Fuzzy State Observer Design for Engine Torque Control System of Spark Ignition Engine

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Abstract— **The T-S (Takagi-Sugeno) model approach consists to construct nonlinear or complex dynamic systems that cannot be exactly modeled, by means of interpolating the behavior of several LTI (Linear Time Invariant) sub models. Since T-S observer based controller has been considered to develop some systematic design algorithms to guarantee the stability and specific performances for the T-S model based systems, this paper represent a new design of fuzzy state observer to estimate the states of engine torque control system of spark ignition engine.**

Keywords – T-S model, T-S observer, fuzzy state observer, engine torque, spark ignition engine.

I. INTRODUCTION

The design of state feedback control, as well as the design of state observer, for nonlinear systems, has been actively considered during the last decades in many works using the Takagi-Sugeno (T-S) models [1]–[4]. The T-S model approach consists to construct nonlinear or complex dynamic systems that cannot be exactly modeled, by means of interpolating the behavior of several LTI (Linear Time Invariant) sub models. Each sub model contributes to the global model in a particular subset of the operating space [2], [5], [6].

Note that this modeling approach can be applied for a large class of physical and industrial processes as automotive control [7] and robot manipulators [8]. Recently, T-S observer based controller has attracted increasing attention, because it can provide a suitable solution to the control of plants that are complex and ill defined and have immeasurable state variables [9]–[12]. The T-S observer based controller has been considered to develop some systematic design algorithms to guarantee the stability and specific performances for the T-S model based systems [4], [13], [14].

In this paper, a fuzzy state observer will be designed based on T-S model to represent engine torque control system of spark ignition engine, since spark ignition engine is a high non linear system with wide uncertainties.

II. TAKAGI-SUGENO FUZZY MODEL

A dynamic T-S fuzzy model is described by a set of fuzzy “IF … THEN” rules with fuzzy sets in the antecedents and dynamic linear time-invariant systems in the consequents. A generic T-S plant rule can be written as follows [15]:

i^{th} *Plant Rule*:
IF $x_1(t)$ *is* M_{i1} *and* ... $x_n(t)$ *is* M_{in} THEN $\dot{x} = A_i x + B_i u$

where $x \in R^{nx1}$ is the state vector, r is the number of rules, M_{ij} are input fuzzy sets, $u \in R^{mx1}$ is the input and $A \in R^{nxn}$, $B \in R^{nxm}$ are state matrix and input matrix respectively.

Using singleton fuzzifier, max-product inference and center average defuzzifier, the aggregated fuzzy model can be written as:

$$\dot{x} = \frac{\sum_{i=1}^{r} \omega_i(x)(A_i x + B_i u)}{\sum_{i=1}^{r} \omega_i(x)} \quad (1)$$

with the term ω_i is defined by:

$$\omega_i(x) = \prod_{j=1}^{n} \mu_{ij}(x_j) \quad (2)$$

where μ_{ij} is the membership function of the j^{th} fuzzy set in the i^{th} rule. Defining the coefficients α_i as:

$$\alpha_i = \frac{\omega_i}{\sum_{i=1}^{r} \omega_i} \quad (3)$$

then (1) can be modified as:

$$\dot{x} = \sum_{i=1}^{r} \alpha_i(x)(A_i x + B_i u) \qquad i = 1, \dots, r \tag{4}$$

$$\sum_{i=1}^{r} \alpha_i = 1$$

where $\alpha_i > 0$ and .

Using the same method for generating T-S fuzzy rules for the controller, the controller rules defined in [16] as:

i^{th} Controller Rule:
IF *$x_1(t)$ is M_{i1} and ... $x_n(t)$ is M_{in}* THEN $u = -K_i x$

The overall controller would be:

$$u = -\sum_{i=1}^{r} \alpha_i(x) K_i x \tag{5}$$

Replacing (5) in (4), the equation for the closed loop system would be:

$$\dot{x} = \sum_{i=1}^{r}\sum_{j=1}^{r} \alpha_i(x)\alpha_j(x)(A_i - B_i K_j)x \tag{6}$$

The following theorems are used to achieve the stability of the closed loop system:

Theorem 1 [17]: The closed fuzzy system (6) is globally asymptotically stable if there is exists a common positive definite matrix *P* which satisfies the following Lyapunov inequalities:

$$\begin{cases} (A_i - B_i K_i)^T P + P(A_i - B_i K_i) < 0 & i = 1, \dots, r \\ G_{ij}^T P + P G_{ij} < 0 & i < j \le r \end{cases} \tag{7}$$

where G_{ij} is defined as:

$$G_{ij} = A_i - B_i K_j + A_j - B_j K_i \tag{8}$$

Pre-multiplying and post-multiplying both sides of inequalities in (7) by P^{-1} and using the following change of variables:

$$\begin{cases} Y = P^{-1} \\ X_i = K_i Y \end{cases} \tag{9}$$

the LMIs obtained in [18] by:

$$\begin{cases} Y A_i^T + A_i Y - B_i X_i - X_i^T B_i^T < 0 \qquad i = 1, \dots, r \\ Y(A_i + A_j)^T + (A_i + A_j)Y - (B_i X_j + B_j X_i) \\ \qquad - (B_i X_j + B_j X_i)^T < 0 \qquad i < j \le r \end{cases} \tag{10}$$

If the above LMIs have a common positive definite solution, stability is guaranteed, but in most practical problem stability by itself is not enough, and the controller needs to satisfy certain design objectives.

III. LTI SYSTEM STATE OBSERVER

In reality, not all state information is measured. State observer is a method to estimate state signals from given plant input-output.

Introduce a linear time variant (LTI) plant with state equations as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{11}$$

State estimation of this LTI plant can be shown as:

$$\dot{\hat{x}} = A\hat{x} + Bu + K_e(y - C\hat{x}) \tag{12}$$

where

$\hat{x}$ = state estimation
K_e = estimation gain
y = plant output
u = plant input

and K_e can be easily calculated with pole placement method on eigenvalues of the matrix $[A \quad BK_x]$.

IV. SPARK IGNITION ENGINE WITH ENGINE TORQUE MANAGEMENT STRATEGY

In general, block diagram of the spark ignition engine with transmission control unit depicted in Figure 1. In this research, the model was a 4-step gear transmission [19].

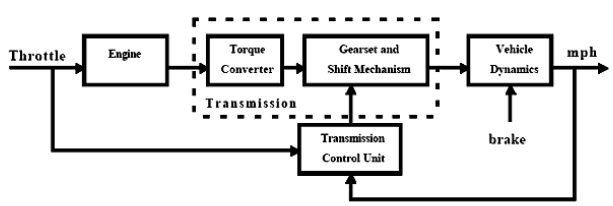


Fig.1. Block diagram of spark ignition engine with transmission control unit [19]

Engine receives input of the throttle opening provided by the driver. The resulting spin machine connected to the impeller of torque converter that is coupled also with the transmission control unit, as:

$$I_{ei}\dot{N}_e = T_e - T_i \tag{13}$$

where

N_e = engine speed
I_{ei} = engine + impeller moment of inertia
T_e = $f_1(throttle, N_e)$ = engine torque
T_i = impeller torque

Input-output characteristics of the torque converter can be expressed with the functions of engine speed and turbine speed, as:

$$T_i = (N_e / K)^2$$
$$T_t = R_{TQ} T_i \tag{14}$$

where
K = $f_2(N_{in} / N_e)$
= capacity of K-factor
N_{in} = turbine (torque converter output) speed
= transmission input speed
T_t = turbine torque
R_{TQ} = torque ratio
= $f_3(N_{in} / N_e)$

Transmission model is expressed as static gear ratios, assumed to have only a small time shift, so that it can be ignored (in fact a matter of this time shift will cause problems robustness), as:

$$R_{TR} = f_4(gear)$$
$$T_{out} = R_{TR} T_{in} \tag{15}$$
$$N_{in} = R_{TR} N_{out}$$

where
T_{in}, T_{out} = transmission input and output torque
N_{in}, N_{out} = transmission input and output speed
R_{TR} = transmission ratio

Vehicle dynamics in this model is influenced by the final drive, inertia, and dynamically varying load.

$$I_v \dot{N}_w = R_{fd}(T_{out} - T_{load}) \tag{16}$$

where
I_v = vehicle inertia
N_w = wheel speed
R_{fd} = final drive ratio
T_{load} = load torque
= $f_5(N_w)$

Load torque includes road load and brake torque. Road load is the summation of frictional and aerodynamic losses.

$$T_{load} = \mathrm{sgn}(mph)(R_{load0} + R_{load2}\, mph^2 + T_{brake}) \tag{17}$$

where
T_{load} = load torque
R_{load0}, R_{load2} = friction and aerodynamic drag coefficients
T_{brake} = brake torque
mph = vehicle linear velocity

Figure 2 provides an illustration of the shift gear ratio schedule. Transmission gear ratio is given in Table I.

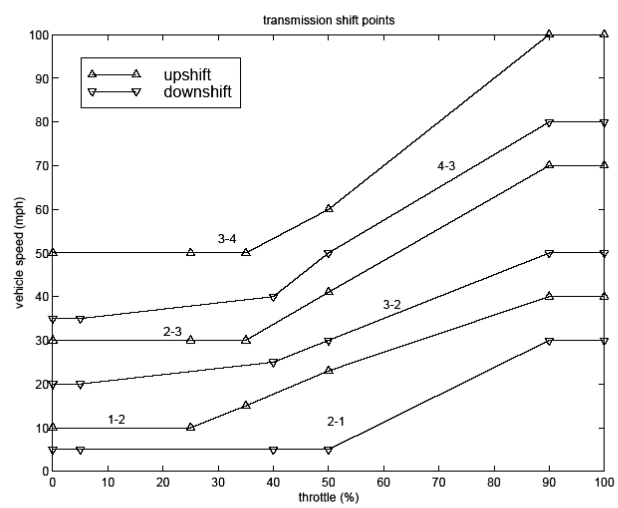


Fig.2. Gear Shift Schedule [19]

TABLE I: GEAR RATIOS [19]

gear	R_{TR}
1	2.393
2	1.450
3	1.000
4	0.677

Engine Torque Management Strategy

Basically, the engine torque management strategy use throttle opening control function, air to fuel ratio (AFR), and ignition timing simultaneously to produce desired engine torque. In practical reality, desired engine torque does not exist, because the input given by the driver on the system is the position of the accelerator pedal (pedal position). For that reason, the engine torque control strategy known as the mapping between the position of throttle opening (pedal position) and engine speed with engine torque command [20]. Figure 3 shows the mapping for the sporty vehicle feel and Figure 4 shows the mapping for economical vehicle feel.

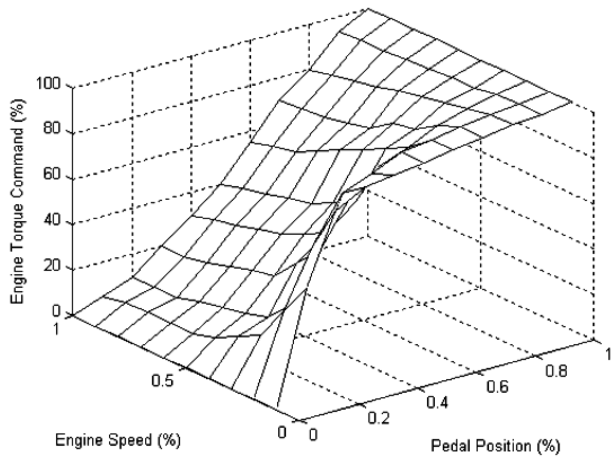


Fig.3. Mapping Pedal Position and Engine Speed with Engine Torque Command for Sporty Vehicle Feel [20]

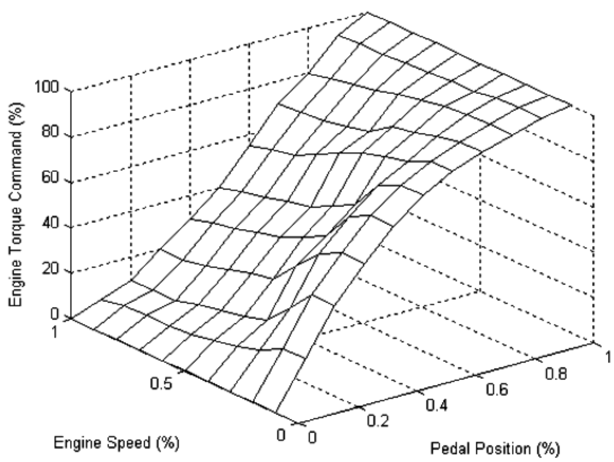


Fig.4. Mapping Pedal Position and Engine Speed with Engine Torque Command for Economical Vehicle Feel [20]

In this research, engine torque control regulation conducted only by controlling throttle plate angle with secondary throttle [21]. AFR and ignition time is left on the standard setting that ideally yield maximum engine torque, i.e. at 14.7 AFR and the spark advance to 15 degree MBT.

V. FUZZY STATE OBSERVER

A non-linear system is expressed as partial set of linear systems with appropriate operating conditions and each linear model is represented by state space equation as follows:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) \\ y_i(t) &= C_i x_i(t) \end{aligned} \qquad (18)$$

where $i = 1, 2, \ldots, n$ (n = sum of possible operating points). If the plant has un-measurable states, then its values can be estimated based on the output and input as (12).

For the case of SI engines in this paper, the initial assumption is by reducing the partial linear plant for each operating point to be second order system (the number of state is 2) with 2 inputs and 1 output. So that the state estimation equation (12) can be derived into:

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_1 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} K_{e1} \\ K_{e2} \end{bmatrix} \left[y - \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} \right] \qquad (19)$$

Fuzzy state observer model is built on the state estimation equation of the plant, where each plant parameters identification and gain estimation calculation results are distributed in each fuzzy rule appropriate with each linear model. The main idea that makes a different result is that fuzzy inference system is operated as soft-switching by adding some operating conditions as trigger inputs to select suitable rules involved to state estimation calculation. Then the input-output observer is designed as follows:

Observer input:

- The operating conditions that trigger the different dynamic characters of the plant (in the case of SI engines in this paper is *throttle degree* and *gear position*).
- Control input (u_1 and u_2).
- Measured output, y.

Observer output:

- $\dot{\hat{x}}_1$ dan $\dot{\hat{x}}_2$ (*derivative states estimation*)

Then output equations expressed by:

$$\dot{\hat{x}}_1 = \begin{bmatrix} 0 & 0 & q_1 & q_2 & B_{11} & B_{12} & K_{e1} & 0 \end{bmatrix} \begin{bmatrix} Cond_1 \\ Cond_2 \\ \hat{x}_1 \\ \hat{x}_2 \\ u_1 \\ u_2 \\ y \\ c \end{bmatrix} \qquad (20)$$

where

$$q_1 = (A)_{11} - K_{e1}C_1$$
$$q_2 = (A)_{12} - K_{e1}C_2$$

and

$$\dot{x}_2 = \begin{bmatrix} 0 & 0 & p_1 & p_2 & B_{21} & B_{22} & K_{e2} & 0 \end{bmatrix} \begin{bmatrix} Cond_1 \\ Cond_2 \\ \hat{x}_1 \\ \hat{x}_2 \\ u_1 \\ u_2 \\ y \\ c \end{bmatrix} \quad (21)$$

where

$$p_1 = (A)_{21} - K_{e2}C_1$$
$$p_2 = (A)_{22} - K_{e2}C_2$$

Figure 5 represent this concept design using Matlab Simulink Toolbox and Matlab Fuzzy Toolbox.

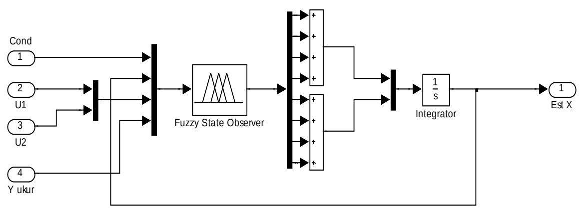
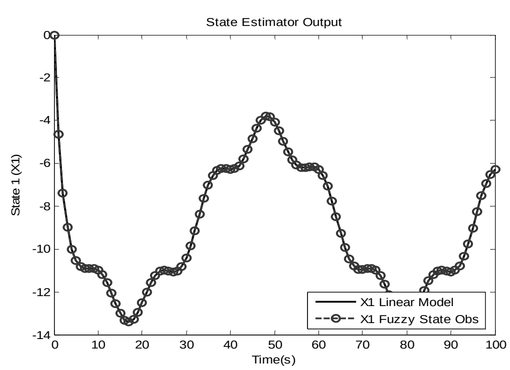


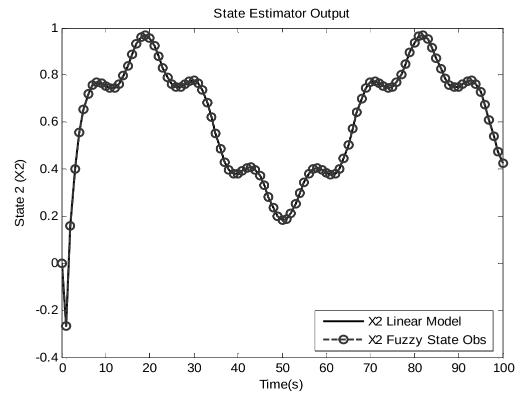
Fig.5. Fuzzy State Observer Structure

VI. SIMULATION AND ANALYSIS

Modeling and simulation performed by Matlab Simulink Toolbox and Matlab Fuzzy Toolbox. Figure 6 – Figure 9 described simulation result under many operating point without any redesign of fuzzy state observer. It can be shown that fuzzy state observer designed by this proposed method work very well for plant with wide uncertainties and high non linear characteristics.

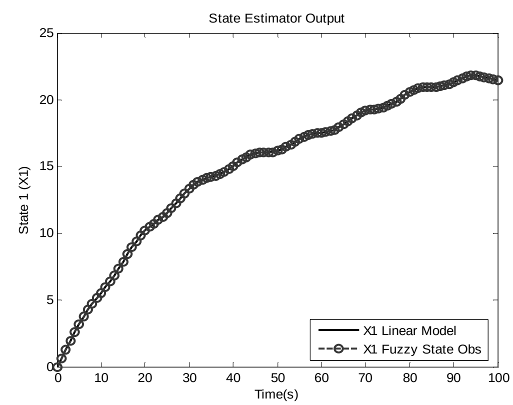


(a)

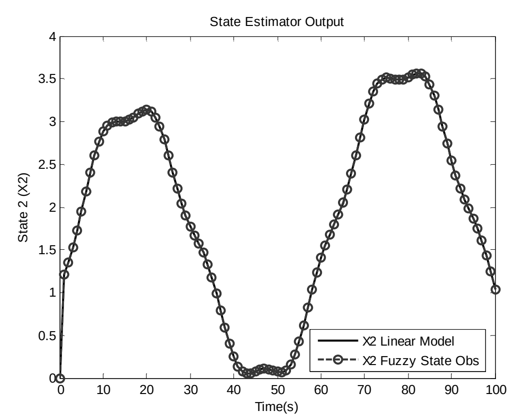


(b)

Fig.6. State estimation with engine operating condition: throttle opening about 10 degrees and first gear application, (a) State 1, and (b) State 2.

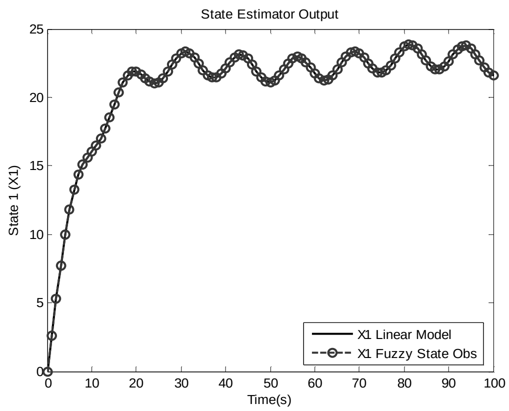


(a)

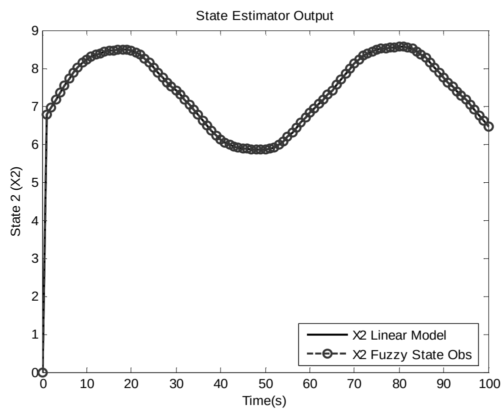


(b)

Fig.7. State estimation with engine operating condition: throttle opening about 10 degrees and second gear application, (a) State 1, and (b) State 2.

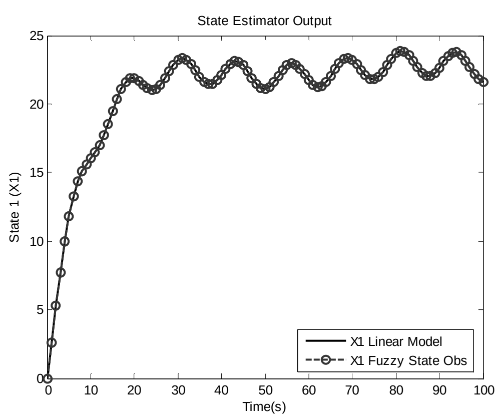


(a)

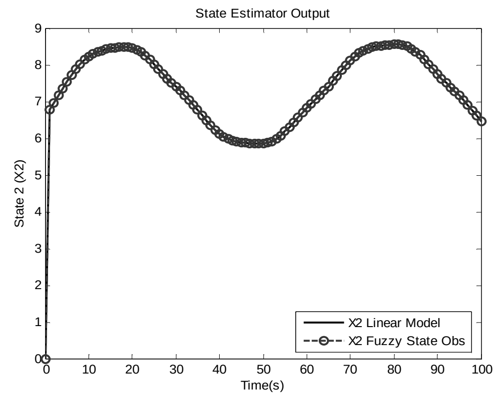


(b)

Fig.8. State estimation with engine operating condition: throttle opening about 10 degrees and third gear application, (a) State 1, and (b) State 2.



(a)



(b)

Fig.9. State estimation with engine operating condition: throttle opening about 50 degrees and third gear application, (a) State 1, and (b) State 2.

VII. CONCLUSION

The papers discuss the problem of the state observer design based on fuzzy system control of an engine torque control system of spark ignition engine. The framework is based on Takagi-Sugeno model, LTI system state observer design, and using fuzzy inference system as soft-switching. The simulations results have illustrated the expected performance and indicate that by using this proposed fuzzy state observer method, states of uncertain non linear system can be estimated easily with only by one integrated fuzzy state observer.

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