Flutter Analysis on the Wing of Typical 4.5th Generation Fighter Aircraft

AH Fauzi[a, *], M. Munadi[b], Nanang Hery S[a], Arief N. Pratomo[a],
NHM Silalahi[a], Dina Aisyah[c], MN Setiawan[d], Apriliandi Nurhidayat [a], Ahmad Syafiq R[a],
[a]Department of Mechanical Engineering, Faculty of Defense Science and Technology,
Republic of Indonesia Defense University
Kawasan Indonesia Peace and Security Center (IPSC) Sentul Bogor Jawa Barat
[b]Department of Mechanical Engineering, Faculty of Engineering, Diponegoro University
Jl. Prof. Jacub Rais, Tembalang, Kec. Tembalang, Kota Semarang, Jawa Tengah 50275
[c]PT Dirgantara Indonesia
Jalan Pajajaran No. 154, Bandung 40174, Jawa Barat - Indonesia
[d]Department of Renewable Energy Engineering, School of Applied STEM, Prasetiya Mulya University
BSD City Kavling Edutown I.1, Jl. BSD Raya Utama, BSD City 15339,Kabupaten Tangerang, Indonesia
*E-mail: ahmad.hasanfauzi05@gmail.com

Abstract

Flutter is a phenomenon of dynamic instability caused by aerodynamic force, inertial force, and structural elasticity. It is recognized by catastrophic oscillation of the structure at a certain flow speed. In aircraft design, flutter analysis must be conducted to ensure that the aircraft does not undergo flutter in its operational region. This work focuses on flutter analysis of the wing of a 4.5th generation fighter aircraft, using the MSC.FLDS/PATRAN/NASTRAN software. The process involves several procedures, including structure modeling, structural dynamics analysis, aerodynamic load modeling, and flutter analysis. The structure is modeled using the finite element method, while the doublet lattice method is used for aerodynamic load modeling in the subsonic regime and ZONE 51 in the supersonic regime. The p-k method is used to solve the flutter analysis. After conducting flutter analysis in the basic configuration and condition, a parametric study is performed to understand how some parameters affect the change in flutter characteristics. The results of the parametric study show that the speed and characteristics of flutter change with changes in flight conditions (altitude and Mach number) and mass configuration (position of the missile in the spanwise direction). Flight conditions change the aerodynamic load in the system, while missile position and structure thickness change the inertia and stiffness of the system. Changes in these parameters - aerodynamic, inertia, and stiffness - affect the flutter speed and characteristics.

Keywords: doublet lattice method; finite element method; flutter; structure dynamics; 4.5 generation fighter jet

Abstrak

Flutter merupakan fenomena ketidakstabilan dinamik yang diakibatkan adanya interaksi antara gaya aerodinamika, gaya inersia, dan elastisitas struktur. Flutter ditandai dengan munculnya osilasi berbahaya pada struktur pada kecepatan aliran tertentu. Pada desain pesawat analisis flutter harus dilakukan untuk menjamin pesawat tidak mengalami fenomena flutter pada daerah operasi terbangnya.Pada penelitian ini, analisis flutter dilakukan pada sayap pesawat tempur generasi 4.5 HF - 1. Analisis flutter dilakukan dengan menggunakan software MSC. FLD/PATRAN/NASTRAN. Dalam pengerjaan analisis flutter, terdapat beberapa prosedur yang dilakukan. Tahapan tersebut adalah pemodelan struktur sayap pesawat, analisis dinamika struktur, pemodelan beban aerodinamika pada sayap, interpolasi antara model struktur dan model beban aerodinamika, dan analisis flutter. Pada pengerjaannya, pemodelan struktur digunakan metode elemen hingga. Metode doublet lattice digunakan untuk pemodelan beban aerodinamika tak stasioner pada daerah subsonik. Sedangkan Metode ZONE 51 untuk pemodelan beban aerodinamika tak stasioner daerah supersonik. Kemudian untuk penyelesaian analisis flutter digunakan metode p-k. Setelah analisis flutter dilakukan pada konfigurasi dan kondisi basis, dilakukan studi parametrik untuk mengetahui bagaimana karakteristik flutter berubah jika suatu parameter diubah. Dari hasil studi parametrik diperoleh bahwa kecepatan dan karakteristik flutter berubah dengan berubahnya kondisi terbang yang meliputi tinggi terbang dan bilangan mach dan konfigurasi massa berupa posisi senjata pada arah span. Kondisi terbang berpengaruh pada beban aerodinamika yang bekerja pada sistem. Letak senjata dan ketebalan struktur berpengaruh pada karakteristik inersia dan kekakuan sistem. Perubahan parameter aerodinamik, inersia, dan kekakuan tersebut akan merubah kecepatan dan karakteristik flutter.

Kata kunci: dinamika struktur; metode elemen hingga; metode doublet lattice flutter; pesawat tempur generasi 4.5

1. Introduction

The 4.5 generation fighter aircraft is a fighter aircraft with wide operational capabilities and range. This generation of aircraft is capable of agile maneuvering and flying at speeds up to 1.8 Mach. Fighter aircraft are not immune to flutter phenomenon, which is a dangerous oscillation in the aircraft structure caused by the interaction between aerodynamic loads and the elastic structure of the aircraft. Therefore, flutter analysis is mandatory in the early stages of aircraft design to ensure that the aircraft is safe within its flight envelope.

According to Frazer et al. [1], "Practically, 'flutter' means oscillation that grows, and eventually either breaks the structure or remains limited to a certain amplitude whose value depends on the nonlinearity of the laws." Aeroelastic instabilities can be categorized into various types depending on how their stability is lost with increasing dynamic pressure or changes in other flight conditions. Divergent flutter can be "explosive" or "hard." A small increase in speed from just below the flutter speed to slightly above the flutter speed will cause extremely divergent oscillations, resulting in the aircraft structure's failure in less than one second. Divergent flutter can also be of the "moderate" type. Here, the loss of stability (as reflected in the reduction of aeroelastic damping in the system) can be identified far below the flutter speed and based on the gradual "sliding" towards instability, the flutter speed can be predicted more reliably by extrapolation tests. Flutter of the "light" type is characterized by the overall loss of aeroelastic damping well before the flutter speed is reached, while the system is still stable but with low damping. As explained by Bisplinghoff [2] beyond the flutter limit, the system is unstable, but the slow-growing divergent oscillation level allows the test pilot to slow back into the stable flight region. The mechanism of "peak mode" flutter will see a gradual loss of damping towards the flutter speed, then very low negative damping, and then, with increasing speed and additional dynamic pressure, an increase in damping back into the stable region. Whether a system will flutter or not in such cases is highly sensitive to the level of damping in the structure and other parameters that affect the structural and aerodynamic dynamic behavior. The stability concepts mentioned above are based on linear aeroelastic and aeroservoelastic theories. Flutter analysis should be carried out overall on all parts of the aircraft. However, in this final project, only the wing section of the aircraft will be analyzed. The analysis in this final project is only a part of a larger flutter analysis task. After flutter analysis of the wing section is completed, flutter analysis can be done on other parts of the aircraft. Component-level flutter analysis is critical for aircraft flutter analysis. This is to separate the characteristics of the overall flutter of the aircraft based on the components that have the most significant influence on the flutter characteristics of the aircraft. Thus, when the flutter characteristics of an aircraft are to be changed, modifications can be made to the aircraft component that has the most dominant flutter characteristics.

2. Research Methodology

The methodology used in this research is numerical simulation methodology. In this analysis, the wing model is built using 3D Design software. Then the CAD model is translated into a finite element model using MSC PATRAN/NASTRAN 2011 software. Next, a structural dynamics analysis is conducted to obtain the structural dynamic characteristics. Then an aerodynamic model is created and combined with the existing structural model to form an aeroelastic model, which will be analyzed to obtain the flutter characteristics of the wing of the 4.5 generation fighter aircraft. Finally, flutter analysis is performed for several variations of structural parameters and flight conditions. The complete flowchart of this research is shown in Figure 1 as shown below.

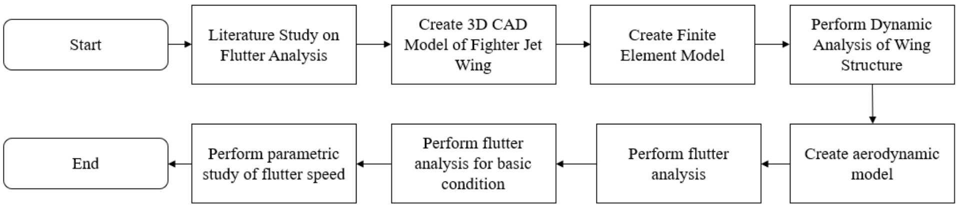


Figure 1 Flow Chart of Research

2.1 Aeroelasticity

Flutter is a dynamic aeroelasticity system. The response of the system does not only depend on one input at a certain time, but it will still depend on the beginning when the system receives input. The response of the dynamic system is expressed as a linear combination of each normal mode - the natural frequency of the system as the following as reported by Albano and Roddena [3]:

$$\{\boldsymbol{x(t)}\} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \boldsymbol{\phi_i q_i(t)} = [\boldsymbol{\phi}]\{\boldsymbol{q(t)}\} \qquad (\mathbf{1})$$

ϕ_i is the mode shape of vibration for i.
$q_i(t)$ is the time function of the mode shape for i.

The equation of motion of an aeroelastic system can be written as a differential equation:

$$[\boldsymbol{m}]\{\ddot{\boldsymbol{x}}(\boldsymbol{t})\} + [\boldsymbol{c}]\{\dot{\boldsymbol{x}}(\boldsymbol{t})\} + [\boldsymbol{k}]\{\boldsymbol{x}(\boldsymbol{t})\} = \boldsymbol{A}(\boldsymbol{x}(\boldsymbol{t}), \boldsymbol{\rho}, \boldsymbol{V},) \qquad (2)$$

Where:
$[\boldsymbol{m}]$s the structural mass matrix.
$[\boldsymbol{c}]$is the structural damping matrix.
$[\boldsymbol{k}]$ is the structural stiffness matrix.
$[\boldsymbol{A}]$ is the aerodynamic load matrix.
$\{\boldsymbol{x}(\boldsymbol{t})\}$ is the system response with finite degree of freedom.

The aeroelastic dynamic equation is expressed in modal coordinates. The transformation from physical coordinates to modal coordinates is obtained by subtituting Equation 1 into Equation 2.

$$[\boldsymbol{m}][\boldsymbol{\phi}_i]\{\ddot{\boldsymbol{q}}_i(\boldsymbol{t})\} + [\boldsymbol{c}][\boldsymbol{\phi}_i]\{\dot{\boldsymbol{q}}_i(\boldsymbol{t})\} + [\boldsymbol{k}][\boldsymbol{\phi}_i]\{\boldsymbol{q}_i(\boldsymbol{t})\} = [\boldsymbol{p}][\boldsymbol{\phi}_i]\{\boldsymbol{q}_i(\boldsymbol{t})\} \qquad (3)$$

Then multiply the equation above by the transpose of the mode shape matrix $[\boldsymbol{\phi}_i]^{-1}$ to obtain:

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{q}}_i(\boldsymbol{t})\} + [\boldsymbol{C}]\{\dot{\boldsymbol{q}}_i(\boldsymbol{t})\} + [\boldsymbol{K}][\boldsymbol{q}_i(\boldsymbol{t})] = [\boldsymbol{\phi}_i]^{\boldsymbol{T}}\,[\boldsymbol{p}][\boldsymbol{\phi}_i][\boldsymbol{q}_i(\boldsymbol{t})] \qquad (4)$$

Where it is defined the generalized mass matrix, damping, and stiffness as below:

$$[\boldsymbol{M}] = [\boldsymbol{\phi}]^{\boldsymbol{T}}[\boldsymbol{m}][\boldsymbol{\phi}]$$
$$[\boldsymbol{C}] = [\boldsymbol{\phi}]^{\boldsymbol{T}}[\boldsymbol{c}][\boldsymbol{\phi}]$$
$$[\boldsymbol{K}] = [\boldsymbol{\phi}]^{\boldsymbol{T}}[\boldsymbol{c}][\boldsymbol{\phi}]$$

By algebraic manipulation, the following equation is obtained:

$$\{\ddot{\boldsymbol{q}}_i(\boldsymbol{t})\} + [\boldsymbol{M}]^{-\boldsymbol{1}}[\boldsymbol{C}]\{\dot{\boldsymbol{q}}_i(\boldsymbol{t})\} + [\boldsymbol{M}]^{-\boldsymbol{1}}[\boldsymbol{K}][\boldsymbol{q}_i(\boldsymbol{t})] - [\boldsymbol{M}]^{-\boldsymbol{1}}[\boldsymbol{\phi}_i]^{\boldsymbol{T}}\,[\boldsymbol{p}][\boldsymbol{\phi}_i][\boldsymbol{q}_i(\boldsymbol{t})] = \boldsymbol{0} \qquad (5)$$

Equation 5 is the aeroelastic dynamic equation that consists of the inertia aspect indicated by the matrix [M], the structural damping aspect indicated by the matrix [C], the structural stiffness aspect indicated by the matrix [K], and the aerodynamic load aspect indicated by the matrix $[\boldsymbol{\phi}_i]^{\boldsymbol{T}}[\boldsymbol{p}][\boldsymbol{\phi}_i][\boldsymbol{q}_i(\boldsymbol{t})$. Thus, if mass, structural damping, structural stiffness, and aerodynamic load change, the solution of the equation will also change, resulting in a change in flutter characteristics. This forms the basis for parameter variation in flutter analysis.

2.2 Aerodynamic Load

In flutter analysis, aerodynamic modeling is a crucial component. The modeling is necessary to determine the aerodynamic forces and moments. The most important aspect in determining the aerodynamic terms in the equation of motion is the pressure difference (delta Cp) between the upper and lower surfaces of the structure. In this research, unsteady aerodynamic modeling is used, but the thickness effect of the airfoil is not included in the model. In MSC NASTRAN software, the solution for unsteady aerodynamic loads is performed using the Doublet Lattice method for the mach number regime of $0 < M < 0.9$ and ZONE 51 method for the low supersonic regime ($1.1 < M < 3.0$). Both the doublet lattice and zone 51 methods have similar matrix structures. The matrix equations that summarize the relationships required to define several aerodynamic coefficients are described by Equation 6, 7, and 8. These three equations describe the relationship between lifting pressure and the normal or vertical velocity induced by the inclination on the surface with respect to the flow or downwash as described by Nastran[5]:

$$\{w_j\} = [A_{jj}]\left\{\frac{f_j}{q}\right\} \qquad (6)$$
$$\{w_j\} = [D_{jk}^1 + ikD_{jk}^2]\{u_k\} + \{w_j^g\} \qquad (7)$$
$$\{P_k\} = [S_{kj}]\{f_j\} \qquad (8)$$

Where:

w_j	downwash
$\{w_j^g\}$	static aerodynamic downwash: generally, includes the distributed static incidence that may arise due to initial angle of attack, camber, and twist
f_j	pressure on element j
q	dynamic pressure

k	reduced frequency, $k = \omega b/U$ where ω is the angular frequency, b is the reference semichord, and U is the freestream velocity.
$A_{jj}(m,k)$	matrix of aerodynamic influence coefficients, function of Mach number and reduced frequency (k)
u_k, P_k	displacement and force at aerodynamic grid point
D^1_{jk}, D^2_{jk}	real and imaginary parts of the substantial differential matrix, respectively (dimensionless)

And finally, by joining Equation 6, 7, and 8, matrix of aerodynamic coefficient can be obtained as described in Equation 9 below.

$$[\boldsymbol{Q_{kk}}] = [\boldsymbol{S_{kj}}][\boldsymbol{A_{jj}}]^{-1}[\boldsymbol{D^1_{jk}} + \boldsymbol{ikD^2_{jk}}] \tag{9}$$

2.3 Flutter Analysis Method

Characteristic of aeroelastic dynamic system causes the complexitiy on physical phenomenon modelling. According to Scanlan[4], there are various approaches to overcome the difficulties in solving the flutter problem such as PK Method, K method, and PKNL Method. In this work, PK Method is selected to solve the flutter problem. PK method assumes the equation of motion as follow:

$$\boldsymbol{x(t) = xe^{pt}} \tag{10}$$

Then the aerodynamic terms can be written as:

$$\{\boldsymbol{A(real + kompleks}\} = \left(\frac{\boldsymbol{\rho V^2}}{\boldsymbol{2}}[\boldsymbol{Q}] + \frac{\boldsymbol{\rho \bar{c} V}}{\boldsymbol{4}}[\boldsymbol{Q}]\boldsymbol{p}\right)\{\boldsymbol{q}\} \tag{11}$$

Equation 10 and 11 combined to obtain:

$$\left[[\boldsymbol{M}]_{\boldsymbol{nxn}}\boldsymbol{p^2} + \left[[\boldsymbol{C}] - \left[\frac{\boldsymbol{\rho cV}}{\boldsymbol{4}}(\boldsymbol{Q^i})\right]\right]\boldsymbol{p} + \left[[\boldsymbol{K}] - \left(\frac{\boldsymbol{\rho V^2}}{\boldsymbol{2}}[\boldsymbol{Q^R}]\right)\right]\right]\{\boldsymbol{q(t)}\}_{\boldsymbol{nx1}} = \boldsymbol{0} \tag{12}$$

$$([\boldsymbol{M}]_{\boldsymbol{nxn}}\boldsymbol{p^2} + [\boldsymbol{B}]_{\boldsymbol{nxn}}\boldsymbol{p} + [\boldsymbol{K}]_{\boldsymbol{nxn}})\{\boldsymbol{q(t)}\}_{\boldsymbol{nxn}} = \boldsymbol{0} \tag{13}$$

And the solution for equation above as reported by Vepa [6] can be written in form of second order equation of motion as follow:

$$\boldsymbol{A - pIu = 0} \tag{14}$$

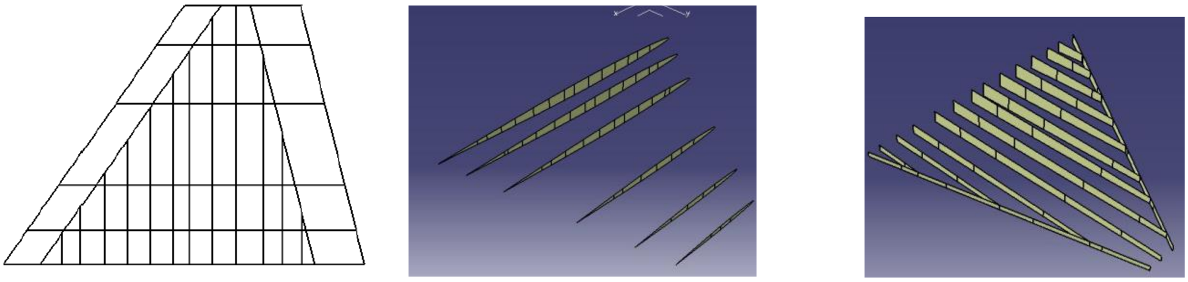
Where

$$[\boldsymbol{A}] = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{1} \\ \dfrac{[\boldsymbol{K}]}{[\boldsymbol{M}]} & \dfrac{[\boldsymbol{B}]}{[\boldsymbol{M}]} \end{bmatrix} \tag{15}$$

where matrix [A] is real matrix and u (generalized vector coordinates) which involves modal displacement and speed. Eigenvalues of matrix real [A] can be real or complex. The real part shows the convergence or divergence of the structure.

2.4 Wing Model

The data of the 4.5th Generation Jet Fighter Wing is obtained by estimation from other aircrafts in the same family or generation with reference data from Mahasti[8] and Ndaomanu[9]. Figure 1 shows the layout of the wing. The construction of the wing consists of skins, Spasr, and Ribs as shown in Figure 2 (a) and (b).



(a) (b) (c)

Figure 2 (a) General Layout of Wing (b) and (c) Ribs and Spars Configuration of Wing

The wing is assumed to to use Aluminium 2024 with the properties shown in Tabel 1 below. To accurately represents actual conditions on the wing, additional distributed weight is added into the model. The distributed weight represents the weight of fuel tank and weapons carried by aircraft in the wing. The mass of the distibuted weight can be seen in Tabel 2 below.

Table 1. General Configuration of the Wing

Parameters	Size
Span	10.10 meter
Chord Root	6.47 meter
Chord Tip	1.67 meter
LE Swept	35 derajat
TE Swept	-14 derajat
Incidence	0
Dihedral	0
Twist	0

Table 2. Additional Distributed Mass in Wing

	Mass (Kg)
Fuel Tank	500
Weapons	152

Further, additional concentrated mass is added to represent the mass of control surfaces as shown in Figure 3 below. The mass of the control surfaces is described in Table 3 below.

Table 3. Mass of Control Surface

Control Surfaces	Massa total (kg)	Massa perkoneksi (Kg)	Jumlah koneksi
Slat	22	7.33	3
flap	30	10	3

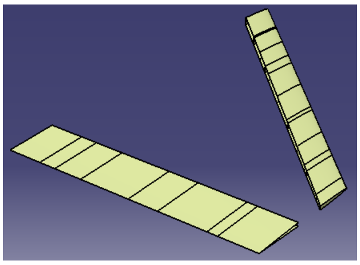


Figure 3 Control Surface Model

Other than wing structure configuration and position of structure components, one of the most important and defining factors is thickness of the structure. The thickness of the structure will directly affect the mass inertia and stiffness, hence the flutter speed. The thickness of the structure is described by Figure 4 and Table 4.

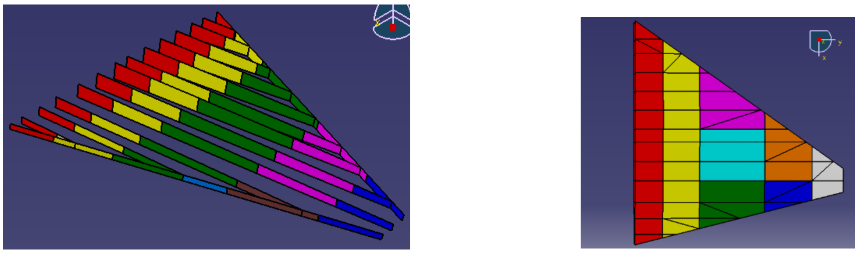


Figure 4 (a) Thickness distribution of Spars and Ribs (b) Thickness distribution of skin

Table 4 (a) Spar Thickness

Spar Thickness	
Segment	Thickness (mm)
🟥	4.9
🟨	3.8
🟩	3
🟪	3
🟦	3.5
🟫	3.4
🟦	4.54

Table 4 (b) Skin Thickness

Ketebalan Skin	
Segment	Thickness (mm)
🟥	7.1
🟨	6.35
🟩	7.1
🟪	8
🟦	6.7
🟧	6.3
🟦	7.7
⬜	4.064

3. Results and Discussion

3.1 Structure Dynamic of Wing

The phenomenon of flutter is closedly related with ratio between bending frequencies and torsion. Structure dynamics analysis is performed to determine the significant mode shapes and natural frequencies. In this research, structure dynamic analysis is performed for the first 20th mode shapes. From 20 mode shapes obtained, only significant mode shapes will be used for flutter analysis. Below presented 1st and 2nd mode shape of the wing structure and the first eight natural frequencies of the wing.

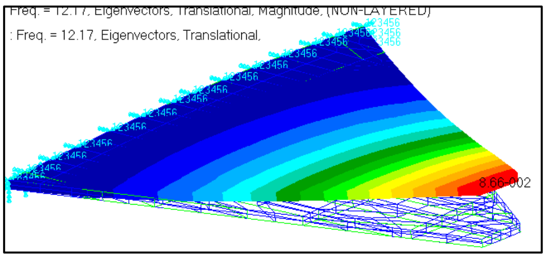


Figure 5 Mode Shape 1 (first bending)

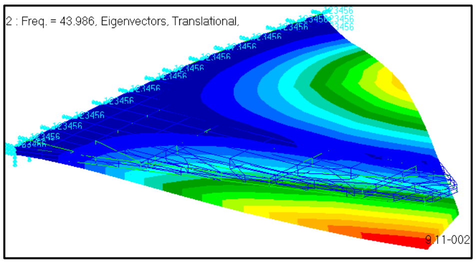


Figure 6 Mode Shape 2 (first torsion)

Table 5 Natural Frequencies of Wing Structure

Mode No.	Eigen Value	Frequency (radian)	Frequency (Hz)	Generalized Mass	Generalized Stiffness
1	1.00E+04	1.00E+02	1.59E+01	1.00E+00	1.00E+04
2	9.64E+04	3.11E+02	4.94E+01	1.00E+00	9.64E+04
3	1.31E+05	3.62E+02	5.76E+01	1.00E+00	1.31E+05
4	3.34E+05	5.78E+02	9.20E+01	1.00E+00	3.34E+05
5	4.71E+05	6.86E+02	1.09E+02	1.00E+00	4.71E+05
6	6.36E+05	7.97E+02	1.27E+02	1.00E+00	6.36E+05
7	7.01E+05	8.37E+02	1.33E+02	1.00E+00	7.01E+05
8	8.22E+05	9.06E+02	1.44E+02	1.00E+00	8.22E+05

3.2 Flutter Speed of Basic Configurations

The Flutter analysis for basic condition has been performed. Base condition means the flight is performed near the sea level. It can be seen in Figure 7 and Figure 8 that mode 2 intersects the velocity axis, indicating that mode 2 experiences flutter. In this base configuration, mode 2 interacts with the first mode. This is shown in the frequency vs. velocity curve in Figure 8, where the frequencies of mode 1 and mode 2 are seen to approach each other.

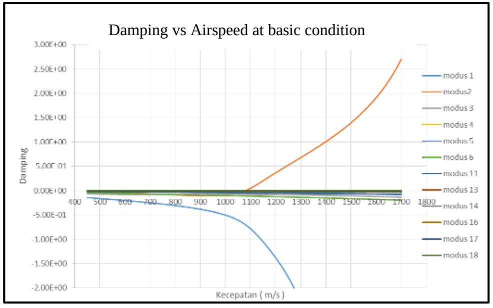


Figure 7 Damping vs Flight Speed at basic configuration

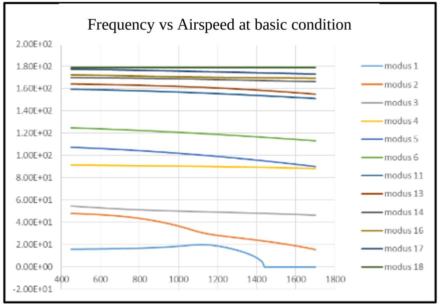


Figure 8 Frequency vs Flight Speed at basic Condition

By incorporating the structural damping factor of g = 0.03, the flutter velocity in the base configuration is obtained to be 1100 m/s.

3.3 Flutter Speed with Variation of Flight Condition

After flutter speed of the wing at basic condition has been obtained, a parametric study is performed to observe the variation of flight speed relative to flutter speed. The variations of flight speed were conducted at sea level altitude, 15000 feet, and 30,000 feet. Then, variations in the Mach number were performed at sea level and 30,000 feet altitude as shown in Figure 9.

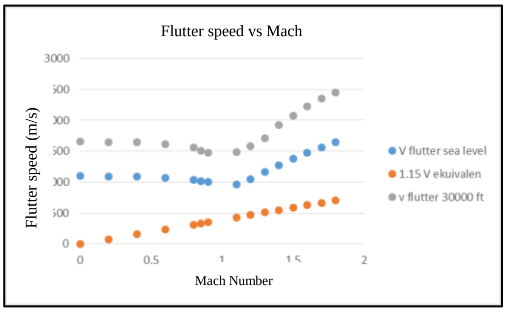


Figure 9 Flutter Speed vs Mach Number at sea level and 30000 ft

It can be shown that with increasing of altitude, the flutter speeds increase as well. It is already predicted because with increase of height, density decreases, hence the aerodynamic load decreases as well. It can be shown too from Figure 9 too that the structure undergoes signicant dip in flutter speed around Mach 1. According to Fung[10] and Dowell[11] this phenomenon is caused by the increase of aerodynamic load around Mach 1 which inversely proportional with $\sqrt{1-M^2}$, hence the decrease of flutter speed around Mach number of 1.

3.4 Flutter Speed with Variation of Missile Position (Spanwise)

In this work, the position of missile is varied along the span direction. From simulation, it was found that the flutter velocity will increase if the missile position moves towards the tip as shown in Figure 10. Modes 1 and 2 contribute to the occurrence of flutter. The flutter velocity increases because the frequency of these two modes decreases at different rates. The first bending mode decreases more than the torsional mode, which results in the second mode frequency approaching a higher velocity. However, it is not conclusive if the variation in mass towards the tip of the span will always increase the flutter velocity. Further investigation needs to be conducted in future research to study which point will produce an optimal point to produce maximum flutter velocity. Since in this analysis, the missile mass is discreetly moved with a sufficiently large distance, the results obtained do not fully represent the actual sensitivity of flutter.

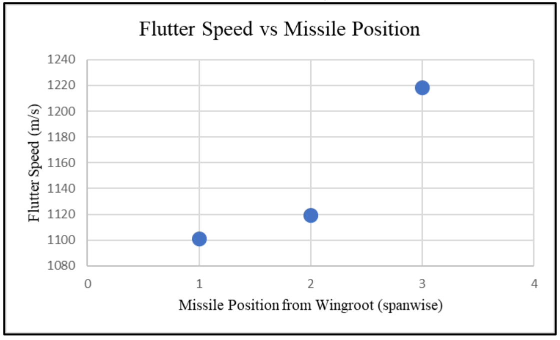


Figure 10 Flutter Speed vs variation of missile position (spanwise)

4. Conclusion

From the analysis, flutter of 4.5 generation fighter aircraft wing at Mach 0 at sea level occurs at the speed of 1100 m/s. Variation of Mach numbers at sea level, the flutter velocity obtained from the analysis is greater than 1.15 times the equivalent velocity, so it can be said that the 4.5 generation fighter aircraft wing is free from flutter at sea level. Further, parametric study of changing the missile position along the span will increase the flutter velocity. However, it cannot be generally concluded that variations towards the tip at span direction will increase the flutter velocity. This is because the variation was only conducted at three discrete points. Further investigation to conducted for more conclusive results. Variation of flight altitude results in decrease of air density, hence the increase of flutter speed.

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