

## **MOMENT RESISTANCE ANALYSIS ON VARIOUS CONFIGURATION OF MULTIPLE-BOLT TIMBER CONNECTION USING BEAM ON ELASTIC FOUNDATION THEORY AND RIGID PLATE ASSUMPTION**

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### **ABSTRAK**

*Kayu merupakan bahan struktur yang masih banyak diminati, mungkin salah satunya karena nilai estetikanya yang belum dapat tergantikan. Untuk mendukung aplikasi Kayu dalam struktur yang lebih luas, maka sajian mengenai sambungan kayu perlu mendapat perhatian. Dalam paper ini disajikan hasil eksperimen mengenai momen tahanan sambungan kayu yang disambung dengan berbagai konfigurasi penyambung. Ada 4 (empat) konfigurasi sambungan yaitu 4H, 6H, 6V dan 6C. Dari hasil eksperimen, diperoleh bahwa jenis konfigurasi 6H menghasilkan momen tahanan yang paling kecil. Konfigurasi 6H ini juga mengalami kegagalan akibat geser.*

**Kata kunci** : momen tahanan, sambungan kayu, konfigurasi sambungan

### **INTRODUCTION**

Connection, such as mechanical connection, is still an interesting issue for many wood engineers. The interaction between the fastener and the wood member is complex, particularly beyond the elastic region (Gatesco 1998). Some potential reasons are: orthotropic behavior of wood, splitting phenomenon, and stress concentration at contact area between bolt and wood member. In general, the strength of bolted connection is depending on some parameters: the dowel bearing strength of wood member, the bending strength of bolt, and the geometry of the connection itself.

The first attempt to discover the design value for bolted timber joint was conducted by Trayer (1932). Trayer's work focused on developing the load-slip curve that can be used to find three important values: the proportional limit, yield load, and ultimate load. Trayer's conclusion was derived as an empirical fit of experimental result. Some of the major conclusion reached by Trayer

were: (1) the slenderness ratio, the thickness of main member divided by the diameter of bolt, was the governing parameter, (2) connection with steel side plates achieves 20% higher capacity than connection with wood side members for parallel to grain loading, and (3) connection with steel side plates and connection with wood side members loaded perpendicular to wood grain have equal capacities.

Yield model for bolted timber connection was developed by Johansen (1949). The bearing capacity of connection is attained when either the compressive strength of wood beneath the bolt is exceeded or simultaneously the timber under the bolt becomes plastic and one or more plastic hinges formed in the bolt. Based on this yield model, ultimate load and failure mode of connection with dowel type fastener can be evaluated. Currently, this yield model is known as European Yield Model (EYM) and it is adopted by many codes.

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Analysis of bolted wood-joint can also be done using beam on elastic foundation theory (Foschi 1974; Hirai et al. 1982; Hirai 1985). In This analysis, bolt and wood member of bolted wood-joint referred as the beam and the elastic foundation, respectively. Since not only the ultimate lateral load can be obtained but also the whole response (load-slip curve) of the connection, this theory is quite versatile. Model of elastic foundation has been inputted in many comprehensive finite element analyses, nowadays.

**OBJECTIVES**

The objectives of this study are to evaluate the moment resistance of steel to wood connection using beam on elastic foundation theory and rigid plate assumption, and to investigate the moment-rotation curve of connection up to failure condition. Experimental results are obtained from previous research.

**ANALYSIS**

Consider the internal forces of bolt under bending in Figure 1. Wood will behave as an elastic foundation to the bolt, and the bolt will behave as a beam. Governing equation to this problem is quite similar to ordinary beam problem, except additional term due to elastic foundation from wood member [see Equation (1)].

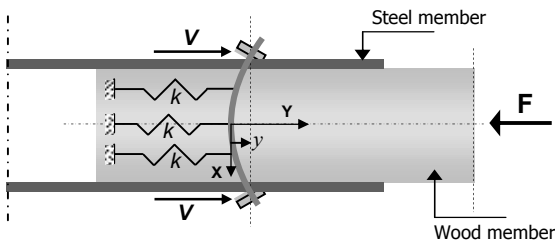


Figure 1. Bolt under bending in steel to wood connection

$$E_s I_s \frac{d^4 y}{dx^4} + k_\alpha y = 0$$

$$\frac{d^4 y}{dx^4} + \frac{k_\alpha}{E_s I_s} y = 0 \dots\dots\dots(1)$$

General solution to Equation (1) has been given by many text books of advance mechanic. Hirai et al (1982) suggested Equation (2) as the general solution to Equation (1).

$$y = \alpha A + bB + cC + dD \dots\dots\dots(2)$$

Where :

$$A = \cosh \beta x \cos \beta x$$

$$B = \frac{1}{2} (\cosh \beta x \sin \beta x + \sinh \beta x \cos \beta x)$$

$$C = \sinh \beta x \sin \beta x$$

$$D = \frac{1}{2} (\cosh \beta x \sin \beta x - \sinh \beta x \cos \beta x)$$

$$\beta = \sqrt[4]{\frac{k_\alpha}{4E_s I_s}}$$

Note that :

$$\text{Slope of bolt : } \frac{dy}{dx} = y' \dots\dots\dots(3)$$

$$\text{Bending moment : } M = -E_s I_s \frac{d^2 y}{dx^2} \dots\dots\dots(4)$$

$$\text{Shear force: } V = -E_s I_s \frac{d^3 y}{dx^3} \dots\dots\dots(5)$$

Substituting Equations (3), (4) and (5) into Equation (2) yields three following equations:

$$y' = -2\beta\beta a + \beta bA + 2\beta\beta c + \beta dC \dots\dots\dots(6)$$

$$\frac{2\beta^2}{k_\alpha} M = aC + bD - cA - dB \dots\dots\dots(7)$$

$$\frac{2\beta}{k_\alpha} V = 2aB + bC + 2cD - dA \dots\dots\dots(8)$$

At mid-span of bolt ( $x = 0$ ), the slope of bolt and the shear force are equal to zero ( $y' = 0$  and  $V = 0$ ). This condition is satisfied if and only if  $b = 0$  and  $d = 0$ . Therefore, Equations (2), (7) and (8) can be rewritten as follows.

$$y = \alpha A + cC \dots\dots\dots(9)$$

$$\frac{2\beta^2}{k_\alpha} M = aC - cA \dots\dots\dots (10)$$

$$\frac{2\beta}{k_\alpha} V = 2aB + 2cD \dots\dots\dots (11)$$

At  $x$  equals to  $t_m/2$ , the restraining force from nut and washer can be considered. However, due to shrinkage of wood member, the restraining force from nut and washer might be disappeared. These two conditions determine the values of  $a$  and  $c$  as follows.

1. When the restraining force from the nut and washer is considered ( $y = s$  and  $y' = 0$ )

$$a = \frac{B^1}{D^1C^1 + A^1B^1} s \dots\dots\dots (12)$$

$$c = \frac{D^1}{D^1C^1 + A^1B^1} s \dots\dots\dots (13)$$

2. No restraining force from the nut and washer ( $y = s$  and  $M = 0$ )

$$a = \frac{A^1}{A^1A^1 + C^1C^1} s \dots\dots\dots (14)$$

$$c = \frac{C^1}{C^1C^1 + A^1A^1} s \dots\dots\dots (15)$$

Finally, shear force ( $V$ ) and lateral load resistance ( $F$ ) can be obtained from these following Equations.

$$V = \frac{k_\alpha}{\beta} (aB^1 + cD^1) \dots\dots\dots (16)$$

$$F = 2V = \frac{2k_\alpha}{\beta} (aB^1 + cD^1) \dots\dots\dots (17)$$

For a moment-resisting connection subjected to applied bending moment  $M$  as shown in Figure 2, fastener  $i$  will resist lateral load  $F_{\alpha i}$  with angle  $\alpha$  to wood grain. Lateral load resistance ( $F_i$ ) for any direction to wood grain is obtained from Equation (17). The applied bending moment will be balanced by couple moment of lateral force of fasteners as shown in Equation (18). Since the side members are much stiffer than main member, Awaludin (2004) assumed that rigid plate assumption can be applied. Therefore, the angle of rotation of all fasteners is the same as stated in Equation (19) where  $ds_i$  is slip of fastener  $i$ .

$$M = \sum_{i=1}^n F_{\alpha i} r_i \dots\dots\dots (18)$$

$$\phi = \frac{ds_i}{r_i} = \frac{ds_j}{r_j} \dots\dots\dots (19)$$

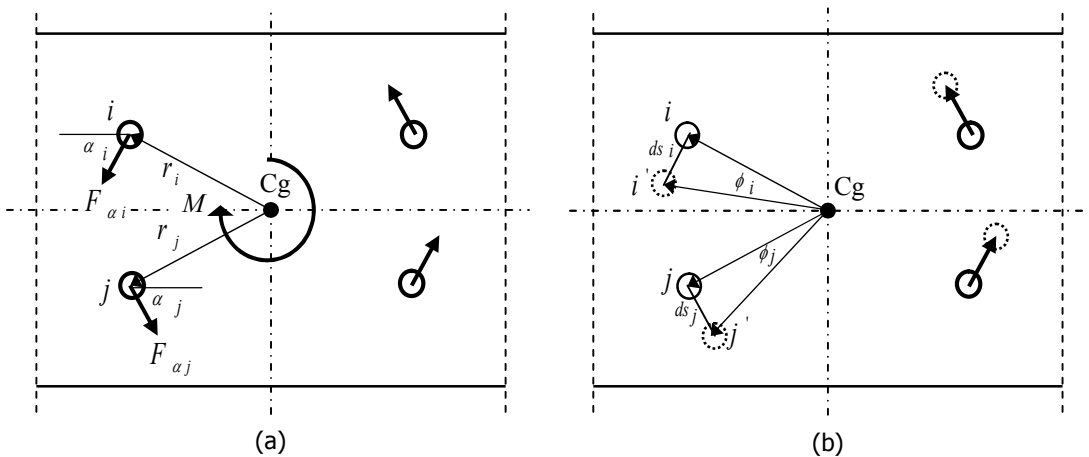


Figure 2. Moment resisting connection; (a) acting forces, (b) joint rotation [Awaludin 2004]

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Two failure criteria that can be used to evaluate the maximum moment resistance of multiple-bolt connection are: yielding of outer-most fastener, and shear failure between two adjacent fasteners. The first criteria can be applied when the distance among fastener are sufficient to prevent early wood splitting. When the distance among fastener is short, on the other hand,

connection tends to fail by wood splitting in low capacity (shear failure).

Multiple-bolt connection is tested under pure bending using test set-up as shown in Figure 3. Several multiple-bolt configuration as shown in Figure 4 was examined by Awaludin (2004). Spacing between fasteners was designed with NDS of U.S (1997).

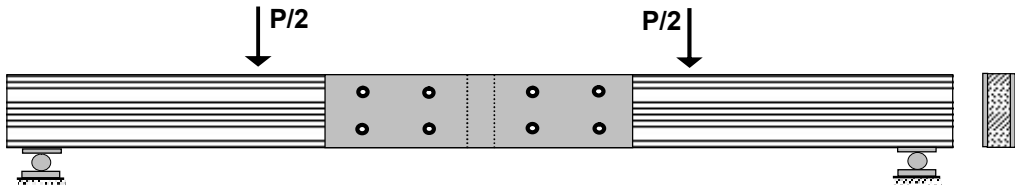


Figure 3. Test set-up of multiple-bolt connection (Awaludin 2004)

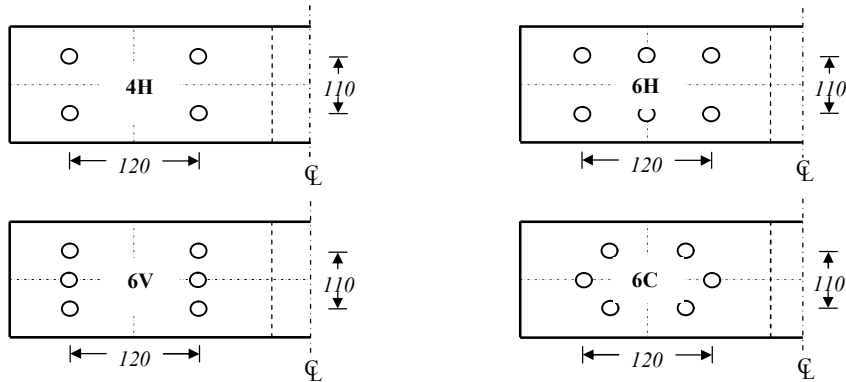


Figure 4. Multiple-bolt configurations (unit in mm)

## RESULTS AND DISCUSSION

Lateral load of fasteners is analyzed with Equation (17) without restraining force from nut and washer. Information of fastener slip ( $s$ ) is obtained from Awaludin (2004). Elastic foundation of wood member was modified into elastoplastic foundation model as shown in Figure 5. Hankinson formula is used to calculate the elastic slip modulus in direction  $\alpha$  to wood grain ( $k_{e\alpha}$ ). For plastic slip modulus ( $k_{p\alpha}$ ) average value between

direction parallel and perpendicular to wood grain is used. Finally, theoretical prediction of maximum moment resistance ( $M_{th}$ ) was calculated by Equation (18) and yielding of outer-most fastener as the failure criteria. Prediction moment resistance based on shear failure between two adjacent fasteners can be evaluated since the shear strength data of wood is not available. Prediction and Experimental maximum moment resistances of multiple-bolt connection are presented in Table 1.

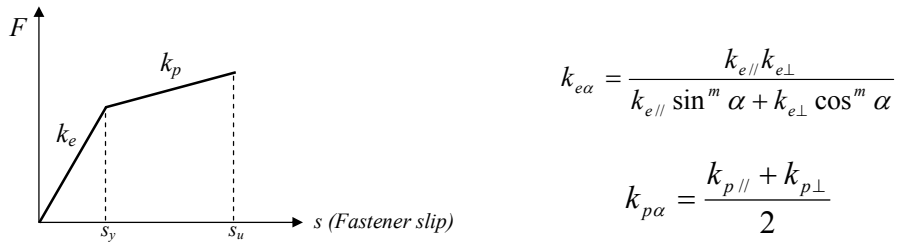


Figure 5. Elastoplastic foundation model

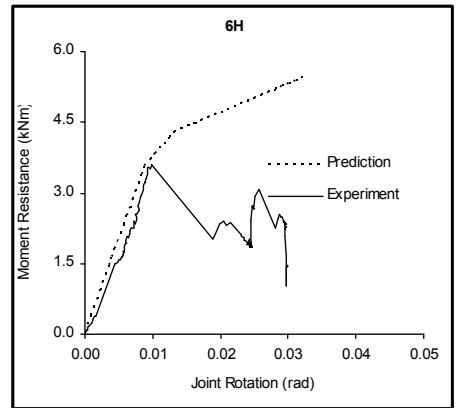
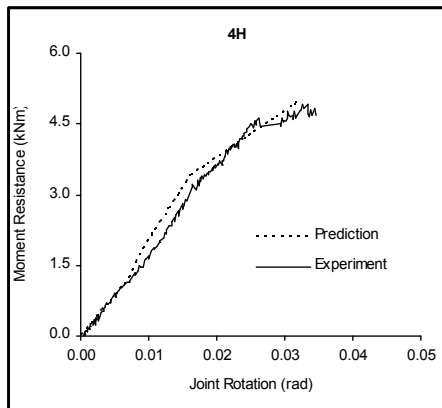
Theoretical moment predictions show small deviation with the experimental results. The deviation is larger when early wood splitting as the failure mode such as in bolt configuration 6H ( $M_{ex} / M_{th} = 0.66$ ). It would be less discrepancy when the maximum moment resistance is evaluated based on the shear capacity between two adjacent fasteners. From moment-rotation

curve (see Figure 6), theoretical moment predictions show good agreement with the experimental results for all bolt configurations except bolt configuration 6H. Bolt configuration 6H shows poor ductility and the load suddenly decreases after the proportional limit. This tendency is caused by insufficient distance between fasteners.

Table 1. Maximum moment resistance of multiple-bolt connection

Bolts configuration	$M_{ex}^*$ (kNm)	$M_{th}$ (kNm)	$M_{ex} / M_{th}$
4H	4.93	4.98	0.99
6H	3.61	5.44	0.66
6C	4.11	4.54	0.91
6V	4.40	5.18	0.85

\* Experimental maximum moment resistances (Awaludin 2004)



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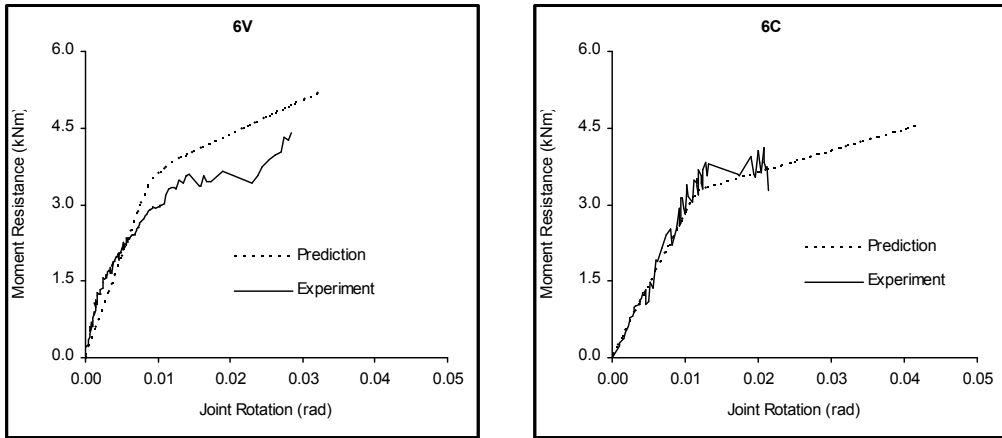


Figure 6. Moment-rotation curve for different bolt configurations

## CONCLUSIONS

Predicted maximum moment resistances show small deviation with the experimental results. Bolt configuration with short distance between fasteners tends to yield higher maximum moment resistance when predicted by yielding of outer-most fastener as the failure mode. Moment-rotation curves obtained from analysis show good agreement with the experimental results.

## REFERENCES

- Awaludin A, (2004). *Flexural behavior of timber connection with various multiple-bolt configurations*, Master Thesis. Civil Engineering Dpt of Chulalongkorn University. Bangkok.
- Gatesco N, (1998) "Strength and local deformability of wood beneath bolted connectors", Journal of Structural Engineering, ASCE, 124/2, 195-202.
- Hirai T, and Sawada M, (1982) "Nominal bearing-stresses of bolted wood-joints at apparent proportional-limits", Mokuzai Gakkaishi, 28/9, 543-547.

Hirai T, (1985) "Nonlinear load-slip relationship of bolted wood-joints with steel side-members III", Mokuzai Gakkaishi, 31/3, 165-170.

Johansen K.W, (1949) "Theory of timber connections", International Association of Bridge and Structural Engineering, 9, 249-262.

Trayer G. W, (1932) "The bearing strength of wood under bolts", Tech. Bulletin. 332, U.S. Dept. of Agric., Washington, D.C., Oct.

## NOTATION

- $E_s I_s$  : Bending stiffness of bolt
- $k_\alpha$  : Slip modulus
- $t_m$  : Wood thickness
- $s, y$  : Fastener slip
- $A^1, B^1, C^1, D^1$  : The value of A, B, C, D when  $x = t_m / 2$
- $\phi, \theta$  : Joint rotation