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DETERMINATION OF INSURANCE PREMIUMS FOR CHILI PLANTATION USING THE BLACK-SCHOLES MODEL WITH CLAYTON COPULA APPROACH

Sarah Sutisna¹, Sukono², Herlina Napitupulu²

¹Masters Program of Mathematics, Padjadjaran University, Jawa Barat, Indonesia ² Department of Mathematics, Padjadjaran University, Jawa Barat, Indonesia

e-mail: sukono@unpad.ac.id

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Abstract: Agriculture is a vulnerable sector to the risk of crop damage due to climate change and other environmental factors. One source of risk in agriculture is rainfall, which significantly affects productivity and farmers' income. Traditional insurance premium calculations often rely on assumptions of normal distribution and linear dependency, which may not accurately capture the complex and non-linear relationships between climatic and agricultural variables. This research presents a novel contribution to agricultural risk management by applying the Clayton Copula to model the dependency structure between rainfall and chili crop production output in the context of crop insurance pricing. The estimation of Copula parameters was conducted using Maximum Likelihood Estimation, yielding a parameter θ value of -0.1252, which indicates the dependency structure between the variables. The predictive accuracy of the Copula Clayton model was evaluated using the Mean Absolute Error, with a result of 0.01291, demonstrating strong relevance in describing the dependency between precipitation and yield. Furthermore, the research integrates the Copula-based rainfall modeling with the Black-Scholes model for determining insurance premiums. The findings reveal that premium prices depend on rainfall index values, where higher rainfall percentages correspond to higher premium costs.

1. INTRODUCTION

The agricultural sector plays a vital role in enhancing national well-being by ensuring food supply, creating employment opportunities, contributing to the economy, supporting regional development, and promoting environmental sustainability (Mulyo, 2016). Among the various branches of agriculture, the horticultural subsector is one of the most influential (Adhiana, 2021). According to the Central Bureau of Statistics (BPS), the national demand for cayenne pepper reached 569.65 thousand tonnes in 2022, an increase of 7.86% or 41.51 thousand tonnes from the 2021 consumption level of 528.14 thousand tonnes. Despite its economic importance, horticultural farming, particularly chili cultivation, remains vulnerable to various uncontrollable risks. Therefore, effective risk management mechanisms are essential to safeguard farmers from potential losses.

Agricultural insurance has emerged as an important risk management tool (Mao et al., 2023), offering protection against yield losses, price fluctuations, and other agricultural uncertainties. Accurate premium pricing plays a crucial role in ensuring the long-term sustainability of agricultural insurance programs (Chen et al., 2022), as it directly influences farmers' participation, affordability, and the financial viability of insurance providers. A number of studies have examined agricultural insurance from various perspectives. Research conducted by Estiningtyas (2015) explored climate index—based agricultural insurance and found that the climate insurance index has the potential to be developed and implemented in Indonesia. Similarly, Sari (2023) investigated premium calculation based on a rainfall index using the exponential distribution power plant method. However, both studies relied on the assumption that the data followed a specific probability distribution.

To address this limitation, a more flexible modeling approach was introduced through the Copula Model. According to Nelsen (2006), the Copula model has gained attention in statistics because it enables the study of dependency structures between variables on a free (scale-invariant) basis and serves as a foundation for constructing bivariate distribution families. Well-known copula families include the Elliptical and Archimedean Copulas. Among these, the Clayton Copula, a member of the Archimedean family, is particularly effective in modeling extreme dependencies between variables. Since rainfall and agricultural output data often exhibit extreme values, the Clayton Copula is suitable for capturing the dependency structure between them. This copula is characterized by its ability to model lower-tail dependency, which is useful in describing co-movements in adverse conditions.

The use of the copula method in this research is essential because it allows for a more accurate modeling of the dependency structure between rainfall and output data, especially under extreme conditions; the combination of the copula approach with traditional statistical methods provides a more comprehensive understanding of both linear and nonlinear relationships, capturing tail dependencies that conventional correlation measures often fail to represent. Fang and Madsen (2013) investigated insurance applications using the Gaussian Copula model, one of the most popular copulas for financial and insurance risk modeling, to measure dependencies among agricultural variables. Shi et al. (2016) further applied the Gaussian Copula to model insurance claims, identifying both cross-sectional and temporal dependencies between layered claims. Their results demonstrated that copula models improve the management of insurance claims. Additionally, Hoyer and Kuss (2018) compared the Gaussian Copula with the Vine Copula in a simulation study of type 2 diabetes, concluding that the Vine Copula provided superior performance.

Parallel to these developments, extensive research has also utilized the Black-Scholes method in the pricing of insurance premiums. Chicaíza and Cabedo (2009) applied the Black-Scholes model to estimate high-cost health insurance premiums in Colombia, finding that the estimated premiums were comparable to those obtained through traditional actuarial methods. Similarly, Suarjana et al. (2017) used the Black-Scholes model to determine the value of agricultural business contracts based on international coffee price parameters and local farmer price levels using data from the Bali Provincial Agriculture Service (2001–2015). Okine (2014) applied the Black-Scholes framework to price index-based insurance in Ghana, using rainfall data from the Tamale district as the index parameter. The study concluded that rainfall data showed a strong correlation with corn yield, validating the use of rainfall as an insurance index. Furthermore, Lestari et al. (2017) extended Suarjana's work by employing a mean reversion model with jumps to determine insurance premium values through Monte Carlo simulations. Their findings indicated that premiums varied depending

on the trigger value, with premium rates decreasing between 5% and 25%, suggesting the model's sensitivity to different loss thresholds.

Based on these studies, this paper focuses on analyzing the dependency structure between rainfall and agricultural output using the Clayton Copula approach. After estimating the parameters of the Clayton Copula, the Cumulative Distribution Function (CDF) of the copula is derived. The resulting CDF is then transformed into a Lognormal distribution, consistent with the assumption that rainfall data follows a lognormal pattern. Finally, the Black-Scholes model is applied to determine the appropriate premium price based on the transformed rainfall data obtained from the Clayton Copula CDF. This integrated approach aims to provide a more accurate and theoretically sound framework for agricultural insurance premium determination.

2. LITERATURE REVIEW

2.1. Uniform Distribution

Uniform distribution is one of the important things to know when doing research using the Copula approach (Ly et al., 2004) by finding the value of the CDF (Cumulative Distribution Function). A continuous random variable X is said to be a uniform distribution that is noted with U(a,b), has a density function of probability given by Equation (1)

$$f(x) = \frac{1}{b-a} \text{ for } a < x < b \tag{1}$$

2.2. Random Variable Transformation to Uniform Distribution [0, 1]

Transforming a random variable to a uniform domain [0, 1] is the first step in performing a Copula analysis. The marginal distribution of an unknown random variable can be determined through Equation (2),

$$F_{x}(x) = \frac{1}{n+1} \sum_{i=1}^{n} I(X_{i} \le x); x \in \mathbb{R}$$
 (2)

with $I(X_i \le x)$ is an indicator function that is worth 1 if $X_i \le x$ and value 0 if not.

The transformation process is done by creating a rank for each random variable. R_1, R_2, \ldots, R_p is the rank of X_1, X_2, \ldots, X_p which has previously been converted into the respective matrix divided by n + 1 as follows,

$$\left(\left(\frac{R_1}{n+1}\right), \left(\frac{R_2}{n+1}\right), \dots, \left(\frac{R_n}{n+1}\right)\right)$$

Therefore, the copula equation with the transformation as follows

$$C(u_1, u_2, ..., u_p) = \frac{1}{n} \sum_{j=1}^{n} I\left(\frac{R_1}{n+1} \le u_1, \frac{R_2}{n+1} \le u_2, ..., \frac{R_n}{n+1} \le u_n\right); \tag{3}$$

 $u_1, u_2, \dots, u_p \in (0.1)$

with I(.) on Equations (2) and (3) is an indicator function if respectively $X^j \le x$ and $\frac{R_i}{i+1} \le u_i$; i = 1, 2, ..., p. The variables $u_1, u_2, ..., u_p$ are the marginal cumulative distribution function (CDF) values of the p random variables $X_1, X_2, ..., X_p$.

2.3. Copula

Copula is used to describe the joint distribution of random variables (Cherubini et al., 2004). The definition of n-dimensional copula by (Nelsen, 2006) noted by C is the F-

multivariate distribution function of the random variable $X_1, X_2, ..., X_n$ with its marginal distribution $F_1, F_2, ..., F_n$ is a standard uniform distribution, given by the Equation (4),

$$F_i \sim U(0,1); i = 1, 2, ..., n$$
 (4)

This Copula function is a function that has a domain $[0,1]^n$ and a range [0,1], which is represented by $C: [0,1]^n \to [0,1]$. Mathematically, copula C is expressed by the Equation (5),

$$C(u, v) = Pr[U \le u, V \le v] \tag{5}$$

Copula as a joint distribution of continuous random variables X and Y has distribution functions in sequence F and G, then a new random variable can be formed namely U = F(X) and V = G(Y). Based on the Sklar theorem (Nelsen, 2006) the distribution function of bivariate X and Y is defined by the Equation (6),

$$H(x,y) = Pr[X \le x, Y \le y]$$

$$= Pr[F(X) \le F(x), G(Y) \le G(y)]$$

$$= Pr[U \le u, V \le v]$$

$$= C(u,v)$$
(6)

for a copula C. If H is a bivariate distribution function with marginal distribution functions F and G, then there is a copula C for all (x, y) such that,

$$H(x,y) = C(F(x),G(y)) \tag{7}$$

The bivariate probability density function corresponding to copula C can be expressed in the Equation (8),

$$h(x,y) = f(x)g(y)c(F(x),G(y))$$
(8)

where

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v} \tag{9}$$

is the copula density function, with c(u, v) = 1, if X and Y are mutually free.

2.4. The Clayton Copula

The Clayton Copula belongs to the Archimedean Copula group. The Archimedean family of copulas has a characteristic that a copula has a single parameter of dependence (θ) and can be formed from a function of producing copula φ . The Archimedean family of Copulas is commonly used to model the common cumulative distribution function of two random variables whose distribution tail is fat (Chao & Zou, 2018). The Archimedean dome family is a family of functions $C: [0,1] \times [0,1] \rightarrow [0,1)$ defined by the Equation (10),

$$C(u,v) = \varphi^{-1}(\varphi(u) + \varphi(v)) \tag{10}$$

where $\varphi: [0,1] \to [0,\infty)$ is a continuous function, strictly decreasing, convex, and $\varphi(1) = 0$. While the Clayton Copula is defined by the following Equation (11)

$$C_{C,\theta}(u,v) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{-\frac{1}{\theta}} \tag{11}$$

with $\theta \in (0, \infty)$. The Clayton Copula generator function is given by the Equation (12)

$$\varphi_{\theta}(t) = \frac{1}{\theta} \left(t^{-\theta} - 1 \right) \tag{12}$$

The Copula Clayton density function is defined by the Equation (13),

$$c_{C,\theta}(u,v) = \frac{\theta+1}{(uv)^{\theta+1}} \left(u^{-\theta} + v^{-\theta} - 1 \right)^{-\left(\frac{2\theta+1}{\theta}\right)}$$
(13)

The parameters θ of the Clayton Copula are given by the Equation (14),

$$\theta = \frac{2\tau}{1-\tau} \tag{14}$$

2.5. Copula Parameter Estimation

The prediction of the copula parameters can be obtained using the method of Maximum Likelihood Estimation (MLE). By describing the given parameter copula and the marginal spread, the MLE is obtained by maximizing the log likelihood function. Nis given a random variable n-dimensional vector of the multivariate distribution, $\hat{x}_1, \dots, \hat{x}_N$, with $\hat{x}_j = (\hat{x}_{j,1}, \dots, \hat{x}_{j,n}), j \in \{1, \dots N\}$ a parametric model for the marginal distribution F_1, F_2, \dots, F_n with the parameter $\alpha_1, \alpha_2, \dots, \alpha_n$ and the Copula parameter is θ so that the density of the multivariant distribution f can be written as follows

$$f(x_1, x_2, \dots, x_p) = c(F_1(x_1), F_2(x_2), \dots, F_p(x_p) \prod_{i=1}^n f_i(x_i).$$
 (15) with c is the copula density and f_1, f_2, \dots, f_n is the density of the marginal distribution.

The parameters of the marginal distribution, α_1 , α_2 , ..., α_n and parameters from copula θ , can be estimated from data using MLE as follows (Joe and Xu 1996)

$$\arg \max_{a_n, \theta} \sum_{i=1}^n \ln[c\left(F_1(\hat{x}_{j,1}; \alpha_1), F_2(\hat{x}_{j,2}; \alpha_2), \dots, F_n(\hat{x}_{j,n}; \alpha_n); \theta \prod_{i=1}^n f_i(x_{j,i}; \alpha_i)\right]$$
(16)

2.6. Copula Error Checker

The selection of the best models in this study was based on the level of prediction error. The smaller the error rate, the more accurate the prediction will be. In this study, the Mean Absolute Error (MAE) method will be used in the calculation of errors. A good model has an MAE of less than 10%. The MAE indicates the magnitude of the error from the theoretical population to the empirical cupola. Given C(u, v) which the theoretical population will test its compatibility, in this case, the Clayton Copula. The MAE value can be obtained using the Equation (17)

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |C(u_i, v_i) - C_n(u_i, v_i)|.$$
 (17)

With (u_i, v_i) for i = 1, 2, ..., n is the pair data and $C_n(u_i, v_i)$ is the empirical copula. Basically, the empire copula is used to approach the theoretical copula.

2.7. Black-Scholes Model

The price of the European type put option determined by the Black-Scholes formula is as follows (Ariyanti et al., 2020):

$$P = Ke^{-rT}N(-d_2) - S_0N(-d_1)$$
(18)

with,

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma_T \sqrt{T}} \tag{19}$$

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma_T \sqrt{T}} = d_1 - \sigma_T \sqrt{T}$$
(20)

where P is put option price, S_0 is initial stock price, K is option strike price, r is annual risk-free interest rate, σ_T is standard deviation of stock price, T is time until maturity, $N(-d_1)$ is standard normal cumulative distribution function of d_1 , and $N(-d_2)$ is standard normal cumulative distribution function of d_2 .

There are several similarities between option pricing and index insurance (Purwandari et al., 2024). Therefore, index insurance can be formulated the same as the option price. In

determining the price of index insurance using the Black-Scholes method, the following can be considered:

- 1) The benchmark value for index insurance is R_T .
- 2) The payment structure for index insurance is one at a time.
- 3) The index follows a Lognormal distribution.

In the Black-Scholes model adapted for rainfall insurance, rainfall (R) is considered the underlying asset underlying the insurance contract. Therefore, R_0 which is the latest rainfall value or the average historical rainfall value relevant at the time of premium calculation, replaces S_0 . So, the agricultural insurance premium value can be calculated by first finding the cumulative distribution value d_2 with Equation (21),

$$d_2 = \frac{\ln\left(\frac{R_0}{R_T}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{21}$$

where R_0 is the latest rainfall values, R_T is benchmark value (rainfall selected as index), σ is standard deviation of rainfall index, r is risk free interest rate, and T is time.

The transformation process for calculating the agricultural insurance premium based on the rainfall index involves adapting financial option pricing principles, particularly from the Black-Scholes model, to quantify weather-related risks. First, the key variables are identified, including the coverage value (P), the risk-free interest rate (r), the time to maturity (T), and the probability that rainfall will fall below the trigger level, represented by $N(-d_2)$. Historical rainfall data are analyzed to estimate the likelihood of a rainfall deficit, and this probability is expressed using the cumulative distribution function of a standard normal distribution. The expected coverage value (P) is then discounted to its present value using the exponential factor e^{-rT} , which accounts for the time value of money by reflecting how future payouts are worth less in present terms. The final premium value is obtained by multiplying the discounted coverage value by the probability of rainfall falling below the trigger threshold, resulting in the Equation (22),

Premiums =
$$Pe^{-r(T)}N(-d_2)$$
 (22)

where P is the value of coverage, $N(-d_2)$ is the probability that the rainfall is less than the trigger value of rainfall, r is the risk-free interest rate, and T is time.

3. MATERIAL AND METHOD

This study adopts a quantitative research approach to determine fair insurance premiums for chili plantations by integrating the Black–Scholes model with the Clayton copula approach. The observation data in this study are monthly data on cane agricultural output and rainfall from 2012 to 2022. Cane agricultural output data is taken from the Department of Agriculture of Lake Malaya District and the rainfall data are taken from NASA. Cane farming yield data is indicated by the random variable Y and the data of rainfall is expressed by the random variable X.

The Clayton copula is applied to model the dependency structure among these risk variables, capturing lower-tail dependence that reflects extreme losses in production. The Black–Scholes model is then adapted to calculate expected losses as option values, using the volatility of chili prices and yields to estimate fair premium rates. Model performance is validated through back-testing and statistical error metrics such as MAE, ensuring reliability in premium estimation for agricultural risk management.

The computational analysis is performed using R software with libraries for time-series analysis, copula modeling, and stochastic simulation.

4. RESULTS AND DISCUSSIONS

The observation data in this study are monthly data on cane agricultural output and rainfall from 2012 to 2022. Cane agricultural output data is taken from the Department of Agriculture of Lake Malaya District and the rainfall data are taken from NASA. Cane farming yield data is indicated by the random variable Y and the data of rainfall is expressed by the random variable X. Descriptive statistics of the data can be seen in Table 1.

Table 1. Descriptive Marginal Data Statistics

	Mean	Median	Maximum	Minimum	Std. Dev.	Variance	Kurtosis	Skewness
X	10.4179	10,207	30.8354	0.19193	7.18409	51.6112	-0.49825	0.40910
Y	300,701	278	679.2	114.19	115,268	13286.7	1.0727	0.98177

Based on Table 1, the skewness values of rainfall and sequential harvest are 0.40910 and 0.98177, both positive values indicating provincial and farmer price data sliding to the right. For kurtosis on rainfall data and sequential harvest yields -0.49825 and 1.0727, both values are less than 3 which indicates that both data have peaked flat tendencies. The data can be said to be distributed normally if the value of the curve is equal to three and the value for the skewness is equal to zero.

The relationship between rainfall and harvest can be described in the form of a scatterplot, can be seen in Figure 1.

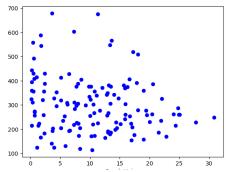


Figure 1. Scatterplot Rainfall and Production

The resulting plot of the output data range with rainfall forms an unlinear pattern in Figure 1, making it difficult to identify the relationship between variables. Therefore, further analysis was carried out using the copula method, where theoretically it has been proven that the copula is a parameter that can explain relationships better than a plot of data frames. Copula is a method that is not limited to the assumptions, especially the normal assumption.

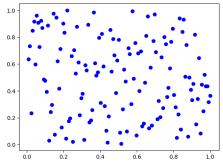


Figure 2. Scatterplot Pseudo Rainfall and Production

The pseudo log likelihood method transforms empirical observations of rainfall and production into pseudo observations of u and v without taking the functional form of the marginal distribution. To calculate pseudo-observations, Equation (3) is used, and scatterplot observation pseudo data of precipitation and production can be seen in Figure 2.

Based on Figure 2, no search data is visible. Some surveillance data is nearby, but patterns of data are not yet visible. So, Copula can be used to further analyze related structures of dependency between rainfall and production. The copula used was from the Clayton family because there was a supposedly extreme incident and a connection at the extreme point

The estimation of the Copula Clayton parameters has been done using maximum likelihood, using R software based on Equation (16). In brief, the steps are as follows:

- 1. Determine the Kendall's Tau correlation between the pairs of rainfall and production data.
- 2. Determine the Clayton copula parameter based on its relationship with the Kendall's Tau correlation from step 1. This stage is performed using the R software.

The value of the copula Clayton parameters $\hat{\theta} = -0.1252$. To compare observation data with simulation data through empirical dome distribution, the simulation is done by constructing the following data series using the Clayton Copula model using previously obtained parameters. To make the distribution clearer, this simulation uses R software to simulate 5000 data. The distribution plot is shown in Figure 3.

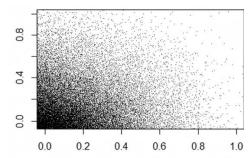


Figure 3. Scatterplot Clayton Copula Simulation Data

The pattern of relationship between rainfall and output that follows the Clayton copula describes that there are extreme events at low values, and there is a relationship between the two variables when both values are low, the higher the observed values on the variable, the weaker the relationship between them is, because the Clayton Copula has a relationship tail at the bottom (Cherubini et al., 2004).

The sample data was observed closer to the log-normal distribution. Next, check Copula's prediction error using MAE; the MAE value shows the magnitude of the error from the theoretical population to the empirical copula (see empirical excerpt in Table 2).

Rainf	all	Product	Emminio	
CDF	Rank	CDF	Rank	Empiric
0.560964	79	0.593377	127	0.325758
0.559063	33	0.807314	95	0.454545
0.533256	93	0.609724	128	0.325758
:	:	:	:	:
0.720210	101	0.591413	5	0.393939

Table 2. Copula Empiric

The MAE value for Copula Clayton can be seen in Table 3 based on equations (17) with R software.

Table 3. Copula MAE Values

Copula	MAE
Clayton	0.01291

Based on the table, the Copula Clayton error value is 0.01291. A copula error value below 1% is still relevant. Therefore, the Clayton Copula can be used to model the dependency between the rainfall variable and the yield of chili.

Agricultural insurance liabilities are the basis for the calculation of premiums and the maximum limit in compensation for losses. The accountability value is based on the cost of cane production, fixed costs and operating costs in one hectare from the beginning of the farmer's planting to the harvest obtained from the Food and Agriculture Service of Tasikmalaya District. Production costs include seeds, fertilizers, pesticides, POT, ZPT, and rapia. Costs still include land rental and cutting tools. Operating costs include from preplanting processing to post-harvesting. The cost details can be seen in Table 4.

Table 4. Chili Resource Value

	T. 00		TD 1
No	Type of Cost		Total
1	Production Costs	IDR	57,730,000.00
2	Fixed Costs	IDR	6,000,000.00
3	Operating Costs	IDR	53,220,000.00
	Total Cost	IDR	116,950,000.00

The rainfall observation data obtained through Copula, then distributed into rainfall data per quarter. Division of rainfall Data per quarter is as follows: first quarter is from January to April, the second quarter is from May to August, and the third quarter is from September to December. The result of the calculation of rain data can be seen in Table 5.

Table 5. Data on Quarter Rainfall in Tasikmalaya District in 2012-2022 (mm)

	•			J		,
Rainfall	2012	2013	2014	2015	2016	2017
1st Quarter	3.721855	6.33463	3.31509	13.2470125	8.27698	3.8797825
2 nd Quarter	1.5250925	1.65992	2.92924	2.4724225	3.20907	2.8812425
3 rd Quarter	2.4178825	1.23439	5.52669	3.7229775	4.28892	5.188535

Rainfall	2018	2019	2020	2021	2022
1st Quarter	7.68471	4.46623	2.55926	2.08588	2.34535
2 nd Quarter	0.96002	0.66157	1.8493	0.93952	0.57028
3 rd Quarter	3.31035	0.42502	4.85446	3.3777	3.17398

In the calculation of insurance premiums, it is necessary to know the exit value used as the payment limit in case of claims. The exit value is the lowest rainfall data after the preparation, so we got an exit rating of 2.085. Trigger determination based on percentage values of the amount of rainfall in each calendar month. The trigger determination results are presented in Table 6.

Table 6. Trigger and Exit Values

Domoomtilos	<u> </u>	Triggers	
Percentiles -	1st Quarter	2 nd Quarter	3 rd Quarter
40	3.64050	1.412078	3.28308
50	3.87978	1.65992	3.3777
60	4.83990	1.9739225	3.83617
70	6.87465	2.6359505	4.51513
80	8.04007	2.910041	5.0549
90	12,253	3.153106	5.45906

Table 6 shows the trigger values for insurance payouts based on different percentiles of a certain risk metric across three time periods (quarters) in a year. These values represent thresholds used to determine whether insurance coverage should be activated. The determination of agricultural insurance premiums based on rainfall in this study uses the Black-Scholes method. The descriptive statistics of average rainfall per quarter from 2012-2022 are shown for the premium calculation presented in Table 7.

Table 7. Descriptive Statistics of Rainfall Data

Donomatan		Value	
Parameter	1st Quarter	2 nd Quarter	3 rd Quarter
Mean	5.265160227	1.787061591	3.41099
Std. Deviation	3.225977202	0.916114286	1.51642
Min	2.0858775	0.5702775	0.42502
Max	13.2470125	3.2090725	5.52669

Based on the latest actual rainfall data in 2022 (R_0) is 5.1642 mm. R_T is the trigger used for each percentile. T is the selected time period, harvest is assumed to occur as many as 3 times in 1 year so the value $T = \frac{4}{12} = 0.33$. Risk-free interest rate, r = 0.06. The default deviations (σ) from the rains of the 1st quarter, the 2nd quarter, and the 3rd quarter in a row are 3.22597, 0.91611, and 1.51642. Then calculate the cumulative distribution value with Equation (21). The examples of calculations of d_2 and $N(-d_2)$ on the 40th percentile of 1st quarter with a value of $R_T = 3.6405$ are as follows:

$$d_2 = \frac{\ln\left(\frac{R_0}{R_T}\right) + \left(r - \frac{(\sigma)^2}{2}\right)T}{\sigma(\sqrt{T})}$$

$$d_2 = \frac{\ln\left(\frac{5.1642}{3.6405}\right) + \left(0.06 - \frac{(3.22597)^2}{2}\right)0.33}{3.22597(\sqrt{0.33})}$$

$$= -0.7272.$$

Then the value of $N(-d_2)$ is calculated,

$$N(-d_2) = N(-(-0.7272)) = N(0.7272) = 0.76646.$$

Also calculated for the other trigger values of each quarter presented in Table 8.

Table 8. Value Calculation Results $N(-d_2)$

Percentiles -	1 st Quarter		2 nd Quarter		3 rd Quarter	
refcentiles	Triggers	$N(-d_2)$	Triggers	$N(-d_2)$	Triggers	$N(-d_2)$
40	3.64050	0.76646	1.412078	0.0126	3.28308	0.457330767
50	3.87978	0.77685	1.65992	0.02673	3.3777	0.470289164
60	4.83990	0.81082	1.9739225	0.05458	3.83617	0.528526824
70	6.87465	0.85775	2.6359505	0.14631	4.51513	0.602043035
80	8.04007	0.87591	2.910041	0.19368	5.0549	0.651091981
90	12,253	0.91654	3.153106	0.23824	5.45906	0.683164866

Table 8 reflects a time-dependent shift in perceived option risk across quarters. $N(-d_2)$ behaves as expected, it increases with percentiles, implying that as trigger values grow, the probability of option exercise rises. After that, the premiums are calculated using Equation (22) where the amount of liability is IDR116,950,000. Example of the calculation of premiums made on the 40^{th} percentile of the 1^{st} quarter with the value of $N(-d_2)$ of 0.76646 as follows:

```
Premium = Pe^{-rT}N(-d_2)
= (IDR116.950.000)e^{-0.06(0.33)} (0.76646)
= IDR87.880.261
```

So, the premium to be paid at the 40^{th} percentile when the trigger value R_T of 3.6405 is IDR 87.880.261. Also calculated the premium price for other trigger values at each quarter presented in Table 9. Table 9 shows the large farm insurance premiums based on rainfall on chili commodities in the district of Tasikmalaya. It can be seen that the higher the percentage value then the greater the rainfall trigger value. The 90^{th} percentile was chosen because it is sufficiently representative of the highest loss risk faced by chili farmers. Furthermore, if the 95^{th} percentile were included, the chili insurance premium would increase due to the higher losses borne by the insurance companies. As a result, fewer insurance companies would be willing to cover the risks of chili farmers, and fewer chili farmers would be willing to participate in the insurance.

			1
Percentiles		Premium	
Percentnes	1 st Quarter	2 nd Quarter	3 rd Quarter
40	IDR87,880,261	IDR1,444,296	IDR52,436,248
50	IDR89,071,233	IDR3,065,013	IDR53,922,021
60	IDR92,965,936	IDR 6,258,459	IDR60,599,386
70	IDR98,347,687	IDR16,775,988	IDR69,028,547
80	IDR100,429,464	IDR22,207,240	IDR74,652,360
90	IDR 105 087 419	IDR27 316 208	IDR 78 329 746

Table 9. Rainfall-Based Chili Plant Insurance Prices per Quarter

5. CONCLUSIONS

The relationship between production results and rainfall can be explained more specifically using the Clayton Copula approach. The Clayton copula has a tail dependency below, which means that extreme events occur when the rainfall is low and the production output obtained is smaller, the closer the relationship. The Black-Scholes model based on the Clayton Copula, which has a relatively small MAE, can be used to determine insurance premiums for chili crops based on crop yields and rainfall. The premium price obtained varies according to the rainfall percentile and the quarterly period used to index. The greater the percentile value, the greater the premium that farmers must pay.

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