
**VARIANCE GAMMA PROCESS WITH MONTE CARLO SIMULATION AND
CLOSED FORM APPROACH FOR EUROPEAN CALL OPTION PRICE
DETERMINATION**

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Abstract: The Option is widely applied in the financial sector. The Black-Scholes-Merton model is often used in calculating option prices on a stock price movement. The model uses geometric Brownian motion which assumes that the data is normally distributed. However, in reality, stock price movements can cause sharp spikes in data, resulting in nonnormal data distribution. So we need a stock price model that is not normally distributed. One of the fastest growing stock price models today is the *L*évy process exponential model. The *L*évy process has the ability to model data that has excess kurtosis and a longer tail (heavy tail) compared to the normal distribution. One of the members of the *L*évy process is the Variance Gamma (VG) process. The VG process has three parameters which each of them, to control volatility, kurtosis and skewness. In this research, the secondary data samples of options and stocks of two companies were used, namely zoom video communications, Inc. (ZM) and Nokia Corporation (NOK). The price of call options is determined by using closed form equations and Monte Carlo simulation. The Simulation was carried out for various *N* values until convergent result was obtained.

1. INTRODUCTION

A well-known option price model is the Black-Scholes model. This model was developed by Fisher Black and Myron Scholes in 1973 to determine the price of European-type options assuming no dividend payments, no transaction costs, constant risk-free interest rates, and changes in stock prices following a random pattern (Hull, 2002). The Black-Scholes model assumes that the volatility of asset returns is constant and that the asset's log returns are normally distributed. The existence of high volatility is one of the causes of the assumption of normality not being met, because it has excess kurtosis and a longer tail (heavy tail) compared to the normal distribution. Abdurakhman & Maruddani (2018) have conducted research using the Black-Scholes model with equations related to the third and fourth moments, namely skewness and kurtosis. Therefore, a model that is able to control skewness and kurtosis is needed, namely the VG model. Seneta & Madan (1990), Madan & Milne (1991) have argued a Variance Gamma (VG) approach which has

the advantage of addiarameters to the log returns distribution to control volatility and kurtosis in the log returns distribution.

Then, this VG model was generalized by Madan et al. (1998) by developing a three-parameter VG process, namely the addition of a parameter that controls skewness. Several research on options have been carried out such as in Daal & Madan (2005). Permana et al. (2014) which concluded that the VG model gives better results than the Black-Scholes model. Another research that has been submitted by Finlay & Seneta (2006), in which the VG distribution is an excellent model for handling financial data. Hoyyi et al. (2021) have conducted a research on stock price prediction which gives the result that the VG model is better than the Geometric Brownian Motion model. The computational procedure for calculating option prices has been introduced by Avramidis et al. (2003), Fu (2007). The closed form is presented by Ivanov (2018), Ivanov & Ano (2016) which is the development of the form presented by Madan et al. (1998).

2. LITERATURE REVIEW

2.1. Brownian Motion

According to Shreve et al. (1997), a stochastic process $\{W_t\}$ is called Brownian motion or Wiener process if it fulfills the following properties:

- a. $W_0 = 0$,
- b. W_t is continuous function in t
- c. If $0 = t_0 < t_1 < t_2 < \dots < t_n$ and defined increments from, W_t and $Y_1 = W_{t_1} - W_{t_0}$, $Y_2 = W_{t_2} - W_{t_1}$, ..., $Y_n = W_{t_n} - W_{t_{n-1}}$

Then,

1. Y_1, Y_2, \dots, Y_n is independent,
2. $E(Y_j) = 0 \quad \forall j$,
3. $\text{var}(Y_j) = t_j - t_{j-1} \quad \forall j$.

If given a Brownian motion model with a drift term, $B(t) = \mu - \frac{1}{2}\sigma^2 + \sigma W(t); t \geq 0$ with parameters *drift* $\mu - \frac{1}{2}\sigma^2$, variance parameter σ^2 , dan W_t is a Brownian motion process that starts at $W_0 = 0$. On the movement of total assets, the stochastic process $\{V(t); t \geq 0\}$ is called geometric Brownian motion if $R(t) = \ln \frac{V_t}{V_{t-1}}$, with $R(t)$ is the asset's log returns at time t (Dmouj, 2004).

2.2. Geometric Brownian Motion Stock Price Model.

According to Reddy & Clinton (2016) Geometric Brownian Motion is a derivative of the Brownian Motion process which is used as a method to simulate stock prices based on stock returns. The Geometric Brownian Motion model will be effectively applied if the company or agency is in a good and stable condition, the stock price of the company or agency is continuous in time, and the stock returns value is normally distributed. The Geometric Brownian Motion model has two parameters, the first parameter is μ which is the expected value of stock returns, the second parameter σ is the volatility of stock prices.

According to Brigo et al. (2011) Geometric Brownian Motion model is determined as follows:

$$dS(t) = \mu S(t) dt + \sigma S(t) dW(t)$$

where the stock price is denoted by S and time is denoted by t . W is Standard Brownian Motion with Normal distribution with mean 0 and variance equal to $t_j - t_{j-1}$, μ is the expected value of stock returns, and σ is stock price volatility. The solution of the Stochastic Differential Equation to obtain a Geometric Brownian Motion stock price model can be obtained through the $It\hat{o}$ theorem.

If there is an equation:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t)$$

Then according to $It\hat{o}$ theorem, the function $G = G(s, t)$ is as follows:

$$dG = \left(\frac{\partial G}{\partial S(t)} \mu S(t) + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial S(t)^2} \sigma^2 S(t)^2 \right) dt + \frac{\partial G}{\partial S(t)} \sigma S(t) dW(t)$$

e.g. The function of $G = \ln S(t)$, under the condition $\frac{\partial G}{\partial S(t)} = \frac{1}{S(t)}$, $\frac{\partial^2 G}{\partial S(t)^2} = -\frac{1}{S(t)^2}$, $\frac{\partial G}{\partial t} = 0$ so that,

$$dG = \left(\mu S(t) \frac{1}{S(t)} + 0 + \frac{1}{2} \sigma^2 S(t)^2 \left(-\frac{1}{S(t)^2} \right) \right) dt + \sigma \frac{1}{S(t)} S(t) dW(t)$$

$$dG = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma dW(t)$$

Then by integrating both sides from 0 to t , we get:

$$\int_0^t dG = \int_0^t \left(\mu - \frac{\sigma^2}{2} \right) dt + \int_0^t \sigma dW(t)$$

$$\Leftrightarrow \ln(S(t)) - \ln(S(0)) = \left(\mu - \frac{\sigma^2}{2} \right) t + \sigma (W(t) - W(0))$$

$$\Leftrightarrow \ln \frac{S(t)}{S(0)} = \left(\mu - \frac{\sigma^2}{2} \right) t + \sigma (W(t) - W(0))$$

$$\Leftrightarrow \frac{S(t)}{S(0)} = \exp \left(\left(\mu - \frac{\sigma^2}{2} \right) t + \sigma (W(t) - W(0)) \right)$$

$$\Leftrightarrow S(t) = S(0) \exp \left(\left(\mu - \frac{\sigma^2}{2} \right) t + \sigma (W(t) - W(0)) \right)$$

$$\Leftrightarrow S(t) = S(0) \exp \left(\left(\mu - \frac{\sigma^2}{2} \right) t + \sigma W(t) \right)$$

To simulate this process, the discrete-time continuous model $t_0 < t_1 < \dots < t_n$ is solved as follows:

$$S(t_{i+1}) = S(t_i) \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)(t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i}Z_{i+1}\right)$$

Where Z_1, Z_2, \dots, Z_n is generated randomly independent of the standard normal distribution.

2.3. Variance Gamma Process

According to Shreve et al. (1997) the stock price at time t is $S(t)$ following the geometric Brownian motion expressed in the equation:

$$S(t) = S(0) \cdot \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W(t)\right]$$

According to (Madan et al., 1998) Variance Gamma (VG) process was obtained by evaluating Brownian motion (with constant drift and volatility) at random time changes given by the Gamma process.

$$b(t; \theta, \sigma) = \theta t + \sigma W(t)$$

VG process is defined on Brownian motion with drift $b(t; \theta, \sigma)$ and Gamma process with mean rate unit, $\gamma(t; 1, v)$ is:

$$X_{VG} = b(\gamma(t; 1, v); \theta, \sigma)$$

In other words, the VG process can be obtained from Brownian motion by replacing the random variable time t with the Gamma process γ .

The stock price model that follows the VG process is

$$S(t) = S(0) \cdot \exp[(\omega + r)t + X_{VG}(t)] \quad (1)$$

According to Avramidis & L'Ecuyer (2006) if there is dividend q , model (1) becomes

$$S(t) = S(0) \cdot \exp[(\omega + r - q)t + X_{VG}(t)]$$

where:

$$\omega = \frac{1}{v} \ln\left(1 - \theta v - \frac{1}{2}\sigma^2 v\right)$$

r : risk free interest rate

σ : the volatility of the brown motion which controls the volatility

v : variance of gamma time change to control kurtosis

θ : drift on Brownian motion to control skewness

4.2. Parameter Estimation

One method of estimating the Variance Gamma parameter is the moment method. This method is easy to do and has a closed form. According to Madan et al. (1998), $X(t)$ at time interval t is a random variable VG normally distributed with mean θg and variance $\sigma\sqrt{g}$ written as follows:

$$X_{VG}(t) = \theta g + \sigma\sqrt{g}z \quad (2)$$

Where z is a standard normal independent of the gamma distribution g .

The first step taken to estimate the VG parameter is to determine the first four moments (m) of $X(t)$ as follows:

$$m_1 = \theta t$$

$$m_2 = (\theta^2 v + \sigma^2)t$$

$$m_3 = (2\theta^3 v^2 + 3\sigma^2 \theta v)t$$

$$m_4 = (3\sigma^4 v + 12\sigma^2 \theta^2 v^2 + 6\theta^4 v^3)t + (3\sigma^4 + 6\sigma^2 \theta^2 v + 3\theta^2 v^2)t^2$$

The proof of this four moment values can be read in (Madan et al., 1998).

According to t Seneta (2004), for value $\theta^2 \approx \theta^3 \approx \theta^4 \approx 0$, then the Variance Gamma parameters are estimated as follows: $\hat{\sigma} = \sqrt{\text{Var}(X)}$, $\hat{\theta} = \frac{\sigma \text{Skewness}(X)}{3v}$ and $\hat{v} = \frac{\text{Kurtosis}(X)}{3} - 1$.

Another approach to estimate VG parameters is done with maximum likelihood estimation (MLE) as written by Fragiadakis et al. (2013).

Numerical methods have played an increasingly important role in financial mathematics. This is due to the fact that most financial models have analytical solutions in only a few special cases. The Monte Carlo method is often used when analytical solutions are difficult to implement due to the complexity of the problem (Avramidis & L'Ecuyer, 2006).

Option pricing depends on the path based on the VG model. The payoff depends on the value of the process on a finite number of observations, Let's say $0 = t_0 < t_1 < \dots < t_d = T$, where T is the expiration date. According to Madan et al. (1998), European call options price $C(S(0), K, t)$ can be written with a familiar expression as follows

$$C(S(0), K, t) = e^{-rt} E[\max(S(t) - K, 0)]$$

where the expectation is obtained based on the risk-free process equation (1).

According to Hull (2002) There are six factors that affect the price of stock options, that is: the current stock price, S_0 (current stock price), deal price, K (strike price), expired date, T (time to expiration), stock price volatility, σ (volatility), risk free interest rate, r (risk-free interest rate) and dividends expected to be paid. The VG density function is very complex because it involves the Gamma process and the Bessel function, resulting in the determination of the call option price also requiring a fairly complicated process. Closed form European call option prices have been discovered by Madan et al. (1998). In this study, two approaches were used to estimate the value of the call option, namely the Monte Carlo simulation and the closed form. The following is a Monte Carlo simulation algorithm for N to calculate European call options without dividends using the VG model,

a. Monte Carlo simulation of the VG process as a change in Brownian motion to Gamma time

Input:

$$N, S_0, K, r, T$$

Parameter VG : σ, θ and v

$$\omega = \frac{1}{v} \ln(1 - \theta v - 0,5\sigma^2 v)$$

Loop : $i = 1$ to N :

1. Generate $\Delta g_i \sim \Gamma\left(\frac{\Delta t_i}{v}, v\right)$, $Z_i \sim N(0,1)$ mutually independent

$$S(i) = S_0 \exp((r + \omega)T + \theta \Delta g_i + \sigma \sqrt{\Delta g_i} Z_i)$$

2. Return $u(i) = \exp(-rT) \max(S(i) - K, 0)$

$$\text{Calculate: call option} = \frac{1}{N} \sum_{i=1}^N u_i$$

- b. Monte Carlo simulation of the VG process as the difference between the two Gamma processes

Input:

$$N, S_0, K, r, T$$

Parameter VG : σ, θ and v

$$\mu_r = \frac{1}{2} \sqrt{\theta^2 + \frac{2\sigma^2}{v}} + \frac{\theta}{2},$$

$$\mu_s = \frac{1}{2} \sqrt{\theta^2 + \frac{2\sigma^2}{v}} - \frac{\theta}{2}$$

$$\omega = \frac{1}{v} \ln(1 - \theta v - 0,5\sigma^2 v)$$

Loop : $i = 1$ to N :

1. Generate $g_{ri} \sim \Gamma\left(\frac{\Delta t_i}{v}, v\mu_r\right)$, $g_{si} \sim \Gamma\left(\frac{\Delta t_i}{v}, v\mu_s\right)$ mutually independent
 $S(i) = S_0 \exp((r + \omega)T + g_{ri} - g_{si})$

2. Return $u(i) = \exp(-rT) \max(S(i) - K, 0)$

Calculate : call option = $\frac{1}{N} \sum_{i=1}^N u_i$

3. MATERIAL AND METHOD

The stages of data analysis are as follows:

- a. Calculating daily stock data log returns using equation (3)
- b. Exploring daily stock returns log data.
- c. Conducting data distribution tests including normality test and Variance Gamma distribution test.
- d. Estimating VG parameters
- e. Performing a Monte Carlo simulation to obtain a call option value with several N values until a convergent result is obtained
- f. Calculating the value of the call option using the closed form equation.

In this study, secondary data for closing daily stock prices and call option data from Zoom Video Communications, Inc. (ZM) (Finance Yahoo, 2021b) and Nokia Corporation (NOK) (Finance Yahoo, 2021a) were applied. The period of ZM daily stock data used is from April 22, 2019 to February 12, 2021 (459 observations). As for NOK stocks from February 15, 2019 to February 12, 2021 (503 observations). The data was processed using the r4.0.2 software (Team, 2020). Some of the r packages used are: VarianceGamma (Scott et al., 2018), BAS (Clyde et al., 2011) and Bessel (Martin & Maechler, 2019).

4. RESULTS AND DISCUSSION

Data exploration is done to see the characteristics of the data by looking at the shape of the data distribution. One form of data exploration that is often used is the histogram. The histogram provides information about the symmetry and height of the shape of the data distribution. The following histogram of daily stock returns logs ZM and NOK,

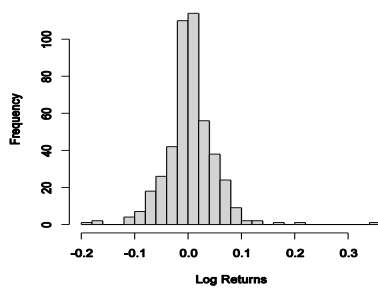


Figure 1. Histogram Log Returns of Daily ZM Stocks

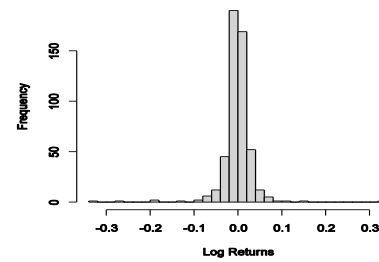


Figure 2. Histogram Log Returns of Daily NOK Stocks

Figure 1 and Figure 2. Show an asymmetric distribution shape (skewness $\neq 0$) and a high curve shape (leptokurtic). These two criteria indicate that the data is not normally distributed. The distribution of data that is not normal can be caused by outliers or spikes from data that are often found in financial data, one of which is stock data. The spike in data can be seen from the plot of data against time (time series plot). The following is a time series plot of the daily stock log returns of ZM and NOK

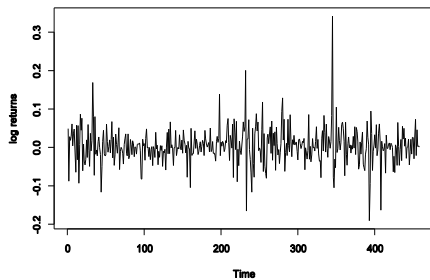


Figure 3. Plot of the Time Series Log Returns of ZM's Daily Stock

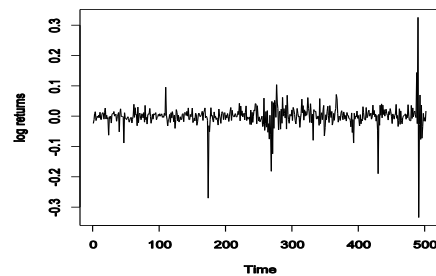


Figure 4. Plot of Time Series Log Returns of NOK's Daily Stock

In Figure 3 and Figure 4. Informing that there has been a spike in the value of stock log returns at several time periods. The increase in value was due to the large number of Zoom and Nokia users which resulted in the increase in the value of ZM and NOK stocks. The time series plots in Figures 3 and 4 show data patterns that have non-constant variances, so there is an assumption that the data are not normally distributed. To ensure that the data distribution is not normal, hypothesis testing is carried out. Testing the normal distribution of daily stock log returns of ZM and NOK. Test results using $\alpha = 5\%$ gave value *p-value* each $0.0007,635 \cdot 10^{-4}$ and 7.426×10^{-13} which concludes that the daily stock returns log distribution is not normally distributed.

Based on the results of data exploration, descriptive statistics and hypothesis testing, it can be concluded that the data are not normally distributed. So that the modeling is carried out for data that is not normally distributed, that is the Variance Gamma model. The VG distribution has three parameters, namely σ , ν and θ . Estimation of these three parameters is carried out using the Maximum Likelihood method. The results are as follows:

Table 2. Parameter Estimation Result of Variance Gamma Distribution

Stock Name	$\hat{\sigma}$	$\hat{\nu}$	$\hat{\theta}$
ZM	0.01763	0.11840	1.60950
NOK	0.04077	0.11840	-0.01120

After the VG distribution parameters have been estimated, the next step is to determine the value of the ZM and NOK stock call options using a Monte Carlo simulation and a closed form. The required values are as follows:

- S_0 , is the current stock price. In this study, the closing price of ZM stocks on February 21, 2021 is used, that is \$ 417.26. Meanwhile, for NOK stocks, the closing price of stocks is \$ 4.0700
- T , is the expiration time. In this study, March 12, 2021 was chosen as the expiration date, so the value of $T = 19/252$
- r , The risk-free interest rate is determined based on the FED central bank interest rate of 0.25%
- K , deal price. The K value used varies for 2 call option contract codes
- Parameter Variance Gamma as listed in Table 2
- N , the number of simulations used various values of N .

The simulation is carried out for various values of N and strike price (K) according to the number of the call option contract. The following are the results of the simulation of the call option prices for ZM and NOK stocks.

Table 3. Call Option Value of ZM stock using Monte Carlo Simulation and Closed Form

Code		Price			Monte Carlo			Closed form
Call Option Contract Code	Last price (\$)	Bid (\$)	Ask (\$)	Strike Price (\$)	N	Call option VG1 (\$)	Call option VG2 (\$)	Call option Closed form (\$)
ZM210312C00250000	171.90	164.20	171.20	250	10	148.9617	210.6376	163.4199
					100	194.9904	162.4577	
					1,000	170.0779	170.0972	
					10,000	166.8949	167.3095	
					100,000	167.0716	167.7997	
					1,000,000	167.1083	167.7887	
ZM210312C00200000	243.50	213.80	221.00	200	10,000,000	167.2059	167.7430	216.3705
					10	198.5891	213.0348	
					100	208.8432	210.7308	
					1,000	216.6191	220.7821	
					10,000	215.5057	218.1523	
					100,000	217.2487	217.9034	
					1,000,000	217.1162	217.6224	
10,000,000	217.1595	217.6176						

Table 3 provides information on the value of the Zoom stock call option model VG1 and VG2. The value of the call option is obtained through a Monte Carlo simulation with several N values and using a closed form. In this study, the value of N is set from 10 simulations to 10,000,000 simulations. At $N = 10$ to $N = 100,000$, it shows the estimated value of the call option that has not converged. The estimated value of the call option has converged in the simulation for $N = 1,000,000$ and $N = 10,000,000$, so that the estimated value of the call option at $N = 10,000,000$ is used. For example, on Zoom stocks with code ZM210312C00250000 with a strike price of 250, the estimated call option value is 167.2059 for the VG1 model and 167.7430 for the VG2 model. The results of the Monte Carlo estimation with two approaches to the VG1 model and the VG2 model show results that are not much different. As well as the value of other Zoom stock call options. While the estimation results using the closed form equation on ZM stocks give slightly different call option results.

Table 4. NOK Stock Call Option Value Using Monte Carlo Simulation and Closed Form

Code	Price				N	Monte Carlo		Closed form
	Call Option Contract Code	Last price (\$)	Bid (\$)	Ask (\$)		Strike Price (\$)	Call option VG1 (\$)	Call option VG2 (\$)
NOK210312C00002000	2.06	2.01	2.12	2.00	10	2.072329	2.053022	2.070377
					100	2.073642	2.064653	
					1,000	2.067947	2.073980	
					10,000	2.070188	2.073750	
					100,000	2.070238	2.073517	
					1,000,000	2.070425	2.073794	
					10,000,000	2.070393	2.073730	
NOK210312C00003000	1.07	1.07	1.10	3.00	10	1.091199	1.048112	1.070565
					100	1.072194	1.068410	
					1,000	1.072100	1.071915	
					10,000	1.070319	1.070427	
					100,000	1.070575	1.070617	
					1,000,000	1.070501	1.070557	
					10,000,000	1.070564	1.070586	

Table 4 provides information on the value of the Nokia stock call option VG1 and VG2 models. The simulation process on Nokia stock converges faster than on Zoom stock. The estimated value of the call option on NOK stocks has converged at $N=100,000$ and $N=1,000,000$. The estimation result of call options for NOK stocks is closer to the last price. The estimation results of the call option obtained from the closed form equation on NOK stocks give result that is not much different from the result of the Monte Carlo simulation.

5. CONCLUSION

Stock price option modeling for log returns data that are not normally distributed can be conducted using the Variance Gamma model. In this study, the estimation result of call options use two approaches, namely the Monte Carlo simulation and the closed form. Through these two approaches, the estimation result of call options on NOK stocks gives result that is not much different.

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