Media Statistika

p-ISSN 1979 – 3693 e-ISSN 2477 – 0647
MEDIA STATISTIKA 15(2) 2022: 175-185
http://ejournal.undip.ac.id/index.php/media_statistika

MANAGING HEART RELATED DISEASE RISKS IN BPJS KESEHATAN USING COLLECTIVE RISK MODELS

Gede Ary Prabha Yogesswara[1], Danang Teguh Qoyyimi[1], Abdurakhman[1]
[1] Department of Mathematics, Universitas Gadjah Mada, Indonesia

e-mail: qoyyimi@ugm.ac.id

DOI: 10.14710/medstat.15.2.175-185

Article Info:
Received: 7 July 2022
Accepted: 9 January 2023
Available Online: 4 April 2023

Keywords:
BPJS Kesehatan; Collective risk model; Heart disease; Lognormal distribution; Poisson distribution
.

Abstract: BPJS Kesehatan is a legal entity established to administer the health service program using the insurance system. Heart related diseases is a disease with the largest coverage cost in Indonesia. It can be calculated by using the collective risk model as an approximation of the aggregate loss model. This model is a compound distribution from claim frequency and claim severity, where claim frequency be the primary distributions. The Poisson distribution can be used to the distribution of the heart disease claim frequency. Whereas, the distribution of the heart disease claim severity has a lognormal distribution. The model obtained can explain the aggregate loss of heart disease claims properly.

1. INTRODUCTION

Badan Penyelenggara Jaminan Sosial (BPJS) Kesehatan is a legal entity established to administer the Jaminan Kesehatan National (JKN) program. The JKN program is a health service program as a form of social security that is held nationally using the insurance system. BPJS Kesehatan participants are required to pay premiums periodically to obtain health insurance benefits that are individual services. This service includes promotive, preventive, curative and rehabilitative services including drug services and medical materials in accordance with the necessary medical needs (DJSN & BPJS Kesehatan, 2021).

One of the diseases covered by BPJS Kesehatan and counted extremely big aggregate claims is heart disease. It is known as catastrophic diseases of the JKN programs. As defined in DJSN & BPJS Kesehatan (2021), catastrophic diseases are those that count the high cost. It comprises of hepatitis, kidney failure, haemophilia, heart disease, cancer, leukemia, stroke dan thalassemia. Catasthtophic disease is a chronic and degenerative disease (Heniwati & Thabrany, 2016). It is called chronic, because the disease is often not realized, but has the potential to appear, and takes a lifetime to control. It is called degenerative, because the disease is more common with age.

In 2019, BPJS Kesehatan reported that heart disease was a catastrophic disease with the most visits to both outpatient and inpatient services. Likewise with the total cost of claims covered by BPJS Kesehatan, heart disease is a catastrophic disease with the highest cost which costs 3 trillion rupiahs or 10 percent of the total cost of JKN claims for outpatient services of 28 trillion rupiahs. Meanwhile, for inpatient services, heart disease costs 8.7 trillion rupiahs or 15 percent of the total cost of inpatient claims of 58 trillion rupiahs. Heart disease is a condition when the heart has problems, such as disorders of the heart's blood vessels, heart rhythm, heart valves, or congenital disorders (Pittara, 2021).

At this stage, it is crucial for BPJS Kesehatan to understand the characteristic of losses associated with heart disease claims. Understanding the losses in health insurance product can be done in two approaches, i.e. life insurance approach and general insurance approach (Pitacco, 2014). In this case we investigated the risk using general insurance approach via collective risk models. Under this approach, the total loss is explained by twofold of the frequency and severity of claims. The frequency captured the number of claim for certain time frame whereas the severity captured the size of each claim. The uncertainty then can be gathered from those two. If a loss can be predicted, then the result can be used as a tool by the insurance company to set the price of insurance products, in this case is the premium (Wang et al, 2019).

The aggregate loss model is one way to calculate losses from insurance companies. This model is a compound distribution of claim frequency and claim severity, where the claim frequency distribution is the primary distribution (Septiany et al, 2020). The purpose of modeling aggregate loss is to build a probability model for total payments by the insurance system. There are two approach models for aggregate loss, namely the individual risk model and the collective risk model (Klugman et al, 2019). In the individual risk model, the model is constructed from a number of individual insurance contract costs. Whereas in the collective risk model, the model is built by the claim frequency and the claim severity of which are random, by assuming that the claim frequency and claim severity random variable are independent of each other and identical distributions on the claim severity random variable.

The collective risk models can be determined by assuming that the number of claims and the amount of claims follow certain distributions. Typically, the distributions of discrete random variable can be used to the claim frequency data and the distributions of continuous random variable can be used to the claim severity data (Shevchenko, 2010). The Poisson distribution is one of the distributions of discrete random variable. The Poisson distribution is very useful to explain the homogeneity of a data since the equivalence of mean and variance (David & Jemna, 2015). However, the Poisson distribution presents significant constraints that limit its use.

The claim frequency of heart disease covered by BPJS Kesehatan can be modeled as Poisson distribution. The aim is to provide an overview of the Poisson distribution for the claim frequency. Furthermore, we determine the best distribution for the claim severity of heart disease covered by BPJS Kesehatan. Thus, it can be determined how the collective risk to models the heart disease covered by BPJS Kesehatan and whether the model can explain the aggregate loss properly.

2. LITERATURE REVIEW

2.1. Collective Risk Model

The collective risk models are fundamental in actuarial to model the aggregate loss of an insurance company (Cossette et al, 2019). The claim covered by BPJS Kesehatan as an insurance company can be estimated by this model for the participants service improvement (Chadidjah et al, 2019).

Let N be a random variable of the claim frequency, X_1 denotes the first payment amounts, X_2 denotes the second payment amounts, and so on.

Definition 1 (Klugman et al, 2019) *The collective risk model represents the aggregate losses as a sum,* S*, of a random number,* N*, of individual payment amounts,* X_k*, for* $k = 1,2,\dots,N$*. In other words,*

$$S = \sum_{k=1}^{N} X_k = X_1 + X_2 + \cdots + X_N \tag{1}$$

where $S = 0$*, when* $N = 0$*.*

In the collective risk model, that is satisfied the following assumptions:

(i) Random variables of individual payment amounts $X_1, X_2, \dots$ are identically distributed.
(ii) A random variable of the claim frequency, N, and random variables of individual payment amounts, $X_1, X_2, \dots$, are mutually independent.

According to Lin & Willmot (2015), the expected aggregate loss is given by,

$$\mathrm{E}[S] = \mathrm{E}[N]\,\mathrm{E}[X] \tag{2}$$

and the variance is,

$$\mathrm{Var}[S] = \mathrm{E}[N]\,\mathrm{Var}[X] + \mathrm{Var}[N]\,(\mathrm{E}[X])^2 \tag{3}$$

2.2. Poisson Distribution

It is assumed that a random variable of the claim frequency, N, has a Poisson distribution with parameter λ. The Poisson distribution is one of the distributions for a discrete random variable for modeling the claim frequency randomly in a given time interval (Omari et al, 2018). The probability function is given by,

$$p(k) = \Pr(N = k) = \frac{\exp(-\lambda)\,\lambda^k}{k!} \tag{4}$$

for $k = 0,1,2,\dots$, where $\lambda > 0$. According to Pramesti (2011), this distribution has the same value of the expected and the variance, that is,

$$\mathrm{E}[N] = \mathrm{Var}[N] = \lambda \tag{5}$$

While the moment generating function of Poisson distribution is,

$$M_N(t) = \exp[\lambda(\exp[t] - 1)] \tag{6}$$

2.3. Truncation at Zero

For insurance count data, it is possible to have the situations in which the minimum observed value of claim frequency is one (Klugman et al, 2019). In order to be interpreted, the zero-truncated distribution can be used in this situation. The probability generating function of zero-truncated distribution is,

$$P_N^T(z) = \frac{P_N(z) - p(0)}{1 - p(0)} \tag{7}$$

where $P_N(z) = M_N(\ln z)$ denotes the probability generating function of the distribution of claim frequency.

By substituting equation (6) to equation (7), the probability generating function of the zero-truncated Poisson distribution is expressed as,

$$P_N^T(z) = \frac{M_N(\ln z) - p(0)}{1 - p(0)} = \frac{\exp[\lambda(z-1)] - \exp(-\lambda)}{1 - \exp(-\lambda)} \tag{8}$$

As the result, the expected of the zero-truncated Poisson distribution is,

$$\mathrm{E}[N^T] = P_N^{T\prime}(1) = \frac{\lambda}{1 - \exp(-\lambda)} \tag{9}$$

and the variance is,

$$\begin{aligned}\mathrm{Var}[N^T] &= \mathrm{E}[N^{T2}] - (\mathrm{E}[N^T])^2 \\ &= \mathrm{E}[N^T(N^T - 1)] + \mathrm{E}[N^T] - (\mathrm{E}[N^T])^2 \\ &= P_N^{T\prime\prime}(1) + P_N^{T\prime}(1) - \left(P_N^{T\prime}(1)\right)^2 \\ &= \frac{\lambda^2 + \lambda}{1 - \exp(-\lambda)} - \left(\frac{\lambda}{1 - \exp(-\lambda)}\right)^2 \end{aligned} \tag{10}$$

2.4. Claim Severity Model

Let $X_k > 0$ denotes the individual payment amounts for the kth claim, where $k = 1,2, \ldots$. It is assumed that $X_1, X_2, \ldots$ are mutually independent and identically distributed. The cumulative distribution function is denoted by $F(x) = \Pr(X_k \leq x)$, with mean μ_X and variance σ_X^2 (Katz, 2002).

According to Katz (2002), the F function is positively skewed distribution function on the interval $(0, \infty)$ in practice. Suppose that the claim severity has a lognormal distribution. If the log-transformation is carried out, then the claim severity has a normal distribution. In other words,

$$Y_k = \ln X_k \sim N(\mu_Y, \sigma_Y^2) \tag{11}$$

Therefore,

$$\mu_X = \exp(\mu_Y + \sigma_Y^2/2), \tag{12}$$

and

$$\sigma_X^2 = \exp[2(\mu_Y + \sigma_Y^2)] - \mu_X^2 \tag{13}$$

2.5. Parameters Estimation

Suppose that the data on the claim frequency of the form n_i for $i = 1,2, \ldots, m$, where n_i denotes the claim frequency occuring in the ith period, available for m periods of the claim occurence. In the case of Poisson distribution, the parameter estimator, λ, can be determined by maximizing the likelihood function (Klugman et al, 2019). It is called the maximum likelihood method. The maximum likelihood estimation of parameter λ is,

$$\hat{\lambda} = \sum_{i=1}^{m} \frac{n_i}{m} = \bar{n} \tag{14}$$

It is given that the data on the claim severity is denoted by $x_k(i)$ for $k = 1,2, \ldots, n_i$ and $i = 1,2, \ldots, m$, where $x_k(i)$ represents the individual payment amounts of the kth claim in the ith period (Katz, 2002). In the case of lognormal distribution, the maximum likelihood estimation of parameters μ_X and σ_X^2 are,

$$\hat{\mu}_X = \frac{1}{n}\sum_{i=1}^{m}\sum_{k=1}^{n_i} \ln x_k(i) \tag{15}$$

and

$$\hat{\sigma}_X^2 = \frac{1}{n}\sum_{i=1}^{m}\sum_{k=1}^{n_i} [\ln x_k(i)]^2 - \hat{\mu}_X^2 \tag{16}$$

respectively.

3. DATA AND METHODOLOGY

3.1. BPJS Kesehatan Sample Data

For this research, we used the sample data published by BPJS Kesehatan (Ariawan et al, 2021). This data is used as a guide in building the JKN system in Indonesia and consists of the universal sample data containing the type of variables and sample selection methodologies. The sample selection of sample data came from three sub-populations of BPJS Kesehatan participants, there are the participants in 2018 or earlier, the participants in 2019 and the participants in 2020.

According to Ariawan et al (2021), the methodology used is stratified random sampling and the process of sample selection of BPJS Kesehatan participants from these sub-populations are carried out independently of each other. The sample data yields a total of 229,216 individual samples in 2019 and 2020. Based on these data, the service datas are obtained and consist of Fasilitas Kesehatan Tingkat Pertama (FKTP, translated as the first-level health facilities) and Fasilitas Kesehatan Rujukan Tingkat Lanjutan (FKRTL, translated as the advanced-level referral health facilities).

3.2. Heart Disease Claims Data

In general, the services for heart disease are provided by the FKRTL service unit, because heart disease is a disease that requires further treatment. FKRTL service data contains information in the form of characteristics of health facilities accessed, types of services, diagnosis codes, and tariffs paid by BPJS Kesehatan to health facilities, where this data has 53 variables with special codes. (Ariawan et al, 2021). There are 25,451 claims for heart disease in Indonesia. The data is known based on the primary diagnosis variable in the FKRTL service data, where all diagnoses that include abnormalities in heart function, heart valves, heart blood vessels, and other heart complications are heart disease.

In this study, the risk model of heart disease will be elaborated based on annually data. First, consider the entry date and exit date variables, the data is divided by year, those are 2019 and 2020. Further, the data on the claim frequency of heart disease in 2019 and 2020 will be fitted with the Poisson distribution. The claim frequencies are calculated from the number of claims submitted per participant. Furthermore, the fit distribution will be determined for the data on the claim severity of heart disease in 2019 and 2020. The claim severities are the final cost that incurred by BPJS Kesehatan for a claim. Based on the Poisson distribution and the distribution of claim severity obtained before, the collective risk model will be determined. Consider the collective risk model in 2019 and 2020, the comparison will be made by year.

4. RESULTS AND DISCUSSION
4.1. Model for Frequency

The claim frequency represents the number of claim event to the insurance company. Therefore, a discrete random variable distribution is used with probabilities only on non-negative integers (Klugman et al, 2019). Thus, the Poisson distribution can be used for the distribution of the claim frequency. The claim frequency of heart disease in 2019 and 2020 can be explained as in Table 1 and Figure 1.

Table 1. Descriptive Statistics of Claim Frequency of Heart Disease

Year	Minimum	Maximum	Mean	Variance	Skewness	Kurtosis
2019	1	25	1.6104	1.6466	3.9395	30.3965
2020	1	12	1.4762	1.0373	3.2752	17.9177

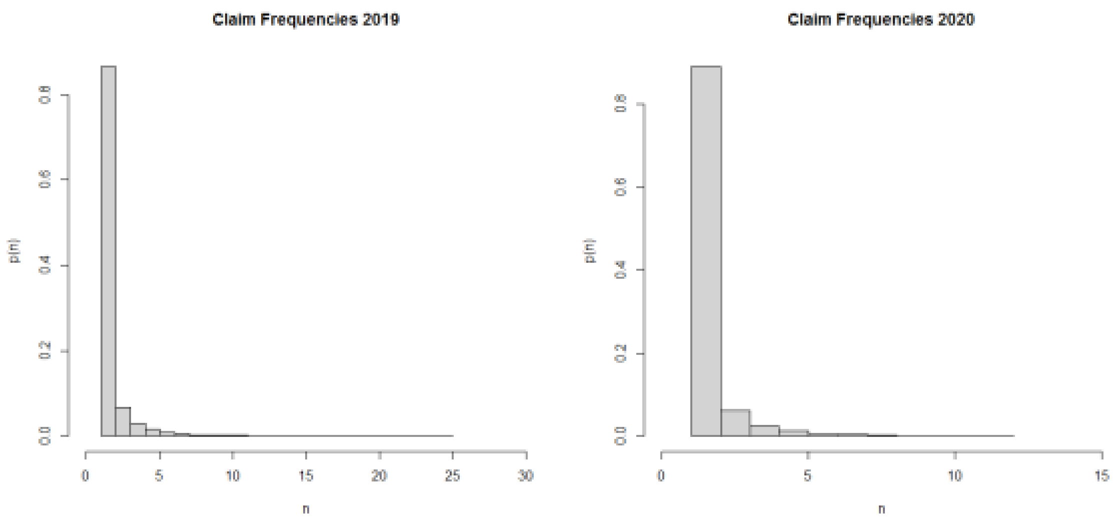

Figure 1. Histogram of Claim Frequency of Heart Disease

Based on **Table 1**, it is known that the mean of the claim frequency of heart disease in 2019 and 2020 are 1.6104 and 1.4762 respectively. Consider equation (12), the mean in each year are the Poisson parameter estimator. Note that the minimum value of claim frequency in both 2019 and 2020 are one. It means, the participants who are not submitted the claim, not recorded in the sample data. So that, we have to do truncation at zero to Poisson distribution. As the result, a zero-truncated Poisson distribution is used to the claim frequency of heart disease in 2019 and 2020. By using equation (9) and equation (10), the calculation of mean and variance of zero-truncated Poisson distribution to the claim frequency of heart disease in 2019 and 2020 are obtained, as in Table 2.

Table 2. Mean and Variance of Zero-Truncated Poisson of Claim Frequency of Heart Disease

Year	Mean	Variance
2019	2.0125	1.2032
2020	1.9135	1.0769

Figure 2 describes the histogram of the claim frequency of heart disease in 2019 and 2020 with the fit of Poisson distribution and zero-truncated Poisson distribution.

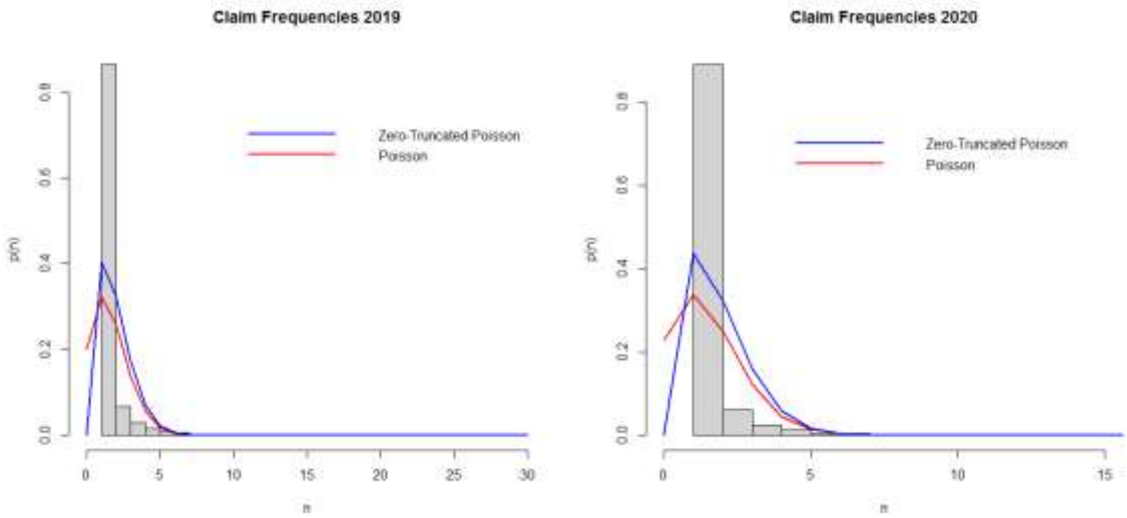

Figure 2. Poisson Distribution dan Zero-Truncated Poisson Distribution to Claim Frequency of Heart Disease

4.2. Model for Severity

The claim severity represents the payment amounts covered by the insurance company when the the event of claims. The claim severity is usually modeled by the distributions of continuous random variable (Klugman et al, 2019). The claim severity of heart disease in 2019 and 2020 can be explained as in Table 3 and Figure 3.

Table 3. Descriptive Statistics of Claim Severity of Heart Disease

Year	2019	2020
Minimum	108700	140000
Maximum	197316000	203235500
Mean	3889093	4011861
Variance	111324700000000	120675400000000
Skewness	7.8299	8.0455
Kurtosis	87.3693	94.3614

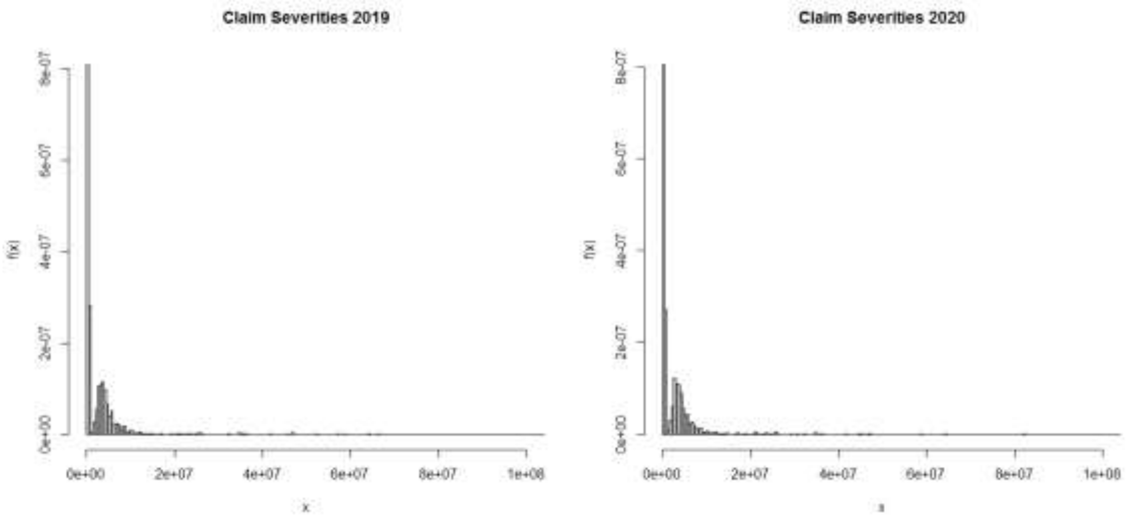

Figure 3. The Histogram of Claim Severity of Heart Disease

Based on Figure 3, the curve of the distribution is right-skewed with long tail to the right. It also shows that the histogram has asymmetrical shape, so that, it indicates the claim severity of heart disease was not normally distributed.

Note that, the data does not contain negative value. In this case, lognormal and Weibull distributions can better fit the data. The lognormal distribution is similar to Weibull distribution in some shape parameters, and some data suitable for Weibull distribution are also appropriate for lognormal distribution (Wang & Gui, 2020). So, we used lognormal and Weibull distribution to model the claim severity of heart disease. By using the maximum likelihood method, the estimated parameter can be obtained as in Table 4.

Table 4. Fit Distributions of Claim Severity of Heart Disease

2019	Parameters	-Loglikelihood
Lognormal	$\mu = 13.9074$ $\sigma = 1.5181$	235889.2
Weibull	$a = 0.6271$ $b = 2486405$	237790.2
2020	**Parameters**	**-Loglikelihood**
Lognormal	$\mu = 13.9224$ $\sigma = 1.5239$	165003.5
Weibull	$a = 0.6231$ $b = 2531141$	166351.5

Based on Table 4, the lognormal distribution is the best fitted distribution for the claim severity of heart disease according to loglikelihood value (Delignette-Muller & Dutang, 2015). The parameters of the lognormal distribution in 2019 are $\mu_{2019} = 13.9074$ and $\sigma_{2019} = 1.5181$, while in 2020 are $\mu_{2020} = 13.9224$ and $\sigma_{2020} = 1.5239$.

4.3. Collective Risk Models of Heart Disease Claims Data

Consider the previous discussions, the zero-truncated Poisson distribution can be used to the distribution of claim frequency with the mean and variance in 2019 are 2.0125 and 1.2032 respectively, while in 2020 are 1.9135 and 1.0769 respectively. The claim severity has a lognormal distribution with parameters in 2019 are $\mu_{2019} = 13.9074$ and $\sigma_{2019} = 1.5181$, while in 2020 are $\mu_{2020} = 13.9224$ and $\sigma_{2020} = 1.5239$. The collective risk model is one of the model approaches of aggregate loss, where the claim frequency and severity are random. This model can be obtained by performing n simulations and the distribution of the claim frequency and severity have been known (Shevchenko, 2010).

The mean and variance of the collective risk models of heart disease in 2019 and 2020 are obtained, as in Table 5. Then, the graph of cumulative distribution the model are obtained, can be illustrated in Figure 4 (Dutang et al, 2008).

To find out whether the model obtained can explain aggregate losses well, it is necessary to pay attention to the error. The error is the difference between the estimated value and the actual value (Septiany et al, 2020). There are two types of error, there are absolute and relative errors. Absolute error is the absolute value of the error obtained from the difference between the analytical results and the simulation results, while the relative error is the comparison between the absolute error and the analytical results. The simulation results have been known in Table 5. The analytical results are obtained using equation (2) and equation (3) for both mean and variance of the collective risk models of heart disease. Table 6 shows the absolute and relative errors based on the mean and variance of the collective risk models of heart disease.

Table 5. Mean and Variance of Collective Risk Models of Heart Disease

Year	Mean	Variance
2019	6,952,569	2.3558×10^{14}
2020	6,788,026.2	2.2743×10^{14}

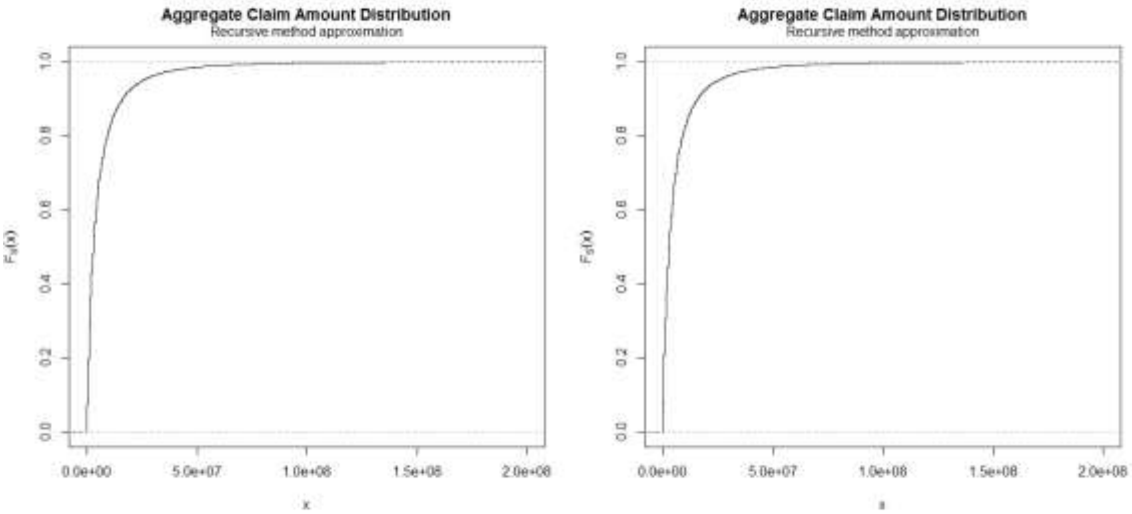

Figure 4. The Cumulative Distribution of Collective Risk Models of Heart Disease

To find out whether the model obtained can explain aggregate losses well, it is necessary to pay attention to the error. The error is the difference between the estimated value and the actual value (Septiany et al, 2020). There are two types of error, there are absolute and relative errors. Absolute error is the absolute value of the error obtained from the difference between the analytical results and the simulation results, while the relative error is the comparison between the absolute error and the analytical results. The simulation results have been known in Table 5. The analytical results are obtained using equation (2) and equation (3) for both mean and variance of the collective risk models of heart disease. Table 6 shows the absolute and relative errors based on the mean and variance of the collective risk models of heart disease.

Table 6. The Errors of Collective Risk Models

2019	Mean	Variance
Simulation Result	6,952,569	2.3558×10^{14}
Analytical Result	6,983,949	2.3314×10^{14}
Absolute Error	31,379.9	2.4324×10^{12}
Relative Error	0.4493%	1.0433%
2020	Mean	Variance
Simulation Result	6,788,026.2	2.2743×10^{14}
Analytical Result	6,800,527	2.3595×10^{14}
Absolute Error	12,501.09	8.5236×10^{12}
Relative Error	0.1838%	3.6125%

It is known in Table 6, the absolute and relative errors in 2019 and 2020 are relatively small. In other words, the collective risk model obtained can explain the aggregate losses properly.

5. CONCLUSION

The Poisson distribution can be used to the distribution of the claim frequency of heart disease with parameter $\boldsymbol{\lambda}$ in 2019 is 1.6104 and in 2020 is 1.4762. Participants who do not make a claim are not included in the data. Therefore, a zero-truncated distribution is used to obtain data suitability. This causes the changes in both the mean and variance of claim frequency of heart disease, in 2019 are 2.0125 and 1.2032, while in 2020 are 1.9135 and 1.0769. The fit distribution to the claim severity of heart disease is lognormal distribution, with parameters in 2019 are $\mu_{2019} = 13.9074$ and $\sigma_{2019} = 1.5181$, while in 2020 are $\mu_{2020} = 13.9224$ and $\sigma_{2020} = 1.5239$. So that the collective risk model is obtained that can explain well the aggregate losses of heart disease claims data in 2019 and 2020, consider the absolute and relative errors.

ACKNOWLEDGEMENT

I would like to thank all who have helped and encouraged me to complete this research. I would also like to extend my thanks to BPJS Kesehatan for the sample data.

REFERENCES

Ariawan, I. et al. (2021). *Data Sampel BPJS Kesehatan Tahun 2015-2020*. Jakarta: BPJS Kesehatan.

Chadidjah, A., Noviyanti, L., and Soleh, A. Z. (2019). Optimal Reinsurance Based on Compound Poisson Distribution. *Journal of Physics: Conf. Series*, Vol. 1317. DOI: 10.1088/1742-6596/1317/1/012022.

Cossette, H., Marceau, E., and Mtalai, I. (2019). Collective Risk Models with Dependence. *Insurance: Mathematics and Economics, 87*, 153–168. DOI: 10.1016/j.insmatheco.2019.04.008.

David, M. and Jemna, D. (2015). Modeling the Frequency of Auto Insurance Claims by Means of Poisson and Negative Binomial Models. *Scientific Annals of the "Alexandru Ioan Cuza" University of Iaşi Economic Sciences*, *62*(2), 151–168. DOI: 10.1515/aicue.

Delignette-Muller, M. L. and Dutang, C. (2015). fitdistrplus: An R Package for Fitting Distributions. *Journal of Statistical Software*, *64*(4), 1–34. DOI: 10.18637/jss.v064.i04.

DJSN & BPJS Kesehatan. (2021). *Statistik JKN 2015-2019*. Jakarta: DJSN and BPJS Kesehatan.

Dutang, C., Goulet, V., and Pigeon, M. (2008). actuar: An R Package for Actuarial Science. *Journal of Statistical Software*, 25(7), 1–37. DOI: 10.18637/jss.v025.i07.

Heniwati and Thabrany, H. (2016). Perbandingan Klaim Penyakit Katastropik Peserta Jaminan Kesehatan Nasional di Provinsi DKI Jakarta dan Nusa Tenggara Timur Tahun 2014. *Jurnal Ekonomi Kesehatan Indonesia*, *1*(2), 18–27. DOI: 10.7454/eki.v1i2.1771.

Katz, R. W. (2002). Stochastic Modeling of Hurricane Damage. *Journal of Applied Meteorology*, 41(7), 754–762. DOI: 10.2307/26185015.

Klugman, S. A., Panjer, H. H., and Willwot, G. E. (2019). *Loss Models from Data to Decisions*. 5th ed. New Jersey: John Wiley and Sons, Inc.

Lin, X. S. and Willmot, G. E. (2015). Collective Risk Models. *Wiley StatsRef: Statistics Reference Online*, 1-7. DOI: 10.1002/9781118445112.stat03558.pub2.

Omari, C., Nyambura, S. G., and Mwangi, J. M. W. (2018). Modeling the Frequency and Severity of Auto Insurance Claims Using Statistical Distributions. *Journal of Mathematical Finance*, 8(1), 137-160. DOI: 10.4236/jmf.2018.81012.

Pitacco, E. (2014). *Health Insurance Basic Actuarial Models*. Trieste: Springer Cham.

Pittara (2021). *Alodokter*. [Online] Available at: https://www.alodokter.com/penyakit-jantung [Accessed 24 February 2022].

Pramesti, G. (2011). Distribusi Rayleigh untuk Klaim Agregasi. *Media Statistika*, 4(2), 105-112. DOI: 10.14710/medstat.4.2.105–112.

Septiany, R., Setiawaty, B., and Purnaba, I. G. P. (2020). The Use of Monte Carlo Method to Model the Aggregate Loss Distribution. *Al-Jabar: Jurnal Pendidikan Matematika*, 11(1), 179–190. DOI: 10.24042/ajpm.v11i1.6599.

Shevchenko, P. V. (2010). Calculation of Aggregate Loss Distributions. *The Journal of Operational Risk*, 5(2), 3–40. DOI: 10.48550/arXiv.1008.1108.

Wang, N., Qian, L., Zhang, N., & Liu, Z. (2019). Modelling the Aggregate Loss for Insurance Claims with Dependence. *Communications in Statistics-Theory and Methods*. DOI: 10.1080/03610926.2019.1659368.

Wang, S. and Gui, W. (2020). Corrected Maximum Likelihood Estimations of the Lognormal Distribution Parameters. *Symmetry,* 12(968). DOI: 10.3390/sym12060968.