# T-S Fuzzy Model Design for Engine Torque Control System of Spark Ignition Engine

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*Abstract*— **On many occasions, fuzzy rule-based systems have been demonstrated to be powerful tools in modeling, decision making and automatic control. The T-S model approach consists to construct nonlinear or complex dynamic systems that cannot be exactly modeled by mathematical model, by means of interpolating the behavior of several LTI (Linear Time Invariant) sub models. In this paper, a new modification fuzzy model based on T-S model will be designed to represent engine torque control system of spark ignition engine, since spark ignition engine is a high non linear system with wide uncertainties that very difficult to be modeled by mathematical model.**

*Keywords* –T-S model, fuzzy model, engine torque, spark ignition engine.

## I. INTRODUCTION

On many occasions, fuzzy rule-based systems have been demonstrated to be powerful tools in modeling, decision making and automatic control. In essence, such a system consists of two main components: a fuzzy rule and an inference mechanism. The choice of an appropriate fuzzy relation modelling the fuzzy rule base and of a compatible inference mechanism is crucial for the proper functioning of the whole system.

The design of state feedback control for nonlinear systems has been actively considered during the last decades in many works using the Takagi-Sugeno (T-S) models [1] – [4]. The T-S model approach consists to construct nonlinear or complex dynamic systems that cannot be exactly modeled by mathematical model, by means of interpolating the behavior of several LTI (Linear Time Invariant) sub models. Each sub model contributes to the global model in a particular subset of the operating space [2], [5], [6]. Note that this modeling approach can be applied for a large class of physical and industrial processes as automotive control [7] and robot manipulators [8].

In this paper, a new modification fuzzy model based on T-S model will be designed to represent engine torque control system of spark ignition engine, since spark ignition engine is a high non linear system with wide uncertainties that very difficult to be modeled by mathematical model. The main idea is by designing fuzzy model using following step designs: (1) linearize non-linear system using partial identification on some operating point that yield plant uncertainties, (2) build T-S fuzzy model based on some linear model from first step, and (3) operate fuzzy inference system as soft-switching by adding some operating conditions as trigger inputs to select suitable rules that describe the right associate model.

## II. SPARK IGNITION ENGINE WITH ENGINE TORQUE MANAGEMENT STRATEGY

In general, block diagram of the spark ignition engine with transmission control unit depicted in Figure 1. In this research, the model was a 4-step gear transmission [9].

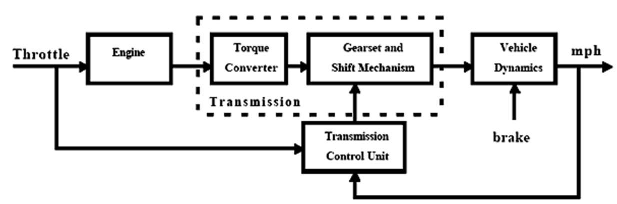


Fig.1. Block diagram of spark ignition engine with transmission control unit [9]

Engine receives input of the throttle opening provided by the driver. The resulting spin machine connected to the impeller of torque converter that is coupled also with the transmission control unit, as:

$$I_{ei}\dot{N}_e = T_e - T_i \tag{1}$$

where

$N_e$ = engine speed
$I_{ei}$ = engine + impeller moment of inertia
$T_e$ = $f_1$(*throttle*, $N_e$) = engine torque
$T_i$ = impeller torque

Input-output characteristics of the torque converter can be expressed with the functions of engine speed and turbine speed, as:

$$T_i = (N_e / K)^2$$
$$T_t = R_{TQ} T_i \qquad (2)$$

where

$K$ = $f_2(N_{in} / N_e)$
= capacity of K-factor
$N_{in}$ = turbine (torque converter output) speed
= transmission input speed
$T_t$ = turbine torque
$R_{TQ}$ = torque ratio
= $f_3(N_{in} / N_e)$

Transmission model is expressed as static gear ratios, assumed to have only a small time shift, so that it can be ignored (in fact a matter of this time shift will cause problems robustness), as:

$$R_{TR} = f_4(gear)$$
$$T_{out} = R_{TR} T_{in} \qquad (3)$$
$$N_{in} = R_{TR} N_{out}$$

where

$T_{in}$, $T_{out}$ = transmission input and output torque
$N_{in}$, $N_{out}$ = transmission input and output speed
$R_{TR}$ = transmission ratio

Vehicle dynamics in this model is influenced by the final drive, inertia, and dynamically varying load.

$$I_v \dot{N}_w = R_{fd} (T_{out} - T_{load}) \qquad (4)$$

where

$I_v$ = vehicle inertia
$N_w$ = wheel speed
$R_{fd}$ = final drive ratio
$T_{load}$ = load torque
= $f_5(N_w)$

Load torque includes road load and brake torque. Road load is the summation of frictional and aerodynamic losses.

$$T_{load} = \mathrm{sgn}(mph)(R_{load0} + R_{load2} mph^2 + T_{brake}) \qquad (5)$$

where

$T_{load}$ = load torque
$R_{load0}$, $R_{load2}$ = friction and aerodynamic drag coefficients
$T_{brake}$ = brake torque
*mph* = vehicle linear velocity

Figure 2 provides an illustration of the shift gear ratio schedule. Transmission gear ratio is given in Table I.

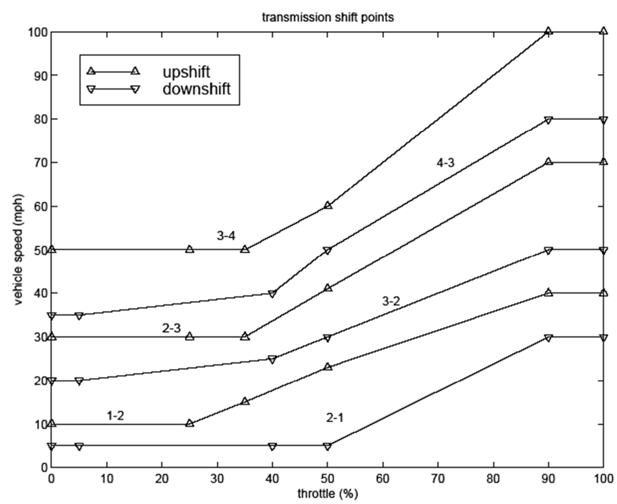


Fig.2. Gear Shift Schedule [9]

TABLE I: GEAR RATIOS [9]

| *gear* | $R_{TR}$ |
|---|---|
| 1 | 2.393 |
| 2 | 1.450 |
| 3 | 1.000 |
| 4 | 0.677 |

*Engine Torque Management Strategy*

Basically, the engine torque management strategy use throttle opening control function, air to fuel ratio (AFR), and ignition timing simultaneously to produce desired engine torque. In practical reality, desired engine torque does not exist, because the input given by the driver on the system is the position of the accelerator pedal (pedal position). For that reason, the engine torque control strategy known as the mapping between the position of throttle opening (pedal position) and engine speed with engine torque command [10].

Figure 3 shows the mapping for the sporty vehicle feel and Figure 4 shows the mapping for economical vehicle feel.

In this research, engine torque control regulation conducted only by controlling throttle plate angle with secondary throttle [11]. AFR and ignition time is left on the standard setting that ideally yield maximum engine torque, i.e. at 14.7 AFR and the spark advance to 15 degree MBT.

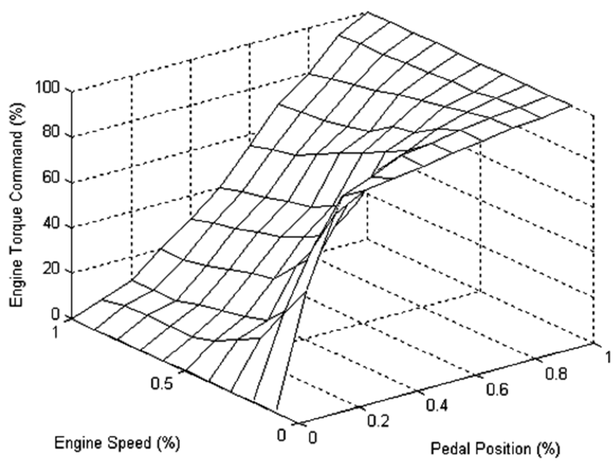


Fig.3. Mapping Pedal Position and Engine Speed with Engine Torque Command for Sporty Vehicle Feel [10]

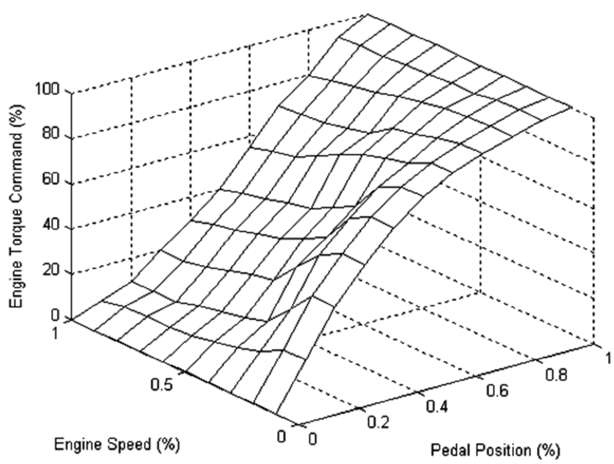


Fig.4. Mapping Pedal Position and Engine Speed with Engine Torque Command for Economical Vehicle Feel [10]

## III. TAKAGI-SUGENO FUZZY MODEL

A dynamic T-S fuzzy model is described by a set of fuzzy “IF … THEN” rules with fuzzy sets in the antecedents and dynamic linear time-invariant systems in the consequents. A generic T-S plant rule can be written as follows [12]:

$i^{th}$ *Plant Rule*:
IF $x_1(t)$ *is* $M_{i1}$ *and ...* $x_n(t)$ *is* $M_{in}$ THEN

where is the state vector, $r$ is the number of rules, $M_{ij}$ are input fuzzy sets, is the input and , are state matrix and input matrix respectively.

Using singleton fuzzifier, max-product inference and center average defuzzifier, the aggregated fuzzy model can be written as:

with the term is defined by:

where is the membership function of the $j^{th}$ fuzzy set in the $i^{th}$ rule. Defining the coefficients as:

then (1) can be modified as:

where and .

Using the same method for generating T-S fuzzy rules for the controller, the controller rules defined in [16] as:

$i^{th}$ *Controller Rule*:
IF $x_1(t)$ *is* $M_{i1}$ *and ...* $x_n(t)$ *is* $M_{in}$ THEN

The overall controller would be:

Replacing (5) in (4), the equation for the closed loop system would be:

The following theorems are used to achieve the stability of the closed loop system:

*Theorem 1* [13]: The closed fuzzy system (11) is globally asymptotically stable if there is exists a common positive definite matrix $P$ which satisfies the following Lyapunov inequalities:

where $G_{ij}$ is defined as:

Pre-multiplying and post-multiplying both sides of inequalities in (13) by $P^{-1}$ and using the following change of variables:

the LMIs obtained in [14] by:

If the above LMIs have a common positive definite solution, stability is guaranteed, but in most practical problem stability by itself is not enough, and the controller needs to satisfy certain design objectives.

## III. FUZZY MODEL DESIGN OF SI ENGINE

A non-linear system is expressed as partial set of linear systems with appropriate operating conditions and each linear model is represented by state space equation as follows:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t)$$
$$y_i(t) = C_i x_i(t) \tag{16}$$

where $i = 1, 2, \ldots, n$ ($n$ = sum of possible operating points).

From the state equation above we will construct two fuzzy models that represent the state equation (*State Fuzzy Model*) and the output equation (*Output Fuzzy Model*) using Matlab Simulink Toolbox and Matlab Fuzzy Toolbox, as shown in Figure 5.

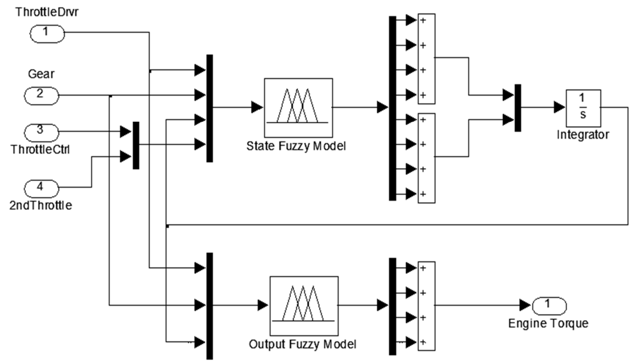


Fig.5. Proposed fuzzy model structure

For the case of SI engines in this paper, the initial assumption is by reducing the partial linear plant for each operating point to be second order system (the number of state is 2) with 2 inputs and 1 output. So that the single state equation in (16) can be derived into:

(17)

*State Fuzzy Model*

Fuzzy state model is constructed based on state equation of the plant, where for each plant parameter identification results (matrices A and B) are distributed in each fuzzy rule appropriate with each linear model. The main idea that makes a different result is that fuzzy inference system is operated as soft-switching by adding some operating conditions as trigger inputs to select suitable rules involved to state estimation calculation. Then the input-output model is designed as follows:

Model input:

- The operating conditions that trigger the different dynamic characters of the plant (in the case of SI engines in this paper is *throttle degree* and *gear position*).
- State of the plant, $x_1$ and $x_2$.
- Control input, $u_1$ and $u_2$.

Model output:

- dan (*derivative states estimation*)

Then output equations expressed by:

and

*Fuzzy Output Model*

As in fuzzy state model, fuzzy output model is constructed based on state equation of the plant too, where for each plant parameter identification results (matrices C) is distributed in each fuzzy rule appropriate with each linear model. The input-output model is designed as follows:

Model input:

- The operating conditions that trigger the different dynamic characters of the plant (in the case of SI engines in this paper is *throttle degree* and *gear position*).
- State of the plant, $x_1$ and $x_2$.

Model output:

- Output, $y$.

Then output equations expressed by:

## IV. SIMULATION AND ANALYSIS

Modeling and simulation performed by Matlab Simulink Toolbox and Matlab Fuzzy Toolbox. Figure 6 – Figure 9 described simulation result under many operating point without any redesign of fuzzy model. It can be shown that fuzzy model designed by this proposed method work very well for plant with wide uncertainties and high non linear characteristics.

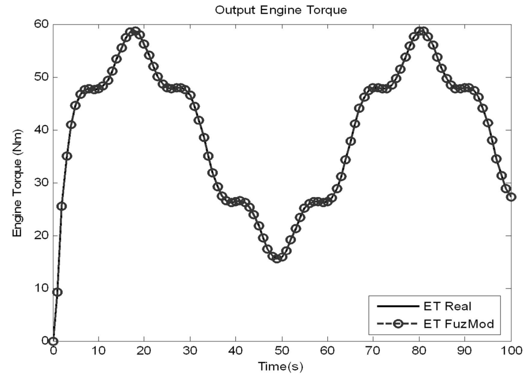


Fig.6. Simulation result with engine operating condition: throttle opening about 10 degrees and first gear application.

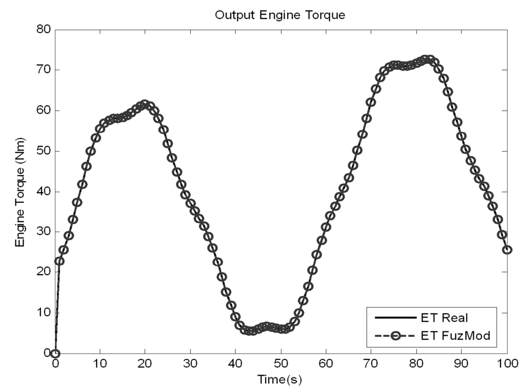


Fig.7. Simulation result with engine operating condition: throttle opening about 10 degrees and second gear application.

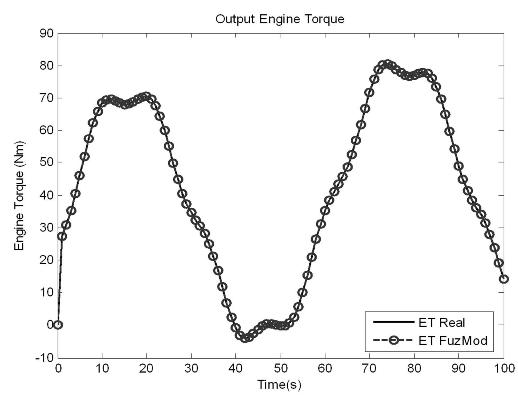


Fig.8. Simulation result with engine operating condition: throttle opening about 10 degrees and third gear application.

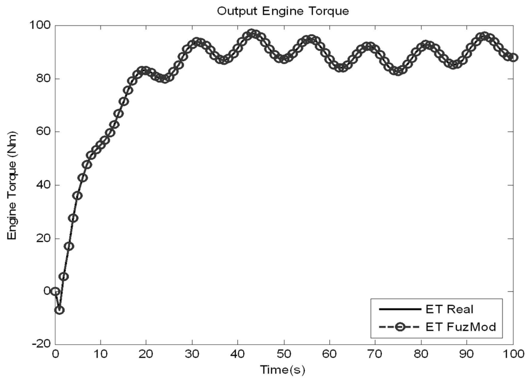


Fig.9. Simulation result with engine operating condition: throttle opening about 50 degrees and third gear application.

## VII. CONCLUSION

The papers discuss the problem of model design based on fuzzy inference system of an engine torque control system of spark ignition engine. The framework is based on Takagi-Sugeno model, linearizing non-linear system using partial identification on some operating point that yield plant uncertainties, and using fuzzy inference system as soft-switching. The simulations results have illustrated the expected performance and indicate that by using this proposed fuzzy model design method, dynamical characteristics of uncertain non linear system can be modeled easily with only by one integrated fuzzy model.

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