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APPLICATION OF DELTA GAMMA (THETA) NORMAL APPROXIMATION IN RISK MEASUREMENT OF AAPL'S AND GOLD'S OPTION

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Abstract: The option value has a nonlinear dependence relationship on risk factors existing in the capital market. Therefore, this paper considered utilizing Delta Gamma (Theta) Normal Approximation (DGTNA) as a nonlinear approach to determine the change of profit/loss of a European call option to assess the option risk. The method uses the second order of Taylor Polynomial around the stock price underlying the option to approximate the option profit/loss, which is crucial to construct the VaR based on DGTNA. VaR based on DGTNA also considered three Greeks, namely Delta, Gamma, and Theta, known as sensitivity measures in option. This research applied VaR based on DGTN approximation to analyze the European call option of Apple Inc (AAPL) and Barrick Gold Corporation (GOLD) for several strike prices. The performance of DGTN VaR analyzed by Kupiec Backtesting summarized that in this case, DGTN VaR provides the best risk assessment over different confidence levels (80, 90, 95, and 99 percent) compared to Delta Normal VaR and Delta Gamma Normal VaR.

1. INTRODUCTION

A call stock option is a right (not an obligation) to buy a specified amount of stock at a specified price and time. Measuring the options risk using the Greek letters of option has been utilized in some research, such as Jones and Schaefer (1999), Dziwago (2016), Lee et al. (2014), and Wang et al. (2017). The Greek letters, namely Delta, Gamma, Theta, Rho, and Vega, in option pricing, are considered as the option price sensitivities relative to changes in the value of risk factors influencing the option price (Hull, 2003).

The option price itself can be quantified by several models, such as the Black Scholes Model (BSM). Yu and Xie (2013) stated that a trader of options would utilize the Greek letters under the Black-Scholes framework (Black-Scholes Greeks) as a standard for appropriately adjusting option holdings to ensure that all risks are acceptable. The Black-Scholes Greeks have been embedded in the Value at Risk (VaR) concept in measuring the risk of the European call option. Different from stock prices which are linearly dependent on market risk factors, the option price under BSM has a nonlinear dependent on market risk factors. Because of those facts, the development of VaR methods that consider the nonlinear dependent is a prominent aspect in order to tackle the investment in the option. The development of option VaR was embarked on VaR based on the first Black Scholes Greek, namely Delta Normal approximation, later called Delta Normal VaR, which linearly measures the options risk. Delta Normal VaR has been utilized by Jones and Schaefer (1999), Ammann and Reich (2001), Duffie and Pan (2001), Sulistianingsih et al. (2019), Di Asih and Abdurakhman (2021), and Devitasari et al. (2023). Delta Gamma Normal VaR, studied by Mina and Ulmer (1999), Jones and Schaefer (1999), Duffie and Pan (2001), and Sulistianingsih et al. (2019), asses the option risk utilizing Delta and Gamma Greek. Meanwhile, Delta Gamma Theta Normal VaR utilizing the first third Greek in BS Greeks has been employed in some research, such as by Jaschke (2001), Cui et al. (2013), and Chen and Yu (2013).

This paper proposes the utilization of the Delta Gamma Theta Normal Approach (DGTNA) as a nonlinear method to assess the stock-option risk. DGTNA employs the second-order Taylor Polynomial to approximate the stock-option profit/loss (P/L). The DGTNA model is recognized and utilized in the field of finance due to its ability to provide accurate results within a short holding period of assets. The model utilized more Greeks rather than the Delta Normal Approach (DNA) and Delta Gamma Normal Approach (DGNA). DNA only uses one Greek, namely Delta, while DGNA utilizes two Greeks, namely Delta and Gamma, in order to measure the risk of individual (portfolio) stock options.

This study examined VaR based on the DGTNA of an individual stock option, Apple Inc (AAPL) and Barrick Gold Corporation (GOLD), at some confidence levels, namely 80%, 90%, 95%, and 99%. The data used for analysis in this study is daily closing prices from 14th April 2022 to 14th April 2023, accessed from https://finance.yahoo.com/.

2. LITERATURE REVIEW

2.1. Black Scholes Models

The price of option using the Black-Scholes Model at time t, which is specified by some aspects as a multivariable function, namely $C_t = f(S_t, K, r, t, \sigma)$ with the maturity time $\tau = T - t$ can be written as follows (Gao, 2009):

$$C_t = S_t N(d_1) - Ke^{-r\tau} N(d_2) \tag{1}$$

where C_t is the call-option price at time t (t indicates the time when investor buys the option), T is the expiration date, S_t is the stock price at time t, K is the exercise price, r is the free risk interest rate, τ is the validity period of the option, and σ is volatility. $N(d_1)$ represents the function of cumulative standard normal distribution of d_1, while $N(d_2)$ is the function of cumulative standard normal distribution of d_2 where $d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$ and $d_2 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$.

2.2. Value at Risk (VaR)

Value at Risk (VaR) is a risk measure of financial investment. VaR can be defined as the estimation of the maximum loss which may occur in the specified time period and at the specified interval of confidence ((Jorion, 2007); (Rosadi, 2009)). Using VaR, it can be estimated the amount of funds that can be allocated to overcome the potential of loss. VaR based on the assumption that the profit/loss is normally distributed at a level of confidence $1 - \alpha$ is as follows (Dowd, 2007):

$$VaR = Z_{1-\alpha} \sigma_{P/L_{t+\Delta t}} - \mu_{P/L_{t+\Delta t}} C_t = S_t N(d_1) - Ke^{-r\tau} N(d_2) \tag{2}$$

where $\mu_{P/L_{t+\Delta t}}$ and $\sigma_{P/L_{t+\Delta t}}$ subsequently are mean and standard deviation of profit/loss. $Z_{1-\alpha}$ is the value of standard normal variate such that $1-\alpha$ from probability density mass located at its left side, and α is probability density mass located on the right side. In application, $\mu_{P/L_{t+\Delta t}}$ and $\sigma_{P/L_{t+\Delta t}}$ are unknown distributed. Therefore, VaR will be estimated based on the estimated value of these parameters. In this research, $\mu_{P/L_{t+\Delta t}}$ and $\sigma_{P/L_{t+\Delta t}}$ are estimated by the formula of sample mean and variance of profit/loss.

2.3. The Option's Greek

In order to utilize options optimally, it is required to calculate the impact of changes in the parameters that influence the value of options (Poncet and Portait, 2022). The changes are quantified using the Greeks of the option, namely Delta, Gamma, and Theta, that analyze the change of the option price as a change of the option indicator.

1. Delta (δ)

   Delta measures the sensitivity of the option price relative to the change of the underlying asset price (in this research is stock). Delta is represented as the first derivative of the option price relative to the stock price (Chen et al., 2010). Delta can be expressed as the following equation (Gao, 2009):

   $$\delta = \frac{\partial C_t}{\partial S_t} \tag{3}$$

   where C_t is the call option price at time t, and S_t is the stock price (at time t) underlying the option. The formula of Delta for the European call option, which assumes there is no dividend during the life of the option is expressed as follows (Yu and Xie, 2013):

   $$\begin{aligned}\delta &= \frac{\partial C_t}{\partial S_t} = \frac{\partial\left(S_t N(d_1) - Ke^{-r\tau}N(d_2)\right)}{\partial S_t} \\ &= N(d_1) + S_t\frac{\partial N(d_1)}{\partial S_t} - Ke^{-r\tau}\frac{\partial N(d_2)}{\partial S_t} \\ &= N(d_1) + S_t\frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}\frac{1}{S_t\sigma\sqrt{t}} - Ke^{-r\tau}\frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}\frac{S_t}{K}e^{r\tau}\frac{1}{S_t\sigma\sqrt{t}} \\ &= N(d_1)\end{aligned} \tag{4}$$

   where $N(d_1) = \int_{-\infty}^{d_1}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dz$.

2. Gamma ($\boldsymbol{\Gamma}$)

   Gamma mesures the change of delta relative to the price change of the stock option price (Chen et al., 2010). Gamma can be formulated as follow (Gao, 2009):

   $$\Gamma = \frac{\partial\delta}{\partial S_t},$$

   where δ is given in Equation (4). The derivation of Gamma is given in the following equations (Yu and Xie, 2013):

   $$\Gamma = \frac{\partial\delta}{\partial S_t} = \frac{\partial(\partial C_t)}{\partial S_t(\partial S_t)} = \frac{\partial(\partial C_t)}{\partial S_t(\partial S_t)} = \frac{\partial\left(N(d_1)\right)}{\partial S_t} = \frac{1}{S_t\sigma\sqrt{t}}N'(d_1) \tag{5}$$

   where $N'(d_1) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}$.

3. Theta ($\boldsymbol{\Theta}$)

   Theta is the price option sensitivity relative to the change of time (Chen et al., 2010).

Theta is formulated as follow (Gao, 2009):

$$\theta = \frac{\partial C_t}{\partial t},$$

where C_t is the call option price at time t and τ is a difference of time t until the expiration time T, namely $\tau = T - t$. Therefore, Theta can be written as follows (Gao, 2009):

$$\theta = \frac{\partial C_t}{\partial t} = -\frac{\partial C}{\partial \tau} < 0$$

The derivation of Theta is given as follows:

$$\theta = \frac{\partial\left(S_t N(d_1) - Ke^{-r\tau}N(d_2)\right)}{\partial \tau}$$

$$= -S_t\left(\frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}\right)\left(\frac{\sigma^2}{2\sigma\tau^{\frac{1}{2}}}\right) - rKe^{-r\tau}N(d_2)$$

$$= -S_t\left(\frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}\right)\left(\frac{\sigma}{2\tau^{\frac{1}{2}}}\right) - rKe^{-r\tau}N(d_2)$$

$$= -\frac{S_t\sigma}{2\tau^{\frac{1}{2}}}N'(d_1) - rKe^{-r\tau}N(d_2) \qquad (6)$$

where $N(d_2) = \int_{-\infty}^{d_2}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}\,dz$.

2.4. Delta Gamma Theta Normal VaR

There are numerous VaR utilized to measure the risk of European call options. Three of the VaR methods are Delta Normal VaR, Delta Gamma Normal VaR, and Delta Gamma Theta Normal VaR. Delta Normal VaR is a risk measure to assess the risk of investment in an option using a linear approximation of option profit loss by the first order of Taylor Polynomial. Meanwhile, Delta Gamma Normal VaR and Delta Gamma Theta Normal VaR are measuring the option risk utilizing the second order of Taylor Polynomial. The latter methods are familiar called nonlinear VaR.

Delta Normal VaR (VaR using Delta Normal Approximation), Delta Gamma Normal VaR (VaR using Delta Gamma Normal Approximation), and also Delta Gamma Theta Normal VaR (VaR using Delta Gamma Theta Normal Approximation) in this research have some assumptions. The first assumption in Delta Normal VaR is that there is a linear relationship between the return of the stock price and the return of the option price. Meanwhile, in Delta Gamma Normal VaR and Delta Gamma Theta Normal VaR, the return of the stock price and the return of the option price are in a nonlinear relationship. Then, the second assumption is that the return of stock price underlying the option is assumed to follow a normal distribution with zero mean and standard deviation (σ) (Sulistianingsih et al, 2019).

The concept of quantifying VaR using Delta Normal Approximation (DNA) and Delta Gamma Normal Approximation (DGNA) based on specified assumptions is provided by Sulistianingsih et al. (2019). VaR calculation using DGTNA also employs Taylor Polynomial of second order to approximate stock option profit/loss around S_t.

The difference between the two methods is the addition of $\theta\Delta t$ in the formula of the profit/loss as expressed in the following equation (Castellacci and Siclari, 2003):

$$\Delta C_t \approx \left(\frac{\partial(C_t)}{\partial S_t}\right)(\Delta S_t) + \frac{1}{2}\left[(\Delta S_t)'\left(\frac{\partial^2(C_t)}{\partial S_t \partial S_t}\right)(\Delta S_t)\right] + \theta\Delta t. \quad (7)$$

Because $\frac{\partial^2(C_t)}{\partial S_t \partial S_t} \approx \left(\frac{\partial^2 f(S_t)}{\partial S_t \partial S_t}\right)$ where $\left(\frac{\partial^2 f(S_t)}{\partial S_t \partial S_t}\right)$ is a matrice of second order derivative from $f(S_t)$ which is relative to the individual risk factors and $\frac{\partial(C_t)}{\partial S_t} = \delta$ as stated in Equation (4), Equation (7) can be written as Equation (8).

$$\Delta C_t \approx \delta\sigma_{\Delta S_t} + \frac{\gamma}{2}(\Delta S_t)^2 + \theta\Delta t \quad (8)$$

Based on Equation (8) and assumptions utilized in DGTNA VaR, it can be obtained mean and variance of the stock option profit/loss as follows subsequently

$$E[\Delta C_t] = E\left[\delta\Delta S_t + \frac{1}{2}\gamma(\Delta S_t)^2 + \theta\Delta t\right] = E[\theta\Delta t] = \theta\Delta t \quad (9)$$

and

$$Var[\Delta C_t] = Var\left[\delta\Delta S_t + \frac{1}{2}\gamma(\Delta S_t)^2 + \theta\Delta t\right] = \delta^2\sigma^2_{\Delta S_t} + \frac{\gamma^2}{4}\sigma^4_{\Delta S_t} \quad (10)$$

VaR based on DGTNA is derived by substituting the value of mean and standard deviation of profit/loss in Equation (2). Therefore, VaR based on DGTNA (VaR_{DGTNA}), for assessing the individual stock option risk can be formulated as Equation (11).

$$\begin{aligned} VaR_{DGTNA} &= Z_{1-\alpha}\sqrt{\delta^2\sigma^2_{\Delta S_t} + \frac{\gamma^2}{4}\sigma^4_{\Delta S_t}} - (\theta\Delta t) \\ &= Z_{1-\alpha}\sqrt{\delta^2\sigma^2_{\Delta S_t} + \frac{\gamma^2}{4}\sigma^4_{\Delta S_t}} - (\theta\tau) \quad (11) \end{aligned}$$

2.5. Kupiec Backtesting

Rosadi (2009) stated that Backtesting is a method utilized to test the validity of the proposed model VaR. Using Backtesting, it can be identified how much the VaR model describes the actual data.

There are some methods to quantify Backtesting, two of the methods are a method based on the number of loss frequencies occurring on the tail of the distribution and based on the forecast evaluation approach. This research only utilizes Backtesting based on the frequency of tail losses, namely Kupiec Backtesting.

Suppose that t^* is the number of loss frequencies occurring in the tail of the distribution (the number of observations that are greater than VaR) and n is the number of observations. Meanwhile, p represents the percentage tolerance limit for the magnitude of deviation from the VaR value of which the amount is determined (by one minus the confidence level of VaR) ((Dowd, 2007); (Jorion, 2007); (Rosadi, 2009)).

Then, t^* has a binomial distribution with

$$P(t^* | n, p) = \binom{n}{t} p^{t^*}(1-p)^{n-t^*}, \qquad t^* = 0,1,2,\dots,n.$$

The hypothesis test which is used in Kupiec Test is as follows (Rosadi, 2009):

$H_0: P(t^*) \leq p$

$H_1 \colon P(t^*) > p,$

where $P(t^*) \leq p$ indicated that the VaR Model is good to work with.

Then, the statistic for Kupiec Test is as follows

$\hat{\alpha} = P(T > t^* \mid p^* = p) = 1 - P(T \leq t^* \mid p^* = p),$

where $P(T \leq t^* \mid p^* = p) = \sum_{t^*=0}^{n} \binom{n}{t} p^{t^*} (1-p)^{n-t^*}$ is a cumulative distribution of Binomial Distribution. Therefore, it can be written that

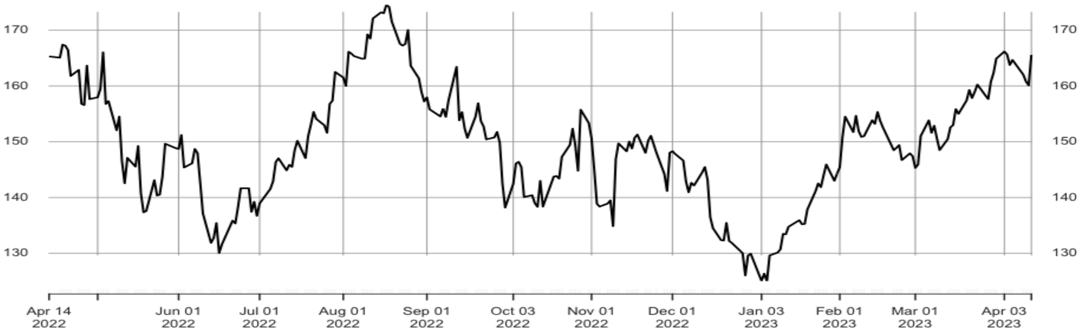
$$\hat{\alpha} = 1 - \sum_{t^*=0}^{n} \binom{n}{t} p^{t^*} (1-p)^{n-t^*} \tag{12}$$

H_0 will be rejected if $\hat{\alpha}$ is less than the level of significance. Then, if H_0 is not rejected, it can be concluded that the VaR model is proper to be employed.

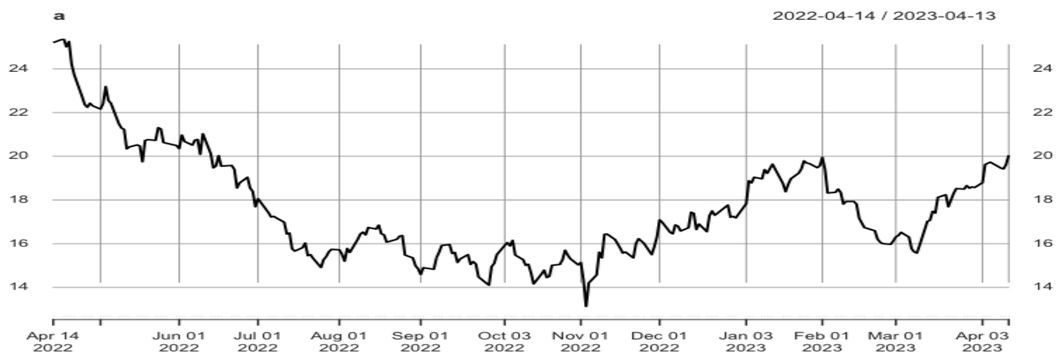
3. MATERIAL AND METHOD

3.1. Data and Source

The option risk that was analyzed in this section was the call options of Apple Inc. (AAPL) and Barrick Gold Corporation (GOLD). The analyzed data of the underlying was taken from 14th April 2022 to 14th April 2023, where the graph of the analyzed stock prices is given in Figure 1. From Figure 1, it can be seen that there is an increasing trend in both stock prices from March 2023 to April 2023.



(a)



(b)

Figure 1. Graphs of The Analyzed Stock Price Underlying the Options
(a) Graphs of AAPL's Stock Price (b) Graphs of GOLD's Stock Price

The price of the stock underlying the analyzed option of AAPL and GOLD subsequently was 160.1 and 19.63 dollars. Both prices are provided at https://finance.yahoo.com. The interest rate used in this section was 4.75%. Then, the strike prices (K) of the call option, which were analyzed, are 95 dollars, 100 dollars, 115 dollars, 165 dollars, 170 dollars, and 175 dollars for AAPL. Meanwhile, the strike prices (K) of the call option, which were analyzed, are 10 dollars, 11 dollars, 12 dollars, 22 dollars, and 23 dollars for GOLD.

3.2. Analysis Steps

This section contains steps of analysis utilized to apply Delta Gamma (Theta) Normal VaR in measuring the option risk. The steps are given as follows:

1. Collected the close price of Apple Inc. (AAPL) and Barrick Gold Corporation (GOLD) at a predetermined time period
2. Calculated the return of close prices of AAPL and GOLD
3. Conducted normality test on the analyzed return data
4. Estimated Parameters of Volatility, Delta, Gamma, and Theta
5. Estimated VaR based on DGTNA at several confidence levels.
6. Testing the validity of VaR based on DGTNA using Kupiec Backtesting at several confidence levels.

4. RESULTS AND DISCUSSION

This section provided the application of Delta Gamma (Theta) Normal Approximation (DGTNA) to measure the risk of individual European Call options. The application to quantify VaR based on DGTNA is conducted using Equation (11). The formula in Equation (11) is closed form so it is easier to utilize rather than the previous research conducted by Chen and Yu (2013). This application is also to complete the previous research conducted by Sulistianingsih et al (2019) which is only to apply VaR using DNA and DGNA which are not yet employing Theta to assess the option risk on the real data.

Before implementing the DGTNA VaR, the normality test is conducted to investigate whether the return of the close stock price of AAPL in the specified period distributes normally or not. The normality test utilized in this research is Kolmogorov-Smirnov Test. The test summarised that the $p - values$ of Kolmogorov Smirnov's statistics are subsequently 0.7372 and 0.2300 for AAPL and GOLD, which are greater than 0.05. Therefore, the analyzed return data are normally distributed.

Then, we need to calculate the volatility of the stock prices underlying the analyzed options. After that, the BS Greeks written in Equations (4), (5), and (6) utilized to quantify the DGTNA VaR, namely Delta, Gamma, and Theta, are quantified. The volatility of the stock price underlying the options is 0.34218, Delta, Gamma, and Theta for the analyzed options are provided in Table 1. In Table 1, K is provided to show that there are various strike prices values in estimating Delta, Gamma, and Theta which are utilized to quantify DGTN VaR.

Individual VaR based on DGTNA for the analyzed options calculated using Equation (11) was summarized in Table 2. Table 2 indicates that the higher level of confidence, the higher VaR based on DGTNA. The performance of VaR based on DGTNAfor this case was measured by Kupiec Backtesting. The results of Kupiec Backtesting are provided in Table 3. TCS in Table 3 is the Ticker Code of Stocks, K is the strike price, PLO is the percentage of tail losses, and P-Value is the value of Equation (12) for the corresponding analyzed VaR.

Table 1. Estimated Parameters of Volatility, Delta, Gamma, and Theta

Ticker Code of Stock	Volatility	K	Delta	Gamma	Theta
AAPL	0.34218	95	0.99990	0.00002	-4.47774
		100	0.99962	0.00006	-4.72244
		115	0.99194	0.00095	-5.60839
		165	0.46991	0.01709	-7.83932
		170	0.38946	0.01647	-7.14912
		175	0.31562	0.01527	-6.33202
GOLD	0.38112	10	1.00000	1.00000	0.00000
		11	1.00000	1.00000	0.00000
		12	1.00000	1.00000	0.00000
		22	0.14406	0.11612	-0.34962
		23	0.06570	0.06542	-0.18368

Table 2. VaR using Delta Gamma (Theta) Normal Approximation for AAPL's Stock Option and GOLD's Stock Option

Ticker Code of Stock	K	80% VaR	90% VaR	95% VaR	99% VaR
AAPL	95	0.72614	1.14903	1.46041	2.04450
	100	0.76031	1.20290	1.52879	2.14011
	115	0.87343	1.38132	1.75529	2.45680
	165	1.10869	1.75158	2.22495	3.11290
	170	0.98345	1.55367	1.97353	2.76112
	175	0.84025	1.32741	1.68611	2.35898
GOLD	10	0.30351	0.49223	0.63119	0.89185
	11	0.30394	0.49280	0.63187	0.89273
	12	0.30439	0.49341	0.63258	0.89366
	22	0.04916	0.07900	0.10096	0.14217
	23	0.02318	0.03720	0.04752	0.06687

Table 3. Kupiec Backtesting of DGTNA VaR for The Analyzed Option's Risks

TCS	K	cl = 80%		cl = 90%		cl = 95%		cl = 99%	
		PLO	PV	PLO	PV	PLO	PV	PLO	PV
AAPL	95	0	1	0	1	0	1	0	1
	100	0	1	0	1	0	1	0	1
	115	0	1	0	1	0	1	0	1
	165	0	1	0	1	0	1	0	1
	170	0	1	0	1	0	1	0	1
	175	0	1	0	1	0	1	0	1
GOLD	10	0	1	0	1	0	1	0	1
	11	0	1	0	1	0	1	0	1
	12	0	1	0	1	0	1	0	1
	22	1.60643	1	0.40160	1	0	1	0	1
	23	17.26908	0.84096	6.42570	0.96773	1.60643	0.9953	0.40160	0.71218

Table 3 indicates that the performance of DGTNA VaR analyzed by Kupiec Backtesting is proper to measure the risk. It also can be checked that DGTNA VaR provides the best risk assessment over different confidence levels (80, 90, 95, and 99 percent) compared to both methods, namely Delta Normal VaR and Delta Gamma Normal VaR.

5. CONCLUSION

Based on the results of the analysis, it can be concluded that VaR based on DGTNA VaR, for this case, can be effective in measuring the risk of AAPL's and GOLD's call options. Moreover, similar to the concept in Delta Normal VaR and Delta Gamma Normal VaR, Delta Gamma (Theta) Normal VaR increases as the level of confidence used in the VaR calculation increases.

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