VOLUME 13, NO. 1, EDISI XXXI PEBRUARI 2005

ANALYSIS OF FLAT PLATE STRUCTURES BY EQUIVALENT GRID METHOD

Ilham Nurhuda[1], Han Ay Lie[1]

ABSTRAK

Penggunaan model lebar efektif balok untuk menganalisis struktur flat-plate yang dikenai beban lateral sudah sering dilakukan. Namun demikian, sampai saat ini model tersebut masih terbatas penggunaannya pada model-model struktur 2 dimensi.

Analisis ini memperluas penggunaan model grid ekuivalen untuk menganalisis struktur secara 3 dimensi. Lebar efektif grid dianalisis secara empirik berdasarkan hasil eksperimen. Perilaku struktur ditinjau dengan melakukan analisis struktur secara linier dan non linier.

Keywords : *flat-plate, model, reinforced concrete.*

INTRODUCTION

A reinforced concrete floor is generally a plate fixed by means of beams, along its four sides. This conventional system is the eldest system and universally well known. However, this plate-beam system has some weaknesses especially in its efficiency and economic aspects. Formwork for example, is time consuming and costly. Due to technology evolvement, the use of this *column-line-beams* system reduces, developing in an advance system where slabs consist of solid plates supported directly by columns. This is called the flat plate [1]. This kind of structure's development is based on the understanding that a slab not only distribute load to the beam but also functions as a structure unity, carrying load.

Difficulty faced in usage of this flat-plate system is in its behavior analysis, especially relate to the effect of lateral loading when the structure is designed to resist strong wind or earthquake. Statical analysis with classical mechanics is not adequate because of the complex stress distribution that occurred in these plates. Advance computer technology development lead to fast progress in solving the problem numerically. The numerical method most common used in structure analysis is the *Finite Element Method.*

The structural model widely used in structural analysis is the *Frame Element Model* or *Bar Element.* This model is often preferred because simplicity and ease application. In the flat-plate structure an analysis by using frame elements is generally carried out by the *Effective Beam-Width Model*. In this method, the plate is modeled as an equivalent beam with a certain effective width. This analysis generally makes use of a *two-dimensional analysis*.

Based on the understanding that the analysis requires modeling of a flat-plates as a *two-directional-single-load supporting unity*, the use of grid model for modelling flat-plate structures, will be studied. This modelling analysis is based on the assumption that slabs consist of grid formations that are able to distribute load to every direction. Effective grid distance will be examined by varying the distance between grids and comparing the result to

[1] Pengajar Jurusan Teknik Sipil Universitas Diponegoro Semarang

experiment data. The effect of *grid-to-slab-deflection number* of is also observed by comparing numerical analysis to experimental result.

GRID MODELING OF FLAT PLATE STRUCTURES

General Behavior of Flat Plate Structures

A flat-plate is a structural system where slabs are supported *directly* by columns without the existance of beams. A flat-plate system is analyzed similarely as a *two-way slab* but differs in behavior to the *two-way slab in slab-beam structures*. In a slab-beam system, the positive elastic moment of plate occurred in the middle of the plate, and the negative moment occurred in the middle of the plate edge connected to the beam. On the contrary, in flat-plate structure the maximum positive and negative moments occurred at the column line.

In the rectangular panel, slabs on rigid beams span predominantly between the long beams, and for an aspect ratio of two the central region is curved practically only in the short span direction. In a flat-plate the central section spans in the longer direction, and the transverse moments are confined to a limited width on either side of the columns, as indicated by fig 1.

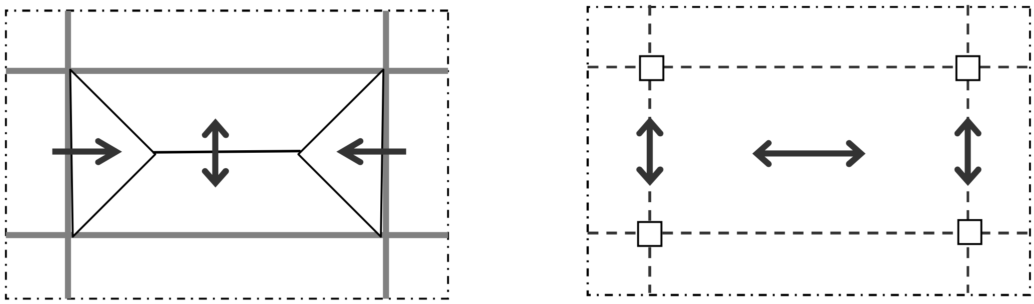


Fig 1. Behavior of Elongated Panels of Continuous Slabs

When the two-way slab is supported by a wall or beam, then shear will not govern the design. Generally, this is because the factored shear force is predominantely smaller than the concrete shear capacity. On the contrary, in a two-way slab system where the slab is supported directly by the column, the slab carries the entire load and distributes it directly to the column. Here shear around the column governs the design. In this case, structure shear design needs special considerations from the designer [5].

Two types of shear behavior requires to be considered in designing flat-plate structures. These are *beam shear* and *punching shear*. Beam shear type *one-way-shear* becomes critical in thin and long plates. While punching shear, which is often the most common mode of failure, is due to the relatively small shear area around the column, inable to detain column point loading. The critical area of punching shear failure is at a distance ½ d from the column face, where "d" is the effective slab depth.

Effective-Beam-Width Model

Concerning the slab-column connection, not all portions of the *slab-width* work effectively as is the case in a flexural element framed to the column. This behavior is due to the fact that the slabs do not experience the same rotation along its entire width. To overcome this, it is common practice to model the slab simply at design phase. The simplest way is by modeling the slab in bar elements known as *slab-beam element*. This slab-beam element rotates uniformly across its transverse width. The beam depth generally is taken equal to the slab thickness. Whereas the effective beam width has been studied and proposed by many researchers.

Banchik (1987), as quoted by Hwang and Moehle [3], presents the effective beam width by using finite element techniques. The solutions apply to interior, edge, and corner connections, for column with square cross sections, and for combinations of c_1/l_1 and l_2/l_1, defined as follows

c_1/l_1 = 0.06, 0.09, 0.12

l_2/l_1 = 0.67, 1.00, 1.50

Where: c_1 is the column width.
l_1 is the column distance in the lateral load direction.
l_2 is the column distance in the transverse direction.

Banchik presented his result using c_1/l_1 as the main geometric variables. The results are divided into two groups: one for the interior frame, and another for the exterior frame. The variations of effective beam width "b" for an interior frame, which includes interior connections and edge connections bending perpendicular to the edge, can be represented as:

$$b = 2.c_1 + \frac{l_1}{3} \quad \text{.....................................} (1)$$

The effective beam width for an exterior frame, which includes corner connections and edge connections bending parallel to the edge, can be represented as

$$b = c_1 + \frac{l_1}{6} \quad \text{..} (2)$$

According to Hwang and Moehle [3], the influence of section crackes is accommodated by means of a reduction factor to the element stiffness, taken as 1/3.

Improving Models by implementing Grid Model for 3D Analysis

Based on the study conducted by Hwang and Moehle [3], the model proposed by Banchik is fair enough to model structures two dimensionally. However, this model fails to represents a comprehensive understanding of structural behavior. Behavior of structure such as *plate deformation*, *internal force distribution*, *influence of structure and loading configuration* can be observed only by using a three-dimension analysis. Moreover, by using nonlinear procedure in the three-dimension analysis, structure inelastic behavior can be examined. To simplify, the grid model where the slab is replaced by an arrangement of equivalent beams in a grid model with a certain width, is proposed.

The questions is how wide the appropriate *effective-grid-width* should be, and how the number of grids influence the analytical result. To answer this question, numerical experiments are conducted. Details of the numerical experiments are explained in the following chapter.

NUMERICAL EXPERIMENTAL PROGRAM

Configuration and Properties

The Test slab configuration and geometries are adapted from the experiment conducted by Hwang and Moehle [2]. This test slab is an idealization of a flat-plate floor on an intermediate multistory office building level. The slab has three bays in each direction with a center-to-center span of 4.6 m and 6.9 m in each direction. Slab thickness is

203 mm and story height is 3.0 m. Gravity loading consists of self-weight and live load. The lateral loads are due to wind.

For experimental purpose the structure is scale to 1:25, thus center to center spans in the two principal directions become 1.8 and 2.7 m. Slab thickness becomes 81 mm, the columns extend 305 mm above the slab and 1220 mm below the slab. The column is pin supported to the model inflection point of moment around column midheight. Configuration and dimension of test slab in cm are as shown in figure 2.

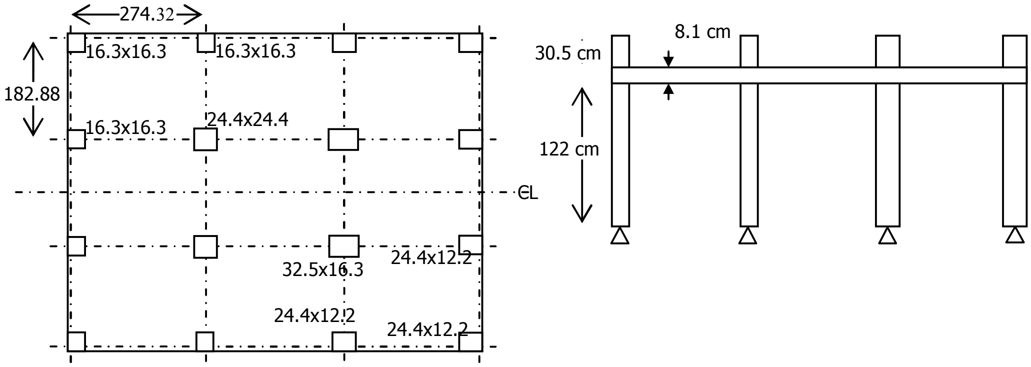


Fig 2. Configuration and Dimension of Test Slab

Structure properties are: ultimate compressive strength of concrete *(f'_c)* 21.8 MPa, Splitting tension *(f_{tr})* 2.6 MPa, with a secant modulus of 17860 Pa. Slab reinforcement are deformed bars No.2, with the a yield strength *(f_y)* of 450 MPa.

Loadings

The test slab is loaded with a combination of vertical and lateral load. The vertical load has a magnitute of 5650 Pa. Lateral loading was applied in the north-south and east-west direction separately, and applied til the structure collapse. In the north-south direction, the lateral load is applied to slab edge at two points, one midway between frame lines "a" and "b", and the other between lines c and d. In the east-west direction, the lateral load is applied to slab edge at midway between frame Lines 1 and 2, and Lines 3 and 4, as shown in fig 3.

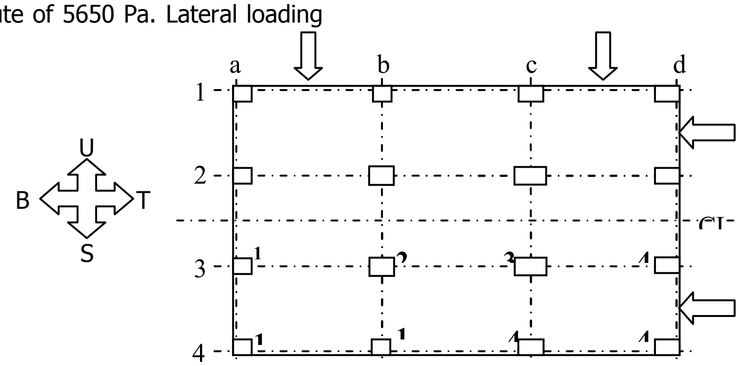


Fig 3. Lateral Loading Direction in Test Slab

Structure Modelling by the Grid Model

For the numerical investigation, four grid models with different *grid-spaces* are made. The column is modelled as a frame element with its section properties as a constant for all the four models. The illustration of the four grid models can be seen in fig 4 (a, b, c, d):

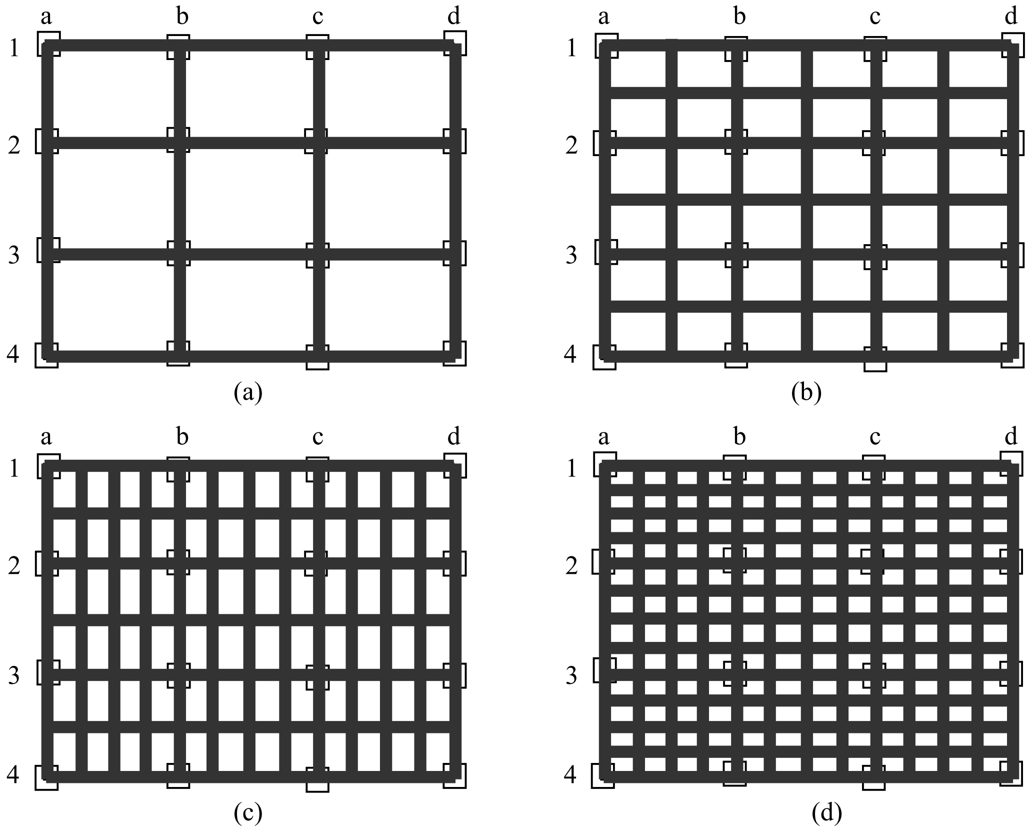


Fig. 4 Illustration of Grid Models (a) Model 1; (b) Model 2; (c) Model 3; (d) Model 4

The distance between grids of each grid model is shown in tables 1.

Table1. Grid-space

Model	Distance Between Column (l_1)		Grid-space (l_g)	
	N – S	E – W	N - S	E - W
1	182.8 cm	274.3 cm	182.8 cm	274.3 cm
2	182.8 cm	274.3 cm	91.4 cm	137.2 cm
3	182.8 cm	274.3 cm	91.4 cm	68.6 cm
4	182.8 cm	274.3 cm	45.7 cm	68.6 cm

The variation of the effective-grid-width will be derived empirically from the four grid models above. The effective-grid-width will be the function of grid-space (l_g) and distance between column in the grid direction (l_1), or b=f (l_1, l_g).

Method Of Analysis

Numerical analysis was governed by two methods, i.e. linear analysis and material-nonlinearity analysis. Linear analysis was done to investigate the effective-grid-width. Nonlinear analyses that count for the material nonlinearity, was governed to observe inelastic behavior of structure after yield occur, and how the models meet the experimental result. Behaviors of structure evaluated here are the relation of the load to lateral displacement, vertical deflection of slab, and shear stress distribution around column. The analysis was conducted by using the MASTAN 2 program [4,6].

RESULTS AND DISCUSSION

Effective Beam Width

Analysis result of effective-grid-width for each model is present in table 2. The result is divide into two groups that is for the interior and the exterior grid.

Table 2.a. Analysis of Effective-grid-width of Interior Grid

b	l_1	l_g	l_g / l_1	b / l_g
61.0	182.8	274.3	1.500547	0.222238
45.7	182.8	137.2	0.750274	0.333358
36.6	182.8	68.6	0.375137	0.533372
36.6	182.8	68.6	0.375137	0.533372
91.4	274.3	182.8	0.666424	0.500182
68.6	274.3	91.4	0.333212	0.750274
68.6	274.3	91.4	0.333212	0.750274
40.0	274.3	45.7	0.166606	0.875274

Table 2.b. Analysis of Effective-grid-width of Exterior Grid

b	l_1	l_g	l_g / l_1	b / l_g
30.5	182.8	274.3	1.500547	0.111119
22.9	182.8	137.15	0.750274	0.166679
18.3	182.8	68.575	0.375137	0.266686
18.3	182.8	68.575	0.375137	0.266686
45.7	274.3	182.8	0.666424	0.250091
34.3	274.3	91.4	0.333212	0.375137
34.3	274.3	91.4	0.333212	0.375137
20.0	274.3	45.7	0.166606	0.437637

Results of analysis are plotted in figure 5, in terms of l_g / l_1. The effective-grid-width is derived by curve fitting.

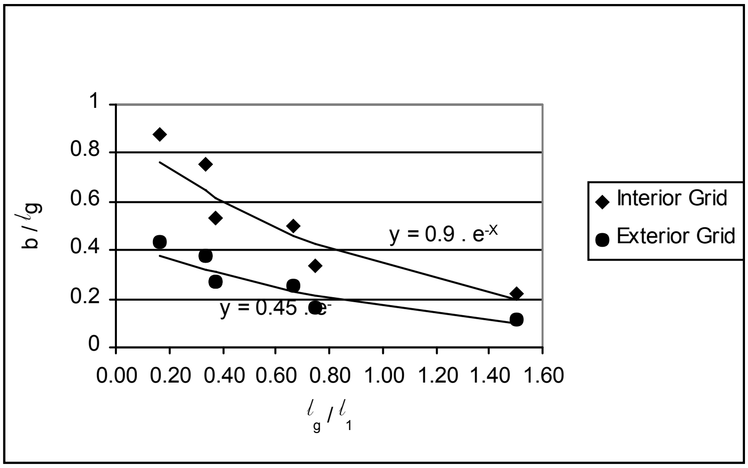


Fig 5. Plot of Effective-grid-width

From the graph above, the variations of the effective-beam-width b for an interior grid, which includes interior connections and edge connections, bending perpendicular to the edge, can be represented as

$$b = 0{,}9.Lg.e^{-Lg/L1}$$

Whereas the effective beam width for an exterior grid, which includes corner connections and edge connections, bending parallel to the edge, can be represented as

$$b = 0{,}45.Lg.e^{-Lg/L1}$$

According to these expressions, the effective-grid-width in exterior grid is *half the value of the interior grid.*

Analysis of Vertical Slab Deflection

Vertical deflection is an important factor in the structure design, especially in service loading conditions. Thereby deflection estimation that may occur, plays an essential rule in the design process. In the fig 6, the vertical deflection resulted from numerical analysis of the three models is plotted together with the experimental result. Due to the limited number of data, the deflection is only controlled at the middle of a_3b_3 column line.

According to the analysis result, it is seen (fig 6) that model 4 gives the best prediction compared to the other models. Model 2 and model 3 shown to overestimate the lateral drift ratio to 1/50. The increasing of grid number in the model will improve the prediction of prevailing vertical deflection. Large number of grid will make the that the stiffness does not change immediately. In model 2, due to its few number of grid, yielding in one element causes the global stiffness drop and makes the vertical deflection to increase fast. Another conclusion is that the three models present a good prediction until a lateral drift ratio of 1/100.

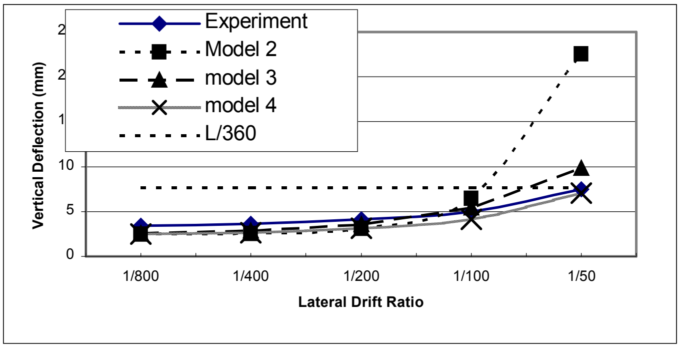


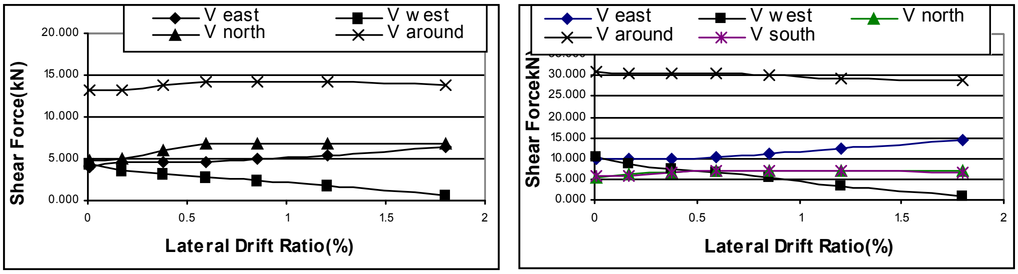
Fig 6. Calculated vertical deflection and experimental result

Shear in Slab

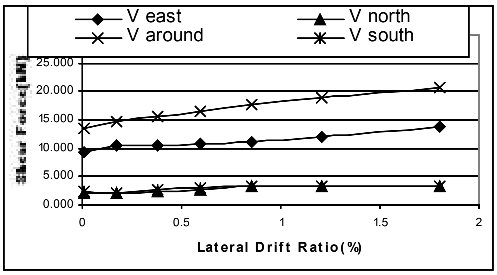
Effect of lateral loading to shear force in the slab is analyzed by a grid model. The shear force around the column is obtained from shear force at the end of each grid element connected to the column joint. Shear strength is calculated in one side of the column where the shear width (bo) is equal to column width (c) plus the effective slab height (h). Shear strength is calculated using an empirical formula [1] as follows:

$$Vc = \frac{1}{3} * \sqrt{f'c} * bo * d \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Result of analysis of the shear force in three locations that is B4, B2, A3, to its corresponding lateral drift ratio are shown in figure 7(a, b, c, d)



(a) (b)



(c)

Column Joint	bo	Shear Strength of Slab
B 4	163mm	14630 kN
B 2	163mm	16575 kN
A 3	163mm	16575 kN

(d)

Fig 7. (a) Shear Force in Slab at Joint B4; (b) Shear Force in Slab at Joint B2
(c) Shear Force in Slab at Joint A3; (d) Shear Strength of Slab at one column side

B4 Joint (fig. 3) is in south edge, whereas B2 joint (fig. 3) is in the middle of the plate, and A3 joint (fig. 3) is on the west edge. During analysis, vertical loading remain constant while the lateral loading increase. It can be seen from fig. 7 that shows the comparison to shear strength, to a lateral drift value ratio of 2% that the calculated shear force remains below its shear strength capacity. These results are congruent with the experimental results by Hwang and Moehle [2] where the shear failure occurs at 4% lateral drift ratio.

CONCLUSION

Based on the result of numerical analysis, the following conclusions are made.

1. The result of numerical simulation shows that the three-dimension grid model can be used to model behavior of flat-plate structures.
2. According to the analysis result, there are relations between the *effective-grid-width*, grid-space, and grid-length from column to column. This analysis derives the variations of *effective-grid-width* as follows:

   $b = 0{,}9.Lg.e^{-Lg/L1}$ for the interior grid,

   and $b = 0{,}45.Lg.e^{-Lg/L1}$ for the exterior,

   Where b is the effective-grid-width, Lg is grid-space and L1 is grid-length from column to column.
3. Analysis result shows that until lateral drift ratio of 1%, the three models (model 2, model 3, and model 4) still perform good prediction. For the elastic analysis purposed, model 2 can be used. Whereas for the inelastic analysis, increasing of grid number is suggested.
4. In this analysis, shear failure is avoided by designing the shear strength capacity almost 3 times of shear from gravitational load. It can be seen that by this way the structure conduct a high degree of ductility.
5. Analysis shows that grid model can be used to reckon shear force and meet the experimental result.

ACKNOWLEDGEMENTS

This research is carried out with the financial support of the Directorate General of Higher Education, Republic of Indonesia through Technological and Professional Skills Development Sector Projects (TPSDP), ADB Loan No 1792-INO, under contract No. 568/TPSDP/TS/K/XII/03. This paper is prepared with the financial aid of Department of Civil Engineering, Diponegoro University.

REFERENCES

Ghosh, S.K., Fanella, D.A., Rabbat, B.G, *Notes on ACI 318-95*, pp. 20.1 – 20.11, Portlad Cement Association, 1996

Hwang, S.J., Moehle, J.P., *Vertical and Lateral Load Tests of Nine Panel Flat-Plate Frame*, ACI Structural journal, vol 97, no.1, pp. 193-202, American Concrete Institute, 2000.

Hwang, S.J., Moehle, J.P., *Models for Laterally Loaded Slab-Column Frames*, ACI Structural journal, vol 97, no.2, pp. 345-352, American Concrete Institute, 2000.

McGuire, W., Gallagher, R.H., Ziemian, R.D., *Matrix Structural Analysis*, 2nd Edition, pp. 216-325, John Willey & Sons Inc, 2000

Regan, P.E., *Behavior of Reinforced Concrete Flat Slab*, pp. 11-19, Construction Industry Research and Information Association, 1981

Ziemian, R.D., Mc Guire, W., *MASTAN 2*, Version 2, John Willey & Sons Inc, 2002.