system with controllability and observability characteristic. The studies about this state space were appeared in [8] including it's controllability and observability characteristic. To simulate the system's response, we can used MPC toolbox for MATLAB that was appeared in [9].

In this paper, we apply MPC to reduce the vibration of two connected buildings that can be caused by some seismic activities. We discretise the continuous model of this plant, design the controller using MPC and simulate it using MATLAB.

2. RESULT

2.1 Mathematical Model of Two Connected Buildings

Dynamical model of two connected buildings will be formulated using basic dynamical model of mass-spring-damper. Consider a mass-spring-damper system given by Figure (1) which has one degree of freedom (d.o.f). Let m be the mass of the plant (kg), k be the spring constant (N/m) and c be the damper coefficient (Ns/m). Let x denote the displacement of the mass measured from the origin and F be the external force. By using Newton's laws, the differential equation of this system is follows which is linear differential equation with two order that was appeared in [1],

$$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + kx(t) = F. \qquad (1)$$

Furthermore, consider two buildings connected by a bridge given by Figure (2). This system contains two buildings said building-A and building-B which were built using base isolation at the bottom as the mass-spring-damper systems. Each building has two d.o.f in the x and y direction. Let x_a and y_a denote the displacements of building-A in the x-direction and y-direction respectively. Similarly, let x_b and y_b denote the displacements of building-B in the x-direction and y-direction respectively.

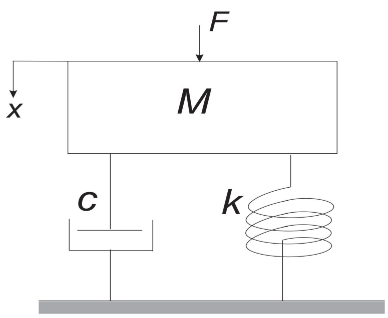

Figure 1. Mass-spring-damper system

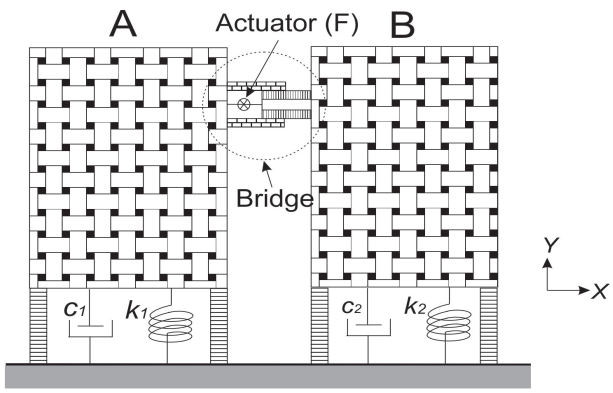

Figure 2. Two connected buildings

Then, the differential equation for these buildings are

$$m_1\frac{d^2x_a(t)}{dt^2} + c_1\frac{dx_a(t)}{dt} + k_1x_a(t) = -F, \qquad (2)$$

$$m_2\frac{d^2x_b(t)}{dt^2} + c_2\frac{dx_b(t)}{dt} + k_2x_b(t) = F. \qquad (3)$$

For simplicity, let $F = u$ is the actuator force. Then (2)-(3) can be rewritten as a linear time invariant state space equation as follows

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}A_1 & 0\\ 0 & A_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}B_1\\ B_2\end{bmatrix}u, \quad (4)$$

$$y(k) = Cx(k) \quad (5)$$

where

$$x = [x_1, x_2]^T,$$
$$x_1 = [\dot{x}_a, \dot{y}_a, x_a, y_a]^T,$$
$$x_2 = [\dot{x}_b, \dot{y}_b, x_b, y_b]^T,$$
$$y = [x_a, y_a, x_b, y_b]^T,$$

$$A_1 = \begin{bmatrix}-\frac{c_1}{m_1} & 0 & -\frac{k_1}{m_1} & 0\\ 0 & -\frac{c_1}{m_1} & 0 & -\frac{k_1}{m_1}\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix},$$

$$A_2 = \begin{bmatrix}-\frac{c_2}{m_2} & 0 & -\frac{k_2}{m_2} & 0\\ 0 & -\frac{c_2}{m_2} & 0 & -\frac{k_2}{m_2}\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix},$$

$$B_1 = \begin{bmatrix}-f\\ 0\\ 0\\ 0\end{bmatrix}, B_2 = \begin{bmatrix}f\\ 0\\ 0\\ 0\end{bmatrix},$$

$$C = \begin{bmatrix}0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\end{bmatrix}.$$

Since we apply discrete time MPC to control this system, we discretise (4)-(5) using MATLAB function *c2d* in the numerical simulation using time step 1 second.

2.2 Model Predictive Control (MPC)

Model predictive control (MPC) is an optimization based control method that it can be applied for controlling of linear discrete system for regulating and tracking problems. Reducing the vibration of TCB is a regulating control problem, then MPC regulator is applicable to control this plant. By following Maciejowski [7], let k denotes the time instant. Consider a discrete linear time invariant system of the form

$$x(k+1) = Ax(k) + Bu(k), \quad x(0) = x_0, \quad (6)$$

$$y(k) = Cx(k), \quad (7)$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^p$ is the input vector, $y(k) \in \mathbb{R}^q$ is the output vector, $x(0)$ is the initial state and A, B and C are real constant matrices with dimension $n \times n$, $n \times p$ and $q \times n$ respectively. Assume that pair (A, B) is controllable.

Let H_p be the horizon prediction length. MPC for regulator can be illustrated by Figure (3). For regulator case, MPC brings the state to the origin by predicting the state and input vector along horizon prediction and minimizing the objective function of the quadratic form of the state and input vectors. At time instant k, let the cost function of MPC has the form

$$J(k) = \sum_{k=0}^{H_p-1} x(k)^T Qx(k) + u(k)^T Ru(k) \quad (8)$$

where Q and R are real symmetric matrices which have properties positive semi-definite and positive definite respectively. Let the constraint has the form $u_{\min} \leq u(k) \leq u_{\max}$. Let $[\hat{x}(k+1), \hat{x}(k+2), ..., \hat{x}(k+H_p)]$ be the state prediction and $[\Delta\hat{u}(k), \Delta\hat{u}(k+1), ..., \Delta\hat{u}(k+H_p-1)]$ be the input prediction where $\Delta\hat{u}(k) = \hat{u}(k) - \hat{u}(k-1)$, by substituting them into (8) and using some algebraic manipulation, (8) can be rewritten as the following constrained quadratic optimization

$$\min_{\Delta\mathcal{U}(k)} J(k) = -\Delta\mathcal{U}(k)^T\mathcal{G} + \Delta\mathcal{U}(k)^T\mathcal{H}\Delta\mathcal{U}(k) \quad (9)$$

subject to :

$$\mathbb{G}\Theta\Delta\mathcal{U}(k) \leq -\mathbb{G}\left[\Psi x(0) + \Upsilon u(k-1)\right] - g$$

where $\mathbb{G}, \Theta, \Psi, \Upsilon,$ and g are real constant matrices with appropriate dimensions. Optimization (9) can be solved using active-set algorithm that was embedded in MATLAB function *quadprog*.

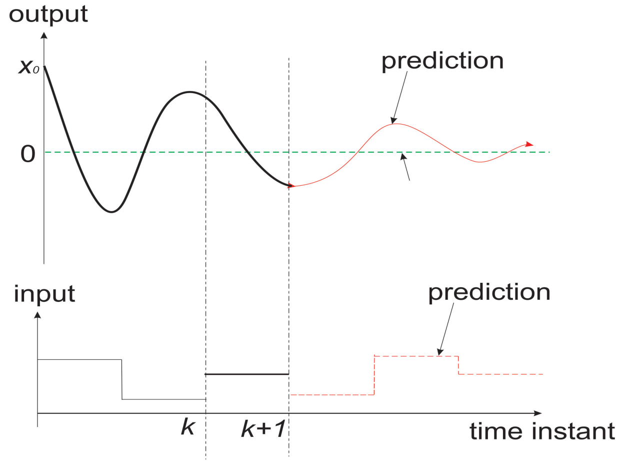

FIGURE 3. MPC principle

TABLE 1. Parameter values for numerical simulation

$m_1[ton]$	$m_2[ton]$	$k_1[N/m]$	$k_2[N/m]$	$c_1[Ns/m]$	$c_2[Ns/m]$
38531	24098	7.62×10^6	3.32×10^6	1.08×10^6	4.29×10^6

2.3 Simulation Results

We simulate TCB system illustrated by Figure (2) using MPC controller. Parameter values for this plant is summarized by Table (1) that was taken from Preumont and Seto [1]. The matrix weightings for objective function (8) are $Q = I$ and $R = I$ where I is identity matrix. Then the matrices of (4) are given by

$$A_1 = \begin{bmatrix} -0.0280 & 0 & -0.1978 & 0 \\ 0 & -0.0280 & 0 & -0.1978 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.1780 & 0 & -13.7771 & 0 \\ 0 & -0.1780 & 0 & -13.7771 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

and matrix C is unchanged. We discretise (4) using MATLAB function *c2d* with sampling time 1 second then the above matrices can be replaced using the following matrices

$$A_1 = \begin{bmatrix} 0.8769 & 0 & -0.1887 & 0 \\ 0 & 0.8769 & 0 & -0.1887 \\ 0.9549 & 0 & 0.9036 & 0 \\ 0 & 0.9539 & 0 & 0.9036 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.7588 & 0 & 1.8303 & 0 \\ 0 & -0.7588 & 0 & 1.8303 \\ -0.1329 & 0 & -0.7825 & 0 \\ 0 & -0.1329 & 0 & -0.7825 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.9539 \\ 0 \\ -0.4873 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} -0.1329 \\ 0 \\ 0.1294 \\ 0 \end{bmatrix},$$

and the pair $\left(\begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}\right)$ is controllable. The length of the horizon prediction is $H_p = 10$. The simulation time is 150s. Finally the control action constraint is $-2 \leq u(k) \leq 2$. The initial state for this plant is $x_1(0) = x_2(0) = [0, 0, 0, 0]^T$. The simulation results are given by Figures (4)-(8).

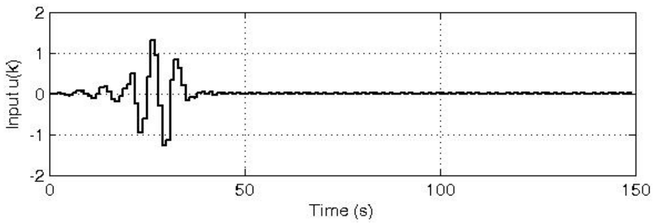

FIGURE 4. Optimal Input Generated By MPC

Figure (4) shows the evolution of the optimal control action generated by MPC. This control action implies the system's responses given by Figure (5,6) for building-A and Figure (7,8) for building-B. From Figure (5) and (6), the controller reduces the vibration of the building-A and bring the state to the origin faster than uncontrolled system. From Figure (7) and (8), similar to building-A, the controller reduces the vibration of the building-B and bring the state to the origin faster than uncontrolled system.

3. CONCLUDING REMARKS AND FUTURE WORKS

In this paper, the application of MPC to reduce the vibration of TCB was considered. MPC was applied to determine the optimal control action of TCB in order to reduce and bring the displacements of TCB to the origin as fast as possible. Numerical simulation was given to observe how this control method is working. From the simulation result, the controller was reduced

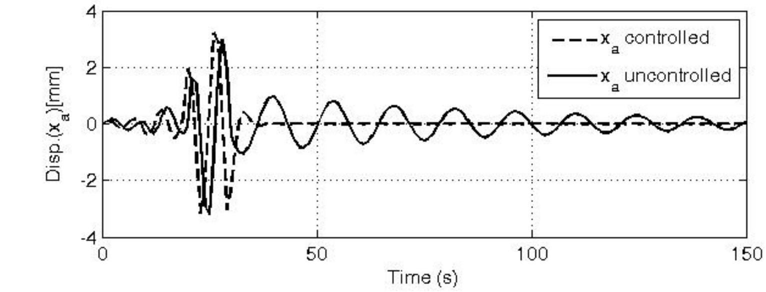

FIGURE 5. Displacement of building-A in the x-direction

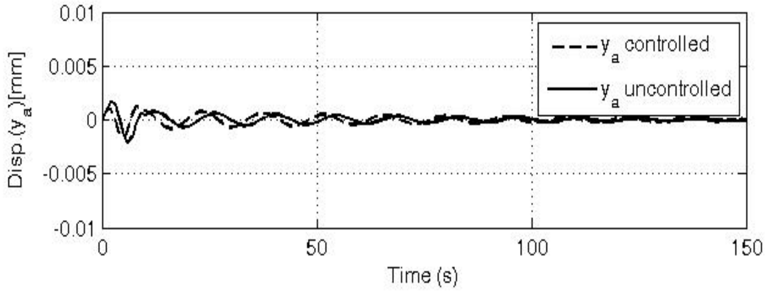

FIGURE 6. Displacement of building-A in the y-direction

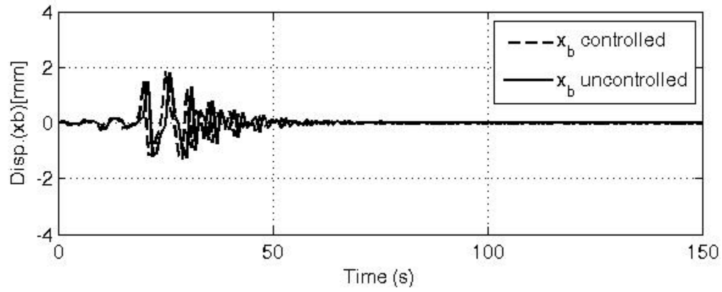

FIGURE 7. Displacement of building-B in the x-direction

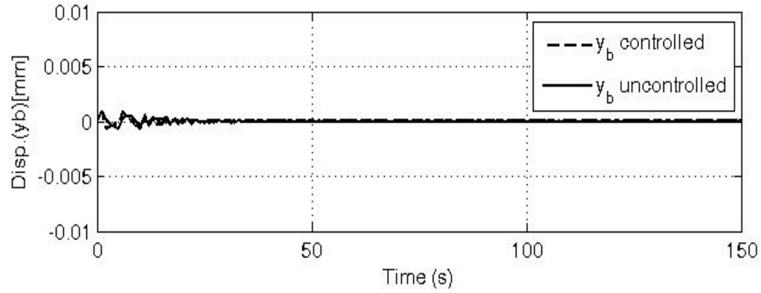

FIGURE 8. Displacement of building-B in the y-direction

the vibration of the buildings and the displacements of the buildings for controlled system was reached the origin faster than uncontrolled system.

In the future works, we will compare our results to the existing results to observe how the performance the controller. And also, we will apply this control method to control the multiconnected buildings that contain three or more connected buildings.

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