VALUATION OF LONG TERM CARE (LTC) HEALTH INSURANCE CONTRACT USING MULTISTATE MODEL

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Abstract. In this paper, we are using multistate model to evaluate Long Term Care (LTC) health insurance contract. Some products in LTC contract such as stand alone Annuity, LTC as a rider benefit, Enhanced pension and Insurance package are discussed. We use Discrete Time Markov Chains (DTMC) to model health status of each of the patients. Every time, a patient is exposed to one of the status that already set in the status space, for example: health, ill and death. We also consider evaluating premiums and reserves generated by this model which also useful for the practioneers and insurance companies.

Keyword*:* Long Term Care, Markov chain, Multistate modeling.

I. INTRODUCTION

The aim of this paper is to give a method for calculating premium and reserve for a health insurance contracts called Long Term care (LTC) and also other insurance contracts. First, we simply focus on some scientific (and practical) contributions which, in our opinion, can be considered as a platform of this research. Secondly, we will concentrate our attention on applying the model on the real data.

Survival models have played a central role throughout the whole history of life insurance and pension plans management. Although in many insurance products the investment component is of great importance, in the last decades concern for mortality issues has grown because of mortality trends experienced in many countries. Recent models for mortality projections and research work dealing with the uncertainty in future trends and the relevant actuarial evidence clearly show this interest. For this reason, the present paper places a special emphasis on some issues concerning survival modeling in a dynamic context.

Daw (1979) addresses the origins of multiple decrement modeling. Starting from the pioneering work of Daniel Bernoulli, in which the effects of inoculation against smallpox are analyzed, the paper illustrates the early contributions concerning the construction of multiple decrement tables and related single decrement tables. The paper by Seal (1977) presents the history of multiple decrement models, from Bernoulli's work up to the contributions of the 1960's. Hald (1987) specifically deals with the early actuarial models, proposed in the latter half of the 17th century for the evaluation of annuities on lives. The paper by Kopf (1926) deals with the early history of life annuities, focusing in particular on seminal contributions to the relevant actuarial structure. The early attempts to consider mortality trends and the relevant actuarial evidence are described by Cramér and Wold (1935). This paper also presents and discusses early projection models. The book by Smith and Keyfitz (1977) collects important contributions to the mathematical development of demography; several contributions, especially in the fields of life tables and parametric curve fitting, are of great interest to actuaries, being related to the development of life insurance mathematics.

II. MULTISTATE MODEL

Consider we have a stochastic process $\{X_t,\ t \in T\}$ where $t \in T$ can be interpreted as the time set. In this case $\{X_t,\ t \in T\}$ is the state space, where all possible status of a patient at time t is recorded. We assumed the process $\{X_t,\ t \in T\}$ is following the Markov property, so we can say that this process is a Markov process. We define transition probability as follow:

Definition 2.1

Given initial state s_0, the conditional transition probability from state s_n at time n to state s_{n+1} at time n+1 is defined as:

$$p_{ij}(n) = P(X_{n+1} = s_{n+1} = j \mid X_n = s_n = i)$$

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This probability plays central role in this paper, as its transition probability matrix $P = \left[p_{ij}(n) \right]$ is the foundation of premium and also reserve calculations. We will give the following case as an example:

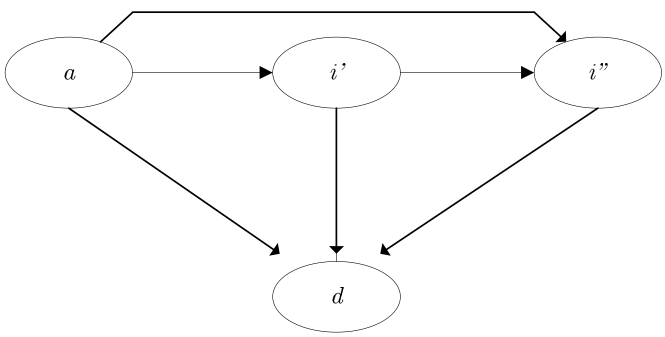

Figure 2.1 A multistate model of a person covered by a health insurance contract

Consider a person who joins a health insurance contract has 3 possibilities of state: healthy (a), death (d), illness first stadium (i') and illness final stadium (i''). The interactions between these states are described at figure 2.1. In this case, at time n, this person has special transition probability:

$$P(n) = \begin{bmatrix} p_{aa}(n) & p_{ai'}(n) & p_{ai''}(n) & p_{ad}(n) \\ 0 & p_{i'i'}(n) & p_{i'i''}(n) & p_{id}(n) \\ 0 & 0 & p_{i''i''}(n) & p_{i''d}(n) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that the lower triangle of the matrix is containing zeros because there are no reverse transitions from higher states to the lower ones. In other words, there is no chance for any patient to recover when he already enter the state.

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III. BENEFITS AND PREMIUMS

As the next step in describing the insurance contract, benefits and premiums must be introduced. First we consider the following figure:

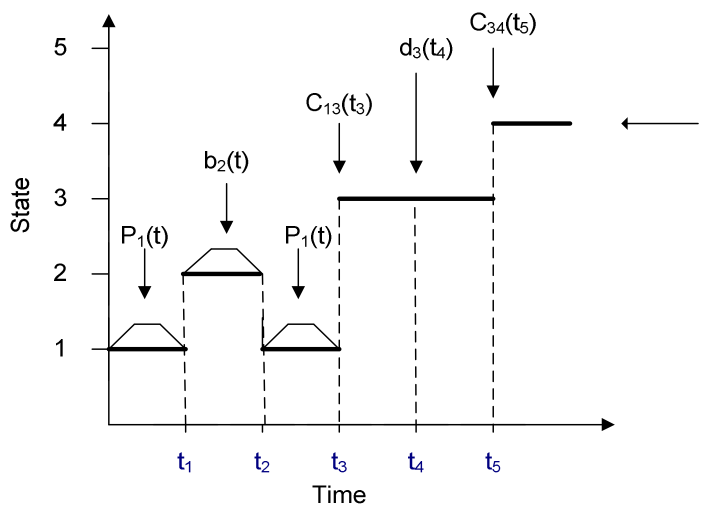

Figure 3.1 A sample path of $S(t)$, benefits and premiums

Some terminologies use in that picture including, $s(t)$ the sample path, $p_1(t)$ the premium paid by the insured while the risk is in state 1, $b_2(t)$ the annuity paid by the insured while the risk is in state 2, $c_{13}(t_3)$ lump sump paid by the insurer at time t_3 because a transition from state 1 to state 3 occurs, $c_{34}(t_5)$ lump sump paid by the insurer at time t_4 because a transition from state 3 to state 4 occurs and $d_3(t_4)$ lump sump paid by the insurer at fixed time t_4 because the risk in state 3.

The premium under this contract can be calculated as follow:

$$B_a^{LTC}(0,\infty) = b'\sum_{j=1}^{\infty} v^j \ {}_{j-1}p_{x+t}^{aa}\ p_{x+t+j-1}^{ai'}\ \ddot{a}_{x+t+j}^{i'} + b''\sum_{j=1}^{\infty} v^j \ {}_{j-1}p_{x+t}^{aa}\ p_{x+t+j-1}^{ai''}\ \ddot{a}_{x+t+j}^{i''} + b''\sum_{j=2}^{\infty} v^j \ {}_{j-1}p_{x+t}^{ai'}\ p_{x+t+j-1}^{i'i''}\ddot{a}_{x+t+j}^{i''} \quad (3.1)$$

$$B_{i'}^{LTC}(0,\infty) = b'\ddot{a}_{x+t}^{i'} + b''\sum_{j=1}^{\infty} v^j \ {}_{j-1}p_{x+t}^{i'i'}\ p_{x+t+j-1}^{i'i''}\ddot{a}_{x+t+j}^{i''} \quad (3.2)$$

$$B_{i''}^{LTC}(0,\infty) = b'\ddot{a}_{x+t}^{i''} \quad (3.3)$$

The reserve under this contract can be calculated as follow:

$$V_t^{\,a} = B_a^{LTC}(t,\infty) - p\,\ddot{a}_{x+t:m-t}^{aa} \text{ if } t < m$$
$$V_t^{\,a} = B_a^{LTC}(t,\infty), \text{ if } t \geq m$$

IV. EXAMPLE

In this example, we present basic calculation of premium and reserve for a health insurance contract base on one specific disease. The contract is annuity as a rider benefit and the disease is heart problems including angina, myocardial infarction, heart murmur, abnormal heart rhythm, stroke and other heart problems. We get data for this example from National Centre for Social Research, Department of Epidemiology and Public Health at the Royal Free and University College Medical School. Commisioned by Department of Health (2004). We also use 1979-82 UK PML80 Male mortality tables for transition to death state.

For annuity as a rider benefit contract, the benefit is not only given to the insured while he was ill but also when he died. In this contract, the duration of benefit payment is limited maximum r years as long as he still under medication. Thus, the level benefit payment is $b = \frac{c}{r}$ each year where c is the death benefit. To construct the probability of an insured get heart problem, we define the following:

$$p_x^{ai} = p_x^{ai}(\text{Angina}) + p_x^{ai}(\text{myocardial infarction}) + p_x^{ai}(\text{he}$$
$$p_x^{ai}(\text{abnormal heart rhythm}) + p_x^{ai}(\text{other heart prob}$$
$$(4.1)$$

For simplicity, we only assume that every time an insured only belong to one of three states, active (a), ill (i) and death (d). Thus the one-step transition probability matrix for this case is as follows:

$$P_x^{ij} = \begin{bmatrix} p_x^{aa} & p_x^{ai} & q_x^a \\ 0 & p_x^{ii} & q_x^i \\ 0 & 0 & 1 \end{bmatrix} \quad (4.2)$$

To fill this matrix out, we need some assumptions:

1. The value of p_x^{ai} is come from Health Survey for England 2003
2. The value of q_x^a is come from use 1979-82 UK PML80 Male mortality table
3. $p_x^{aa} = 1 - p_x^{ai} - q_x^a$
4. There exist k where $q_x^i = (1+k)q_x^a,\ k > 0$
5. $p_x^{ii} = 1 - q_x^i$

Furthermore, in order to compute the next h year transition probability matrix, we use Markov chain properties:

$${}_hP_y^{ij} = \begin{vmatrix} p_y^{aa} & p_y^{ai'} & p_y^{ai''} & q_y^a \\ 0 & p_y^{i'i'} & p_y^{i'i''} & q_y^{i'} \\ 0 & 0 & p_y^{i''i''} & q_y^{i''} \\ 0 & 0 & 0 & 1 \end{vmatrix} \bullet \begin{vmatrix} p_{y+1}^{aa} & p_{y+1}^{ai'} & p_{y+1}^{ai''} & q_{y+1}^a \\ 0 & p_{y+1}^{i'i'} & p_{y+1}^{i'i''} & q_{y+1}^{i'} \\ 0 & 0 & p_{y+1}^{i''i''} & q_{y+1}^{i''} \\ 0 & 0 & 0 & 1 \end{vmatrix} \cdots \begin{vmatrix} p \end{vmatrix}$$

For k = 0.3, we have the following one-step transition probability graph for insured age 20 until 100 years old:

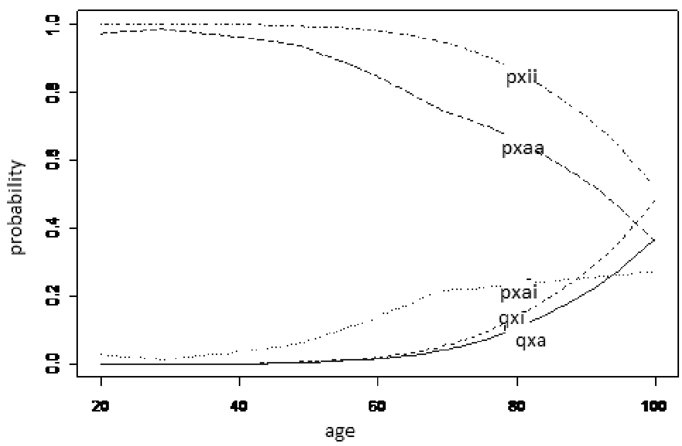

Figure 4.1 Graph of one-step transition probability for insured age 20 until 100 years old, k = 0.3.

Now let us try the model to the specific case. Consider a 30-years-old man join this annuity as a rider benefit contract. The protections are 250 million rupiahs for death and 7 years maximum hospital care. The level premiums are set for 5 years immediately after the deal. These premiums are paid at the beginning of each year as long as this man is healthy. If the effective interest rate is 6% p.a. calculate the rate of level premium and the reserve after 3 years! We have the level premium for this man and the 3rd annual reserve are, respectively, Rp. 26.297.074,- and Rp. 67.271.749,-.

V. CONCLUSIONS AND ACKNOWLEDGEMENTS

This paper gives an alternative way calculating net level premium and reserve for health insurance contract. We present only annuity as a rider benefit contract as an example. We find that Markov process can be used for modeling insured's multistate phenomenon while some assumptions are needed to accomplish the calculation. Special thank to our student, Purwaningtyas, for her help during this research.

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