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IMPLEMENTATION OF NEURAL PREDICTIVE CONTROL TO DISTILLATION COLUMN

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Abstrak

This paper presents a neural predictive controller that is applied to distillation column. Distillation columns represent complex multivariable systems, with fast and slow dynamics, significant interactions and directionality. A phenomenological model (i.e. a model derived from fundamental equations like mass and energy balances) of a distillation column is very complicated. For this reason, classical linear controller, such as PID (Proportional, Integral, and Derivative) controller, will provide robustness only over relatively small range operation because of complexity and nonlinearity of the system. Neural predictive controller employed in this system to increase the range operation without lack of robustness. In this work, a neural network is developed for modelling and controlling a distillation column based on measured input-output data pairs. In distillation column, a neural network is trained on the unknown parameters of the system. The resulting implementation of the neural predictive controller is able to eliminate the most significant obstacles encountered in conventional predictive control applications by facilitating the development of complex multivariable models and providing a rapid, reliable solution to the control algorithm. Controller design and implementation are illustrated for a plant frequently referred to in the literature. Results are given for simulation experiments, which demonstrate the advantages of the neural based predictive controller both at the transient region and at the steady state region to overcome any overshoots.

Keywords : *neural predictive controller; distillation column; complex multivariable models*

Introduction

Distillation is a process of separating two or more miscible liquids by taking advantage of the boiling point differences between the liquids. For methanol and water, heat is added to the mixture of methanol and water and eventually the most volatile component (methanol) begins to vaporize. As the methanol vaporizes it takes with it molecules of water.

The methanol-water vapor mixture is then condensed and evaporated again, giving a higher mole fraction of methanol in the vapor phase and a higher mole fraction of water in the liquid phase. This process of condensation and evaporation continues in stages up the column until the methanol rich vapor component is condensed and collected as tops product (99.5% recovery / 99.5% pure) and the IPA/Water rich liquid is collected as bottoms product.

The distillation column has two inputs into the system, the amount of amperage in the Reboiler and the heat flux percent or (reflux ratio). The output would be the temperature of the distillate and the temperature of the Reboiler.

The demand for pure products of the chemical industry coupled with the increasing attention for greater efficiency, has necessitated continued research in the techniques of distillation. The demand on designers is not only to achieve the desired product quality at the minimum cost, but also to provide a constant purity of products even though there may be some variation in feed composition and flow rate or other disturbances. This is why it is important not to consider a distillation column without its associated control system.

Distillation column controller means product quality control, operation rate and minimum raw material usage. It is important to understand its behavior and character obtain an efficiency, optimal operation and quality product.

However, the most common design procedure requires first an engineering analysis and choice of the operating parameters (number and type of trays, pressure, reflux ratio, column diameter) in order to minimize the total annual cost of the column. Only subsequently the choice of the controlled and manipulated variables and the design of the control system are accomplished. No implications of the operating parameters on the control system characteristics, such as interactions among loops and plant ill conditioning, are generally considered by project engineers.

In this paper a distillation case study is presented in order to show which implications have the choices of the two principle column design parameters, i.e., the reflux ratio and the operating pressure, on the interaction and ill conditioning of a commonly used control structure for quality control. In particular it will be shown that the common engineering guidelines generally lead to a well-

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designed system also from a control point of view even though this important goal does not appear explicitly in the design procedure.

Effective distillation control is usually achieved by means of model based predictive control (MBPC) algorithms. Model based predictive control has established itself in industry as an important form of advanced control (Townsend and Irwin, 2001).

Model Based Predictive Control (MBPC) is a strategy that finds a control trajectory over a future time horizon based on a dynamic model of the process. In the last decades, MBPC has become an important and a distinctive part of control theory and has been widely used in the area of industrial applications. All predictive controllers are based on the fact that the process output can be predicted over a time horizon by using the past process inputs and outputs, if a suitable model of the plant is known.

There are many algorithms proposed in literature for implementing a predictive control, such as: Model Algorithmic Control – MAC (Mehra and Rouhani, 1982), Extended Prediction Self - Adaptive Control – EPSAC (De Keyser and Van Cauwenberghe, 1985), Generalized Predictive Control – GPC (Clarke, et.al., 1987) and Unified Predictive Control – UPC (Soeterboek, 1992). These algorithms are very similar because they are based on the same general ideas: receding horizon principle, plant model as part of the controller, prediction of the system's output and optimization of a cost function. The most important differences consist in the used plant models and in the chosen cost function.

The model used in the predictive controller plays a decisive role for obtaining a successful control strategy that can be applied to a real plant. The model must be capable to accurately follow the process dynamics and in the same time must be simple to implement and fast in simulation. From these points of view, for many plants, a neural network can be a good model to use in a predictive controller. Multiple Layer Perceptrons (MLP) and Radial-Basis-Functions (RBF) are nonlinear neural networks that can be trained in a supervised manner. Both MLP and RBF architectures are universal approximators (Cybenko, 1989; Park and Sandberg, 1991), fact which makes these types of neural networks suitable for constructing nonlinear dynamic models.

This paper analyzes a neural based predictive controller, which eliminates the most significant obstacles for nonlinear MBPC implementation by developing a nonlinear model, designing a neural predictor and providing a rapid, reliable solution for the control algorithm. The proposed method is demonstrated by controlling the transient response of a high purity distillation column.

Distillation Column Design

In this section a short summary of the procedure usually followed by chemical engineers to design a continuous distillation column is reported (Pannocchia, 2001). Given a distillation problem, i.e., a feed mixture and some specifications on the purity of the products, the engineer has to make a number of decisions in order to accomplish this end, including in the first stage the choice of the reflux ratio and of the operating pressure. The reflux ratio is normally defined as

$$R = \frac{\text{flow returned as reflux}}{\text{flow of top product}} = \frac{L}{D} \quad (1)$$

Before choosing the effective reflux ratio two limit conditions must be considered (Sinnott, 1996).

- Total reflux, i.e., the condition when all the condensate is returned into the column as reflux: no product is taken off and there is no feed. In this condition the number of stages for a given separation is the minimum.
- Minimum reflux, i.e., the condition when the reflux is reduced to a value that does not permit to achieve the desired separation even with an infinite number of stages.

Practical reflux ratios lie somewhere between the minimum and total reflux and this place is found by minimizing an overall annual cost function that comprises two terms (Coulson and Richardson, 1993).

- Capital cost, principally determined by the number and diameter of trays. For small values of R, the capital cost decreases with R because the number of trays rapidly decreases, while at very high values of R it rises again because the number of trays is close to the minimum value but the column diameter increases.
- Operating costs, principally determined by the cost of the steam. The operating costs increase with the reflux ratio since an increasing boil-up rate is required.

There is no simple correlation between the optimal reflux ratio and the separation characteristics, but practical values generally lie between 1.1 and 1.5 times the minimum value.

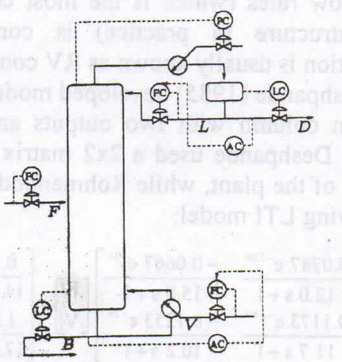

Figure 1. Distillation column scheme

Except for the case of heat-sensitive products, the operating pressure must be chosen in such a way that the dew point of the condensate is above the plant cooling water temperature. The maximum cooling water temperature is usually taken as $30\ ^{\circ}C$ and, therefore, the dew point of the top product should not be below $45 \div 50\ ^{\circ}C$. When this corresponds to very high pressures, the possibility of a refrigerating cycle should be considered. Vacuum operations can be necessary in the presence of heat-sensitive products. With these considerations in mind, the operating pressure should be as low as possible because a lower pressure provides a higher relative volatility and, therefore, a smaller number of stages.

The following assumptions have been accepted to model the column

- Liquid and vapour mole rates are constant (in each section above and below the feed stage).
- Liquid and vapour mixtures are in thermodynamic equilibrium in each stage (i.e. they have the same temperature and their compositions are related by the Rault low).
- The hold up of each stage is assumed to be constant.
- The operating pressure is constant.
- The dynamics of the condenser level control and of the bottom level control are negligible with respect to the composition control dynamics, so that the overall mass balances in these stages are assumed to be always satisfied.

In Figure 1 (adapted from Pannocchia, 2001), a general scheme of a distillation column with its corresponding control loops is presented (Skogestad and Postlethwaite, 1996). Two different levels of control objectives can be identified.

1. Inventory control: safe fluid-dynamic operation of the column is accomplished by the level control loops (LC) on the top product vessel and on the bottom of the column, and through the column pressure control loop (PC).
2. Quality control: the desired product purity is obtained by the product quality control loops (AC).

Here and in what follows, the case of the control of both qualities through the manipulation of reflux and boil-up flow rates (which is the most commonly used control structure in practice) is considered. This configuration is usually known as RV control structure.

Deshpande (1985) developed model of the binary distillation column with two outputs and two control variables. Deshpande used a 2x2 matrix to capture the dynamics of the plant, while Rohmanuddin (1998) used the following LTI model:

$$\begin{bmatrix} X_D \\ X_B \end{bmatrix} = \begin{bmatrix} \dfrac{0.0747\ e^{-3.0s}}{12.0\ s+1} & \dfrac{-0.0667\ e^{-2s}}{15.0\ s+1} \\ \dfrac{0.1173\ e^{-3.0s}}{11.7\ s+1} & \dfrac{-0.1253\ e^{-2s}}{10.2\ s+1} \end{bmatrix} \begin{bmatrix} R \\ V \end{bmatrix} + \begin{bmatrix} \dfrac{0.7\ e^{-5s}}{14.4\ s+1} \\ \dfrac{1.3\ e^{-3s}}{12.0\ s+1} \end{bmatrix} X_F \quad (2)$$

X_D = the top product composition

X_B = the bottom product composition

R = the reflux

V = the boil-up

X_F = the feed composition

Predictive Control Based On Neural Models

The use of neural networks for nonlinear process modeling and identification is justified by their capacity to approximate the dynamics of nonlinear systems including those with high nonlinearities or dead time. In order to estimate the nonlinear process, the neural network must be trained until the optimal values of the weight vectors (i.e. weights and biases in a vector form organization) are found. In most applications, feedforward neural networks are used, because the training algorithms are less complicated.

When it comes to nonlinear models, the most general one, which includes the largest class of nonlinear processes, is doubtless the NARMAX model (Chen and Billings, 1989; Narendra and Parthasarathy, 1990) given by:

$$\begin{aligned} y[k] = f(&y[k-1], \dots, y[k-n], \\ &u[k-d], \dots, u[k-d-m]) \\ &k, m, n, d \in N \end{aligned} \quad (3)$$

where $f : \mathbf{R}^{n+m+1} \rightarrow \mathbf{R}$ is a smooth nonlinear mapping describing the input-output transfer of a static network with adequate topology (i.e. MLP or RBF), d is the dead time, n and m are the orders of the nonlinear system model. A neural-network-based model corresponding to the NARMAX model may be obtained by adjusting the weights of a multi-layer perception architecture with adequately delayed inputs (Chen, et.al, 1990). The neural NARMAX model is briefly represented in Figure 2 (adapted from Lazar and Pastravanu, 2002).

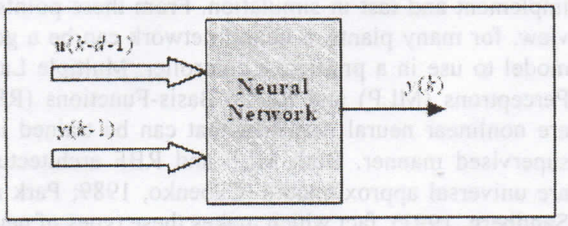

Figure 2. The neural NARMAX model

In this case, the neural network output will be given by:

$$y[k] = f^N(u[k-d-1], y[k-1]) \quad (4)$$

where f^N denotes the input output transfer function of the neural network which replaces the nonlinear function f in (3), and $u[k-d-1]$, $y[k-1]$ are vectors which contain m respectively n delayed elements of u and y starting from the time instant $k-1$, i.e.:

$$u[k-d-1] = [u[k-d-1], u[k-d-2], \cdots, u[k-d-m]]^T \quad (5)$$

$$y[k-1] = [y[k-1], y[k-2], \cdots, y[k-n]]^T$$

The neural NARMAX corresponds to a recurrent neural network, because some of the network inputs are past values of the network output.

If equation (4) is explicitly written for a two-layer network, the following expression is obtained for the network output at an arbitrary discrete-time instant:

$$y[k] = \sum_{j=1}^{N} w_j \sigma_j (w_j^u u[k-d-1] + w_j^y y[k-1] + b_j) + b \quad (6)$$

where the following notations were used:

N – the number of neurons in the hidden layer;

w_j – the weight for the output layer correspon-ding to the $j-th$ neuron from the hidden layer;

σ_j the activation function for the $j-th$ neuron from the hidden layer;

w_j^u – the weight vector (row vector) for the $j-th$ neuron with respect to the inputs stored in $u[k-d-1]$;

w_j^y – the weight vector (row vector) for the $j-th$ neuron with respect to the inputs stored in $y[k-1]$;

b_j – the bias for the $j-th$ neuron from the hidden layer;

b – the bias for the output layer.

Such structures with a single hidden layer are considered satisfactory for most of the cases. In order to obtain the model of a nonlinear process, the vector $u[k-d-1]$ defined by (5) is applied as input to the process. The plant output is stored in a vector, which will be used as the target vector for the neural network. The target vector together with an input vector, which contains the input values applied to the plant, are used to train the neural network. The training procedure consists in sequentially adjusting the network weight vectors, so that the mean squared error between the desired response (the values from the target vector) and the network output is minimized. Thus when a certain stop criteria is satisfied, the training algorithm gives a set of optimal values of the weight vectors.

The neural network with the constant values for the weight vectors, obtained after the training, represents the nonlinear model of the system. Before this model is used to obtain the neural predictors, validation of the neural based model is necessary. Two validation methods are recommended:

- a what-if test, which is a time validation test consisting in a comparison between the output of the neural network and the output of the nonlinear system when an input signal, different from the input signal used to train the network is used. This test uses the following error index to appreciate the quality of the model (with N the number of samples):

$$\text{error}_{index} = \sqrt{\frac{\sum_{k=1}^{N} (y_n[k] - y_p[k])^2}{\sum_{k=1}^{N} (y_p[k])^2}} \quad (7)$$

where y_p is the process output and y_n is the neural model output.

- a correlation test based on five correlation criteria.

If the neural network succeeds in all these tests, it is accepted as a good model of the nonlinear system.

The predictors are necessary for the prediction of future values of the plant output that are considered in the predictive control strategy. The implementation approach proposed in this paper uses neural predictors obtained by appropriately shifting the inputs of the neural based model. The predictive control algorithm utilizes them in order to calculate the future control signal. Neural predictors rely on the neural-based model of the process (Liu, et.al., 1998; Tan and De Keyser, 1994). In order to obtain the model of the nonlinear system, the same structure of the neural network given by (6) is considered. A sequential algorithm based on the knowledge of current values of u and y together with the neural-network system model gives the $i-$step ahead neural predictor. From equation (6), one can properly derive the network output at the k+1 time instant:

$$y[k+1] = \sum_{j=1}^{N} w_j \sigma_j (w_j^u u[k-d] + w_j^y y[k] + b_j) + b \quad (8)$$

where:

$$u[k-d] = [u[k-d], u[k-d-1], \cdots, u[k-d+1-m]]^T \quad (9)$$

$$y[k] = [y[k], y[k-1], \cdots, y[k+1-n]]^T$$

This is the expression of the one-step-ahead predictor, with respect to the notations introduced in equation (6). In Figure 3 (adapted from Lazar and Pastravanu, 2002), the construction of the neural predictors is presented in a suggestive manner. Extending equation (8) one step further ahead, $y[k+2]$ can be obtained and generally, the $i-$ step ahead predictor can be derived:

$$y[k+i] = \sum_{j=1}^{N} w_j \sigma_j (w_j^u u[k-d+i-1] + w_j^y y[k+i-1] + b_j) + b \quad (10)$$

where:

$$u[k-d+i-1] = [u[k-d+i-1], u[k-d+i-2], \cdots, u[k-d+i-m]]^T \quad (11)$$

$$y[k+i-1] = [y[k+i-1], y[k+i-2], \cdots, y[k+i-n]]^T$$

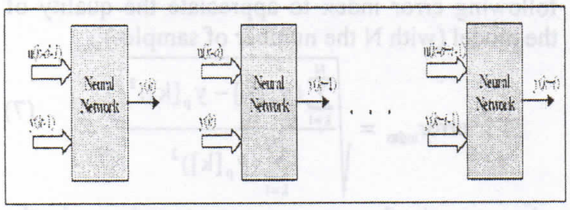

Figure 3. The structure of the neural predictors

The neural predictors will be used by the predictive control algorithm for calculating the future control signal to be applied to the nonlinear system.

An intuitive graphical representation of the predictive control strategy is given in Figure 4 (adapted from Kloetzer and Pastravanu, 2004). At each time moment kT, where T is the sample time and will be omitted in the following formulas for simplicity, the future control policy u is computed in the idea that the process output y will accurately follow the reference trajectory r.

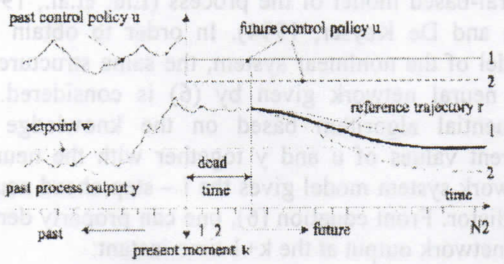

Figure 4. MBPC strategy

In presenting the basics of the standard predictive control, the following notations will be used: N_u – the control horizon; N_1 – the minimum prediction horizon; N_2 – the prediction horizon; λ – the weight factor; r – the reference trajectory.

The objective of the predictive control strategy using neural predictors is twofold: (i) to estimate the future output of the plant and (ii) to minimize a cost function based on the error between the predicted output of the processes and the reference trajectory. The cost function, which may be different from case to case, is minimized in order to obtain the optimum control input that is applied to the nonlinear plant. A possible form of the cost function, used in most of the predictive control algorithms, a quadratic form is utilized for the cost function:

$$J = \sum_{i=N_1}^{N_2} \{r[k+i] - y[k+i]\}^2 + \lambda \sum_{i=1}^{N_u} \{u[k+i-1] - u[k+i-2]\}^2 \quad (12)$$

with additional requirements:

$$\Delta u[k+i-1] = 0, \quad 1 \le N_u < i \le N_2 \quad (13)$$

where the following notations were used:

N_u – the control horizon;

N_1 – the minimum prediction horizon;

N_2 – the prediction horizon;

i – the order of the predictor;

r – the reference trajectory;

λ – weight factor;

Δ – the differentiation operator.

The command u may be subject to amplitude constraints:

$$u_{min} \le u[k+i] \le u_{max}, \quad i = 1, \cdots, N_2 \quad (14)$$

The cost function is often used with the weight factor $\lambda = 0$ and the minimum prediction horizon is $N_1 = 1$. A very important parameter in the predictive control strategy is the control horizon N_u, which specifies the instant time, since when the output of the controller should be kept at a constant value. The output sequence of the optimal controller is obtained over the prediction horizon by minimizing the cost function with respect to the vector $\mathbf{u}$, at each time moment k. This can be achieved by setting:

$$\frac{\partial J}{\partial \mathbf{u}} = 0, \quad \mathbf{u} = [u[k-d], u[k-d+1], \cdots, u[k-d+N_u-1]]^T \quad (15)$$

However, when proceeding further with the calculation of ∂, a major inconvenience occurs. The analytical approach to the optimisation problem needs for the differentiation of the cost function and, finally, leads to a nonlinear algebraic equation; unfortunately this equation cannot be solved by any analytic procedure. This is why a computational method is preferred for the minimization of the cost function, also complying with the typical requirements of the real-time implementations (guaranteed convergence, at least to a sub-optimal solution, within a given time interval).

The computation of the optimal control signal at the discrete time instant k can be achieved with the following algorithm:

- the minimization procedure, performed at the previous time instant gives the command vector:

$$\mathbf{u} = [u[k-d-1], u[k-d], \cdots, u[k-d+N_u-2]]^T \quad (16)$$

At the first time instant, the control input vector will contain some initial values provided by the user. The number of values introduced must be equal to the control horizon.

- the step ahead predictors of orders between N_1 and N_2 are calculated by using the vectors $u[k-d+N_1-1], y[k+N_1-1]$ and $u[k-d+N_2-1], y[k+N_2-1]$ respectively, as well as the neural network based process model.
- the output control signal is obtained by minimizing the cost function J with respect to the command vector:

$$u = \left[u[k-d], u[k-d+1], \cdots, u[k-d+N_u-1]\right]^T \quad (17)$$

The advantage of this nonlinear neural predictive controller consists in the implementation method that solves the key problems of the nonlinear MBPC. The implementation is robust, easy to use and fulfills the requirements imposed for the minimization algorithm. Changes in the parameters of the neural predictive controller (such as the prediction horizons, the control horizon, as well as the necessary constraints) are straightforward operations.

Considering the basic theoretical preliminaries of MPBC presented in this section, a block diagram of a standard predictive controller can be depicted as in Figure 5 (adapted from Kloetzer and Pastravanu, 2004).

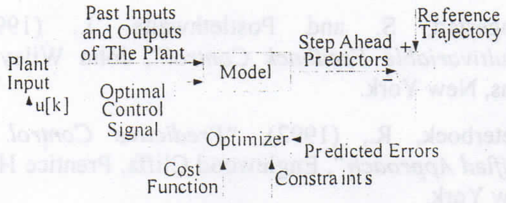

Figure 5. Basic structure of MBPC

Simulation Result And Discussion

The proposed neural based predictive control algorithm was compared with the existing PID control used in industry. In this application, it is important to prevent overshoots which seriously affect the quality of the control system.

The improvement offered over the PID control scheme is purely a result of the ability of the Neural Based Predictive Controller (NBPC) to handle process dead time and interaction. The response to a set point change in the top product composition from 0.96 to 0.98 and the bottom product composition from 0.04 to 0.02 is shown in Figure 6 and Figure 7, respectively. Shorter settling time and less overshoot are obtained by the NBPC.

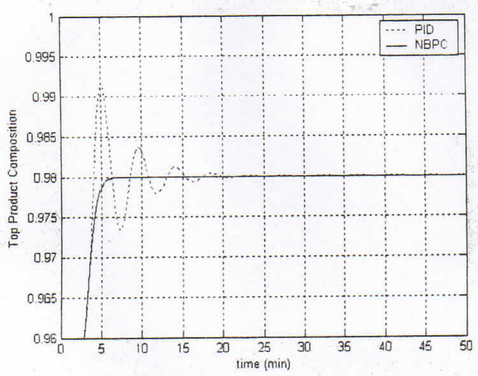

Figure 6. Top Product Response to Set Changes From 0.96 to 0.98

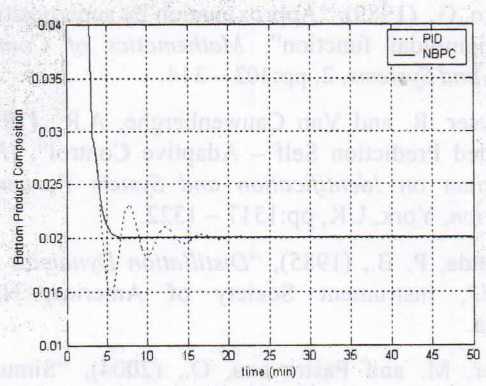

Figure 7. Bottom Product Response to Set Changes From 0.04 to 0.02

For the proposed controller, it could be easily tuned to completely kill the overshoot with a reduction about 20% of the seek time as shown in the Figure 6 and Figure 7 compared to the PID controller. It is clear from Figure 6 and Figure 7 that the proposed controller completely meets the design specifications mentioned above, while, it is not easy to tune the PID controller for such a purpose.

Conclusions

A neural based predictive controller applied in distillation column has been reported in this paper. By applying the proposed controller we can get the advantages of the neural based predictive controller both at the transient region and at the steady state region to overcome any overshoots.

Simulation results using a mathematical model of distillation column show a much better performance using the proposed controller compared to that of the PID controller. Surprisingly, such an enhanced response is accompanied by minimal or even no-overshoot while; the control input limit has not been reached.

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