

APPLICATION OF BIPLLOT ANALYSIS WITH ROBUST SINGULAR VALUE DECOMPOSITION TO POVERTY DATA IN SULAWESI ISLAND

Febriyana Taki, Lailany Yahya, Muhammad Rezky Friesta Payu
Department of Mathematics, Gorontalo State University, Indonesia

e-mail: febriyanataki23@gmail.com

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Abstract: Poverty is defined as an inability of the individual to meet basic needs for a decent life. According to BPS data in 2020, Sulawesi Island ranks fifth as the poorest island in Indonesia. This study aims to find out the mapping of areas and indicators of poverty in Sulawesi Island using Biplot Analysis with Robust Singular Value Decomposition approach for outlier research data. Based on the results of the study, there are five objects that are outlier and the information provided by the biplot amounted 98.45%. District/city that have similar characteristics are divided into 4 groups. The indicator of poverty that has the most diversity is the School Old Expectations Numbers (Var 4) and the one with the least diversity is Poor Households Using Clean Water (Var 8). Indicators of poverty that are positively correlated are Literacy Numbers (Var 1) and Non-Working Poor Population (Var 5), while the negative correlated are The Non-Working Poor Population (Var 5) and Poor Households Using Clean Water (Var 8). There are 19 districts/cities that have literacy values above the average of all districts/cities and 11 districts/cities that have a per capita expenditure value below the average of all districts/cities.

1. INTRODUCTION

Poverty is defined as an inability of the individual to meet basic needs for a decent life. According to data from Badan Pusat Statistik, for one year, in 2019 to 2020, the number of people in Indonesia has increased by 2,691.5 thousand people, where the number of poor people has increased too by 2,763.8 thousand people. Especially in Sulawesi Island which consists of six provinces, the number of poor people in 2020 was recorded at 185,02 thousand people in Gorontalo province, North Sulawesi reached 192,37 thousand people, Central Sulawesi was recorded at 398,73 thousand people, South Sulawesi was recorded at 776.83 thousand people, West Sulawesi was recorded at 152,02 thousand people and Southeast Sulawesi was recorded at 301,82 thousand poor people (Badan Pusat Statistik, 2020). To reduce poverty levels in view of poverty indicators, practical methods are needed to overcome poverty which has always been a social problem faced by the government, therefore, researchers want to know the mapping of areas and indicators of poverty on Sulawesi Island.

One method that can be used to get mapping of objects and variables is biplot analysis. Biplot analysis is a multivariate method that uses rows and columns in a graph. This analysis is used to display the objects and variables studied (Leleury & Wokanubun, 2015). Information obtained from biplots includes objects and variables in a single image (Fitria et al., 2013). Based on that presentation, it can be analyzed diversity, correlation and relative position between the observation object and the variable (Jolliffe, 2002). Biplot analysis is based on Singular Value Decomposition (SVD). SVD is a method used to decipher a matrix associated with its singular value (Srinadi and Sumarjaya, 2016). Biplot analysis with SVD, requires a data matrix that does not have an outlier, if the data used in the study there is outlier data, then biplot analysis using SVD has not provided the results of the actual data. To obtain results that are resistant to outlier data, it can be overcome by using Robust Singular Value Decomposition (RSVD) in biplot analysis by guessing a number of eigen values, right and left vector eigen (Hawkins et al., 2001).

Research about the poverty was conducted by Agung Saputro using Factor Analysis obtained by indicators of poverty in 15 provinces in Indonesia in 2007 including food, education, employment, health, and residential homes. Then another study on Biplot Analysis was conducted by Widowati & Muzdalifah (2017) who discussed mapping private universities in East Java using Biplot Analysis with SVD and RSVD.

Based on these studies, it became a reference for this study to find out the plot of mapping areas and indicators of poverty in Sulawesi Island using Biplot Analysis with Robust Singular Value Decomposition. From the plot, an analysis was conducted to find out the grouping of regions and indicators of poverty, the diversity of each poverty indicator, the relationship between poverty indicators and the value of poverty indicators in districts/cities on Sulawesi Island.

2. LITERATURE REVIEW

2.1. Outlier

An outlier is an observation that is located far from the data center and does not follow most patterns. To identify whether a data is outlier, it can be tested using *Mahalanobis* distance with the formula:

$$MD_i^2 = (x_i - \bar{x})' \mathbf{S}^{-1} (x_i - \bar{x}); i = 1, 2, \dots, n \quad (1)$$

with x = object data; \bar{x} = average vector; \mathbf{S} = covariance variance matrix; n = number of objects.

Mahalanobis distance value compared to chi square table value. If $MD_i^2 > \chi_{\alpha;p}^2$ then x_i is an outlier data (Meirina et al., 2013).

2.2. Robust Singular Value Decomposition

Robust Singular Value Decomposition is one of the decomposition techniques associated with the singular value of a matrix whose results are resistant to outlier data. Suppose X is a median-centered data matrix, then the equation would be:

$$\mathbf{X} = \mathbf{X}^* - (\text{median}_{1 \leq j \leq p} \mathbf{X}^*.j) \quad (2)$$

with \mathbf{X} = matrix sized $n \times p$ centered on median; \mathbf{X}^* = matrix sized $n \times p$, which contain outlier data; $\mathbf{X}^*.j$ = column vector to j of matrix \mathbf{X}^* where $\mathbf{X}^*.j = (x_{1j}^*, x_{2j}^*, \dots, x_{nj}^*)$; $j = 1, 2, \dots, p$

The algorithm of *L1* Robust Singular Value Decomposition is as follows (Hawkins et al., 2001):

1. Determine the initial estimate of the vector u_1
2. For each column, $j = 1, 2, \dots, p$, fit the L_1 regression coefficient c_j by $\min \sum_{i=1}^n |x_{ij} - c_j u_{i1}|$
3. Calculate the resulting estimate of the right eigen vector $a_1 = \frac{c}{\|c\|}$
4. To refine the estimate of the left vector eigen, the right vector eigen estimate result is used. For each row, $i = 1, 2, \dots, n$, fit the L_1 regression coefficient d_i by $\min \sum_{j=1}^p |x_{ij} - d_i a_{j1}|$
5. Calculate the resulting estimate of the left eigen vector $u_1 = \frac{d}{\|d\|}$
6. Repeat steps 2 through 5 again until obtain a convergent estimation of right vector eigen and left vector eigen

To determine the second and next eigen vector and eigen values, the X matrix used is a derivative matrix that aims to reduce the new form:

$$X \leftarrow X^* - \lambda_k \mathbf{u}_k \mathbf{a}'_k \quad (3)$$

with X = new data matrix; X^* = original data matrix which contain outlier data; λ_k = eigen value decreases to k ; \mathbf{u}_k = left eigen vector to k ; \mathbf{a}'_k = right eigen vector to k ; (Hawkins et al., 2001).

2.3. Biplot Analysis

Biplot analysis can display the plot of n and variable p observations simultaneously in a two-dimensional field and can provide additional, better information about the relationship between variables and observations (Jolliffe, 2002). This analysis belongs to the exploration of multiple variables in low-dimensional space, usually two or three so that the behavior of the data is easy to interpret (A.R et al., 2018). Biplot analysis is based on Singular Value Decomposition (SVD). SVD is the decipherment of the singular value of an $n \times p$ matrix in which n is the number of objects observed and p is the number of variables (Jolliffe, 2010). Suppose matrix X has been corrected to its average value and has a rank of r with $r \leq \{n, p\}$ so that it can be written or described into:

$$X = ULA' \quad (4)$$

With X = matrix sized $n \times p$; U = matrix sized $n \times r$, with columns containing vector eigen of matrix XX' ; L = matrix sized $r \times r$, contains the square root of eigen value from matrix $X'X$; A = matrix sized $r \times p$, with columns containing vector eigen of matrix $X'X$

According to Jolliffe (2002), in biplot presentation, it takes a α value to define matrix G and matrix H' , with a condition of $0 \leq \alpha \leq 1$. Frequently used α values are $\alpha = 0$, $\alpha = 0.5$ and $\alpha = 1$ (Nugroho, 2008). The taking of $\alpha = 0$ and $\alpha = 1$ are useful in biplot interpretations. Suppose $G = UL^\alpha$ dan $H' = L^{1-\alpha}A'$, if $\alpha = 0$ then it was obtained $G = U, H' = LA'$ (Mattjik & Sumertajaya, 2011)

2.4. Interpretation of Biplot

There are 4 pieces of information obtained from the biplot view (Mattjik dan Sumertajaya, 2011):

- a. Proximity between objects

Using *Euclidean* distance, it can be measured the proximity between objects g_i and g_j .

b. Variable diversity

The diversity of each poverty indicator can be seen based on the length of the variable vector.

c. Correlation between variables

Information used to see the effect of one variable on another. The angular value between two variable vectors describes the correlation or relationship between the two variables.

d. Variable values on an object

This information is used to see the advantages of each district/city. Districts/cities that are located in the direction of the variable vector, showing that the district/city has a variable value above the average. Conversely, if the district/city that is located opposite the direction of the variable vector shows that the district/city has a variable value below the average.

2.5. Goodness of Fit of Biplot Test

Goodness of Fit of Biplots is calculated by the following formula:

$$\rho^2 = \frac{(\lambda_1 + \lambda_2)}{\sum_{k=1}^r \lambda_k} \times 100\% \quad (5)$$

Biplot can provide an excellent presentation of the data information contained in the actual data if the resulting value of ρ^2 is close to the value of one (Gabriel, 1971).

2.6. Poverty

Poverty is referred to as a person's inability to meet basic needs (Abdillah et al., 2019). The unemployed population tends to reduce economic growth, resulting in poverty (Seran, 2017). While universally recognized basic authorities include meeting the needs of food, housing, clean water, land, health environment, education, employment, a sense of security or peace from violence and participation in social affairs. Bappenas argues that the main indicators of poverty are lack of food, clothing and shelter, lack of expertise in reading and writing, limited land ownership and means of production, lack of security and welfare, difficulties in economic and social aspects, having limited access to science, lack of access to health services, limited employment opportunities, low quality of educational services and limited access to clean water (Bappenas, 2018).

3. MATERIAL AND METHOD

3.1. Data Sources

The data used in this study is secondary data obtained from the website of Badan Pusat Statistik. The data used is data on poverty indicators on the island of Sulawesi in 2020.

3.2. Research Variables

The data used is data on poverty indicators on the island of Sulawesi. The research object is shown in Table 1. The research variables are shown in Table 2. All functions of the package use R Version 4.1 (Team, 2021)

Table 1. Research Object

Number	Object	Number	Object	Number	Object
1	Boalemo	28	Wakatobi	55	Jeneponto
2	Gorontalo	29	Kolaka Utara	56	Takalar
3	Pohuwato	30	Buton Utara	57	Gowa
4	Bone Bolango	31	Konawe Utara	58	Sinjai
5	Gorontalo Utara	32	Kolaka Timur	59	Maros
6	Kota Gorontalo	33	Konawe Kep.	60	Pangkajene
7	Bolmong	34	Muna Barat	61	Barru
8	Minahasa	35	Buton Tengah	62	Bone
9	Kepulauan Sangihe	36	Buton Selatan	63	Soppeng
10	Kepulauan Talaud	37	Kendari	64	Wajo
11	Minahasa Selatan	38	Baubau	65	Sidenreng Rapp.
12	Minahasa Utara	39	Banggai Kep.	66	Pinrang
13	Bolmong Utara	40	Banggai	67	Enrekang
14	Siau T. Biaro	41	Morowali	68	Luwu
15	Minahasa Tenggara	42	Poso	69	Tana Toraja
16	Bolmong Selatan	43	Donggala	70	Luwu Utara
17	Bolmong Timur	44	Toli Toli	71	Luwu Timur
18	Manado	45	Buol	72	Toraja Utara
19	Bitung	46	Parigi	73	Makassar
20	Tomohon	47	Tojo Una Una	74	Pare Pare
21	Kotamobagu	48	Sigi	75	Palopo
22	Buton	49	Banggai Laut	76	Majene
23	Muna	50	Morowali Utara	77	Polewali Mandar
24	Konawe	51	Palu	78	Mamasa
25	Kolaka	52	Kep. Selayar	79	Mamuju
26	Konawe Selatan	53	Bulukumba	80	Mamuju Utara
27	Bombana	54	Bantaeng	81	Mamuju Tengah

Table 2. Research Variables

Variable	Description	Scale
Var 1	Literacy Numbers	Ratio
Var 2	Education Completed Under Elementary School	Ratio
Var 3	Length Average of School	Ratio
Var 4	School Old Expectations Numbers	Ratio
Var 5	Non-Working Poor Population	Ratio
Var 6	Raskin Recipients Household	Ratio
Var 7	Per capita Production	Ratio
Var 8	Poor Households Using Clean Water	Ratio

3.3. Data Analysis Methods

The data analysis stages are as follows:

1. Descriptive analysis of poverty indicator data on Sulawesi Island.
2. Test of outlier using Mahalanobis distance test and chi square test. After an outlier test, if the research data has outlier data, the research data is formed into a matrix X .
3. Robust singular value decomposition. At this stage, vector eigen can be generated using a matrix that is standardized against the median. Once standardized, matrix X is described into matrices U , L , and A using RSVD.
4. Biplot analysis. To create a biplot, it takes a matrix of G and H . Once the G and H matrices are formed, the first two columns of both matrices are drawn to create a plot mapping regions and indicators of poverty.
5. Interpretation of biplot result.
6. Test of the goodness fit of biplot.

4. RESULTS AND DISCUSSION

4.1. Poverty Indicator Data Descriptive Analysis

The data used in this study is data on poverty indicators in Sulawesi Island with 81 districts/cities as objects and eight indicators of poverty as variables. In this research data, there are variables that have missing values, namely Raskin Recipient Household variable Var 6 and mean imputation has been done. The purpose of mean imputation is to fill in the missing data in a variable with an average of all known values in a variable (Acuna & Rodrigues, 2004). Descriptive poverty indicator data is shown in Table 3.

Table 3. Statistics Descriptive

Variable	Min	Mean	Max	Std
Literacy Numbers	87.02	99.60	100	1.63
Education Completed Under Elementary School	7.10	26.24	49.46	9.55
Length Average of School	6.59	8.49	12.20	1.19
School Old Expectations Numbers	11.52	13.04	16.62	0.96
Non-Working Poor Population	27.54	46.62	65.85	7.69
Raskin Recipients Household	3.78	31.13	58.00	10.28
Per capita Expenditure	51.97	66.86	79.89	6.16
Poor Households Using Clean Water	10.39	68.02	97.80	17.58

4.2. Outlier Test

The outlier test on the study used the Mahalanobis distance test and the chi square test. If Mahalanobis distance value exceeds Chi Square value ($MD_i^2 > \chi_{0.05;8}^2$) the object of the research is an outlier. $\chi_{0.05;8}^2 = 15,51$. In accordance with these criteria, five outlier values can be found in Table 4. Because in this research data there is outlier data, then this research data is formed into a matrix X measuring 81×8 and will be continued in the Biplot Analysis process using RSVD.

Table 4. Mahalanobis Distance Test

Object	MD^2	Decision
Kendari	19.53	Outlier
Palu	20.79	Outlier
Bulukumba	61.32	Outlier
Tana Toraja	18.27	Outlier
Mamasa	16.96	Outlier

4.3. Robust Singular Value Decomposition

At this stage, vector eigen can be generated using a matrix that is standardized against the median. The data matrix in this study is a data matrix that has not been standardized against the median, therefore, the data matrix must be transformed first according to the equation (3). Once transformed, the data matrix will then be described into U , L and A matrices using RSVD. Using R software, obtained matrix U measuring 81×8 , matrix L is a diagonal matrix measuring 8×8 , and matrix A measuring 8×8 .

4.4. Biplot Analysis

To get the plot of the results of mapping the region and poverty indicators on Sulawesi island, it takes a G matrix which is the coordinate point of the district or city and the H' matrix which is the coordinate point of the poverty indicator, with a value of $\alpha = 0$, it is obtained $G = UL^\alpha$ dan $H' = L^{1-\alpha}A'$. From these results, the first two columns of the G matrix and the first two columns of the H' matrix will form the G_2 and H'_2 matrices. The plot of the results of mapping the region and poverty indicators on Sulawesi Island formed from the G_2 and H'_2 matrix using R software can be seen in the Figure 1.

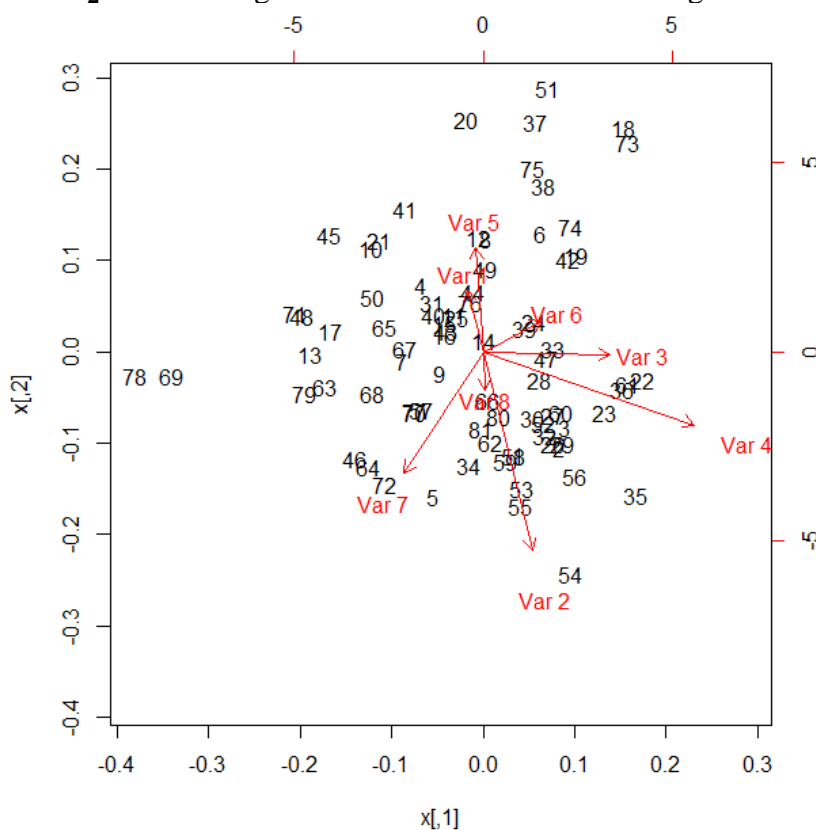


Figure 1. Plot Results of Mapping Areas and Indicators of Poverty in Sulawesi Island

4.5. Interpretation of Biplot

4.5.1. Proximity Between Objects

Proximity between objects can be measured using *Euclidean* distances. Furthermore, it can be seen that the area in the biplot is divided into four quadrants as in Figure 1. Grouping districts/cities in the same quadrant can be seen in Table 5.

Table 5. Grouping Objects by Quadrant

Group	Object	Variable
1	6, 18, 19, 24, 37, 38, 39, 42, 51, 73, 74, 75	Var 6
2	4, 8, 10, 11, 12, 14, 15, 16, 17, 20, 21, 25, 31, 40, 41, 43, 44, 45, 48, 49, 50, 65, 67, 71, 76	Var 1, Var 5
3	5, 7, 9, 13, 34, 46, 57, 63, 64, 68, 69, 70, 72, 77, 78, 79, 81	Var 7
4	1, 2, 3, 22, 23, 26, 27, 28, 29, 30, 32, 33, 35, 36, 47, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 66, 80	Var 2, Var 3, Var 4, Var 8

1. Group 1 consists of 12 districts/cities that have similar properties based on the indicator of Raskin Recipient Households (Var 6).
2. Group 2 consists of 25 districts/cities that have similar properties based on the Literacy Number indicator (Var 1) Non-Working Poor Population (Var 5).
3. Group 3 consists of 17 districts / cities that have similar properties based on Per capita Expenditure indicator (Var 7).
4. Group 4 consists of 27 districts/cities that have similar properties based on Education Completed Under Elementary School (Var 2), Length Average of School (Var 3), School Old Expectations Numbers (Var 4) and Poor Households Using Clean Water (Var 8).

4.5.2. Variable Diversity

This information is used to see the diversity of poverty indicators of each district/city based on the length of variable vectors that can be seen in Table 6.

Table 6. Variable Vector Length

Variable	Vector Length
Literacy Numbers (Var 1)	2.1258
Education Completed Under Elementary School (Var 2)	6.7850
Length Average of School (Var 3)	4.2140
School Old Expectations Numbers (Var 4)	7.3906
Non-Working Poor Population (Var 5)	3.4596
Raskin Recipients Household (Var 6)	2.2000
Per capita Expenditure (Var 7)	4.7984
Poor Households Using Clean Water (Var 8)	1.2820

The variable that has the first longest vector is the variable of School Old Expectations Numbers (Var 4), this means that the school old expectations numbers are the variables that have the most diversity. While the variable that has the shortest vector is the variable of Poor Households Using Clean Water (Var 8), this means that the variables of poor households using clean water have the least diversity.

4.5.3. Correlation Between Variables

Correlations or relationships between poverty indicators can be seen from an angle formed from two variables as in Table 7.

Table 7. Angular Size θ of Each Variable Indicator of Poverty

Variable	Var 1	Var 2	Var 3	Var 4	Var 5	Var 6	Var 7	Var 8
Var 1	0.00°							
Var 2	180.0°	0.00°						
Var 3	105.6°	74.9°	0.00°					
Var 4	123.6°	56.9°	18.0°	0.00°				
Var 5	10.7°	170.0°	95.1°	113.0°	0.00°			
Var 6	76.8°	103.8°	28.9°	46.8°	66.2°	0.00°		
Var 7	132.4°	47.1°	122.0°	104.0°	142.9°	150.9°	0.00°	
Var 8	168.5°	11.0°	85.9°	67.9°	179.2°	114.8°	36.1°	0.00°

Two vectors that form a taper angle or have the same direction describe the two variables have a positive correlation value. Conversely, two vectors that form an obtuse angle describe the two variables have a negative correlation value. While, two vectors that have angles close to right angle describe the two variables have no correlation. The variables that have a positive correlation are the Literacy Numbers (Var 1) variable and the Non-Working Poor Population (Var 5) with an angle between vectors of 10.6. The variables that have a negative correlation are the Non-Working Poor Population (Var 5) variable and the Poor Households Using Clean Water (X8) with an angle between vectors of 179.2. And variables that have no correlation with each other are the Length Average of School (Var 3) variable and and the Poor Households Using Clean Water (Var 8) variable with the angle between vectors almost forming a right angle as large as 85.9.

4.5.4. Variable Values on an Object

The location of districts/cities to the poverty indicator can be seen in Table 8.

Table 8. Location of Districts/cities on Poverty Indicators

Object	Location	Variable
4, 10, 11, 15, 16, 17, 21, 25, 31, 40, 41, 44, 45, 48, 50, 65, 71, 76	Unidirectional	Var 1
1, 2, 3, 30, 52, 53, 54, 55, 56, 58, 59, 60, 80	Unidirectional	Var 2
33, 47	Unidirectional	Var 3
22, 23, 28, 36, 61	Unidirectional	Var 4
8, 12, 14, 20, 49	Unidirectional	Var 5
6, 18, 19, 24, 42, 51, 74, 75	Unidirectional	Var 6
34, 46, 57, 64, 68, 70, 72, 77, 79	Unidirectional	Var 7
26, 27, 62, 66, 81	Unidirectional	Var 8
53, 55, 81	Opposite	Var 1
4, 8, 11, 25, 31, 40, 41, 43, 44	Opposite	Var 2
7, 13, 17, 67, 69, 78	Opposite	Var 3
10, 15, 16, 21, 43, 45, 48, 50, 65, 71	Opposite	Var 4
2, 3, 5, 29, 30, 32, 35, 58	Opposite	Var 5
9, 57, 63, 68, 70, 79	Opposite	Var 6
6, 18, 19, 37, 38, 39, 42, 51, 73, 74, 75	Opposite	Var 7
12, 20, 76	Opposite	Var 8

An object that is located in the same direction as the vector of the poverty indicator can be interpreted that the poverty indicator for that object has a value above the average of all districts/cities. Conversely, if an object is located in the opposite direction to the vector of poverty indicator, the value of the poverty indicator is low or below the average of all districts/cities. This means Bone Bolango (4), Kepulauan Talaud (10), South Minahasa (11), Southeast Minahasa (15), South Mongondow Bolaang (16), East Bolaang Mongondow (17), Kotamobagu (21), North Konawe (25), Kolaka (31), Banggai (40), Morowali (41), Toli-Toli (44), Buol (45), Sigi (48), North Morowali (50), Sidenreng Rappang (65), East Luwu (71) and Majene (76) are in line with the variable vector of Literacy Number (Var 1), this shows that the 19 districts/cities are excel in literacy and have literacy values above the average of all districts/cities. While Gorontalo City (6), Manado (18), Bitung (19), Kendari (37), Baubau (38), Banggai Kepulauan (39), Poso (42), Palu (51), Makassar (73), Pare-Pare (74) and Palopo (75) opposite direction to variable vector Percapita Expenditure (Var 7), this shows that the 11 districts/cities have not excelled in per capita spending and have a per capita expenditure value below the average of all districts/cities.

4.6. Goodness of Fit of Biplot Test

The percentage of accuracy of biplot results is as follows:

$$\rho^2 = \frac{22805.75 + 180.43}{22805.75 + 180.43 + 95.34 + 85.58 + 71.43 + 54.72 + 43.40 + 12.31} \times 100\%$$

$$= 0.9845 \times 100\% = 98.45\%$$

From the results of these calculations, biplot can provide excellent presentation because the value of ρ^2 obtained is close to the value of 1. Information provided by biplots amounted to 98,45% of the overall information provided by objects and variables in this study.

5. CONCLUSION

The results of mapping areas and poverty indicators in Sulawesi Island using Biplot Analysis with RSVD can provide information on 98.45 % of the overall information provided by objects and variables in this study. Areas that have similar characteristics are divided into 4 groups based on quadrants. The variable of School Old Expectations Numbers (Var 4) has the most diversity and can be said to be the most dominant poverty indicators on the island of Sulawesi, while the variable of Poor Households Using Clean Water (Var 8) has the least diversity and can be said to be the least dominant poverty indicators on the island of Sulawesi. The variable of Literacy Numbers (Var 1) and the Non-Working Poor Population (Var 5) have a positive correlation, it means the relationship between the two variables are strong, if Var 1 increases, then Var 5 also increases. While the variables of Non-Working Poor Population (Var 5) and the Poor Households Using Clean Water (Var 8) have a negative correlation, it means the relationship between the two variables are small, if Var 5 increases, then Var 8 decreases.

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