

ANALOG FILTER WIZARD DESIGN TOOL FOR ASSISTING STUDENTS IN DESIGNING AN ACTIVE-RC FILTER CIRCUIT

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Abstrak

Artikel ini merupakan sebuah catatan aplikasi, yaitu suatu tulisan teknik yang menjelaskan suatu alat bantu perangkat lunak yang digunakan untuk mendesain berbagai tipe filter aktif resistor dan kapasitor. Alat bantu ini bernama *Analog Filter Wizard*. Merupakan sebuah perangkat lunak yang diciptakan oleh Analog Devices Inc. Artikel ini ditujukan untuk siswa yang mengambil subyek perancangan filter. Artikel dimulai dengan penjelasan tentang konsep dasar pemfilteran yang berisi teori filter pasif dan filter aktif, fungsi transfer filter, dan filter Butterworth. Selanjutnya, dijelaskan secara detail dan tahap demi tahap prosedur pengoperasian *Analog Filter Wizard*. Sebuah contoh desain filter aktif kemudian dibuat menggunakan tool ini, yaitu perancangan sebuah *Low-Pass* Butterworth filter aktif orde 4 dengan memanfaatkan komponen pasif resistor (R), kapasitor (C), dan suatu komponen aktif yaitu *Operational Amplifier* (*Op-Amp*). Filter didesain menggunakan topologi Sallen-Key, yang merupakan sebuah topologi filter elektronik yang biasanya dipakai untuk mengimplementasikan filter orde 2. Dari hasil percobaan menggunakan sebuah *design example* dapat disimpulkan bahwa tool ini telah berhasil melakukan proses pendesainan filter aktif *RC* yang memenuhi requirement yang sudah ditentukan. Dilihat dari sisi pengguna, tool ini sangat membantu siswa dalam mendesain berbagai tipe filter aktif *RC* hanya dengan mengikuti beberapa tahapan yang mudah dan sederhana, sampai bisa menghasilkan rangkaian lengkap filter. Dan juga membantu siswa untuk mendesain, mengoptimalkan, dan mensimulasikan filter aktif *multi-stage* dalam waktu singkat. Siswa kemudian dapat melakukan pengecekan kembali hasil filter yang sudah terdesain secara otomatis tersebut menggunakan rumus dan teori yang ada di teori pemfilteran.

Kata kunci: filter aktif, resistor, kapasitor, operational amplifier, topologi Sallen-Key, konversi data, pemrosesan sinyal

Abstract

This article is an application note, a technical writing for explaining a software tool that can be used to design various types of active filters using resistors and capacitors. The tool is the Analog Filter Wizard. It is a software created by Analog Devices Inc. This article is intended for students who have subject in filter designing. The first part of this article contains basic concepts of filtering which consists of passive filters and active filters theories, filter transfer function, and Butterworth filter. Next, the step-by-step operating procedure for using this tool, explained thoroughly, is presented. This is then followed by a design example of an active low-pass 4th order Butterworth filters utilising passive component resistors (R), capacitors (C), and operational amplifiers. The filter was designed using the Sallen-Key topology, which is an electronic filter topology usually used to implement second-order active filters. By using a design example it can be concluded that this tool has successfully perform the process of designing an active RC filter that satisfies a set of requirements. This tool is useful for student in helping them to design various type of an active RC filters by following an easy and simple step-by-step procedure, resulting in a complete filter circuit and also lets the students to design, optimize, and simulate complete multi-stage active filter solutions within minutes. The students are then able to compare the results created by this tool with the results obtained from filter calculations.

Keywords: active filters, capacitors, resistors, operational amplifier, Sallen-Key topology, data conversion, signal processing

1. Introduction

Filters are used in modern electronics to obtain dc voltages in power supplies, eliminate noise in communication channels, separate radio and television channels from the

multiplexed signal provided by antennas, and boost the bass signal in a car stereo, to name just a few applications [1]. In short, filter is an important building block of communications and instrumentation systems.

Filters circuits may be very simple, consisting of a single capacitor or inductor whose addition to a given network leads to improved performance. They may also be fairly sophisticated, consisting of many resistors, capacitors, inductors, and op amps in order to obtain the precise response curve required for a given application.

In this research, an active-RC low-pass 4th-order Butterworth filter were designed by using a computer program (software) called Analog Filter Wizard. The design process involves entering various filter parameters to fulfill the design specification. The software will automatically create the filter circuit, including its components, according to the parameters we have entered. In other words, the tool will create the complete filter circuit according to the specifications.

Some other research that use some other filter design wizard can be mentioned as follows. A research done by V. Artuhov and O. Brytov titled "Digital filter design by Micro-Cap tools," proposed digital filter design procedure using Micro-Cap capabilities. The procedure is illustrated by an example [2]. Another research by M. A. Uslu and L. Sevgi titled "Matlab-based filter design program: From lumped elements to microstriplines," introduced a systematic filter design procedure, from lumped (LC) elements to transmission lines (TL) and microstriplines (ML). A Matlab based filter tool, MWFilterDesigner which automates all the design and analysis steps is developed. It just needs filter specifications; the rest is performed automatically and corresponding lumped and distributed filter schematics are plotted together with filter characteristics [3]. Another work was done by O. Selçuk and O. Kızılbey, titled "Design and realization of different microstrip low pass filter topologies by aid of AWR microwave office ifilter wizard,". In this work; 7th order Butterworth, Chebyshev, Cauer (elliptic), Bessel, Legendre and Gaussian low pass filters designed, simulated, realized and measured with aid of iFilter tool of AWR Microwave Office. First, all the filters were designed with lumped L-C elements. Secondly, all lumped components were converted to the microstrip lines as series lines and shunt stubs. All microstrip line widths and lengths were optimized with AWR simulation tool [4].

The oldest technology for realizing filters makes use of inductors and capacitors, and the resulting circuits are called passive LC filters. It can also be made using just a single resistor in series with a non-polarized capacitor connected across a sinusoidal input signal, in which the resulting circuits are called passive RC filter [5].

Such passive filters work well at high frequencies; however, in low-frequency applications (dc to 100 kHz) the required inductors are large and physically bulky, and their characteristics are quite nonideal. The amplitude of the output signal is less than that of the input signal and that the load impedance affects the filters characteristics. For

passive filter circuits containing multiple stages, this loss in signal amplitude called "Attenuation" can become quiet severe.

Therefore, there has been considerable interest in finding filter realizations that do not require inductors. Of the various possible types of inductorless filters is the active-RC filters. Amplification through the use of this active filters can be used for restoring or controlling the loss of signal

Active filters are important in modern electronics; every data acquisition system need them for bandwidth limiting signals before ADCs as anti-aliasing filters, or after DACs as anti-imaging filters. Instrumentation also relies on them for accurate signal measurements. Active filters are used for cutoff frequencies that range from sub 1 Hz to 10 MHz,

The filters which are about to design here are linear circuits that can be represented by the general two-port network shown in Fig. 1. The filter transfer function $T(s)$ is the ratio of the output voltage $V_o(s)$ to the input voltage $V_i(s)$. That is [5],

$$T(s) \equiv \frac{V_o(s)}{V_i(s)} \quad (1)$$

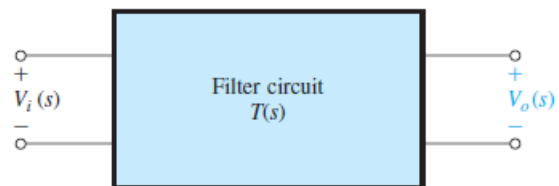


Figure 1. The filters are linear circuits represented by the general two-port network shown. The filter transfer function $T(s) \equiv \frac{V_o(s)}{V_i(s)}$ [5].

The filter transmission is found by evaluating $T(s)$ for physical frequencies, $s = j\omega$, and can be expressed in terms of its magnitude and phase as

$$T(j\omega) = |T(j\omega)|e^{j\varphi(\omega)} \quad (2)$$

The magnitude of transmission is often expressed in decibels in terms of the gain function

$$G(\omega) \equiv 20 \log|T(j\omega)|, \text{ dB} \quad (3)$$

or, alternatively, in terms of the attenuation function

$$A(\omega) \equiv -20 \log|T(j\omega)|, \text{ dB} \quad (4)$$

A filter will shape the frequency spectrum of the input signal, $|V_i(j\omega)|$, according to the magnitude of the transfer

function $|T(j\omega)|$, thus providing an output $|V_o(j\omega)|$ with a spectrum

$$|V_o(j\omega)| = |T(j\omega)||V_i(j\omega)| \quad (5)$$

Filters which are of our interest are those that perform a frequency-selection function: passing signals whose frequency spectrum lies within a specified range, and stopping signals whose frequency spectrum falls outside this range. Such a filter has ideally a frequency band (or bands) over which the magnitude of transmission is unity (the filter passband) and a frequency band (or bands) over which the transmission is zero (the filter stopband). Figure 2 depicts the ideal transmission characteristics of the four major filter types: low-pass (LP) in Fig. 2(a), high-pass (HP) in Fig. 2(b), bandpass (BP) in Fig. 2(c), and bandstop (BS) or band-reject in Fig. 2(d). These idealized characteristics are known as brick-wall responses. In this research, we will discuss and design the first type, the low-pass (LP) filters.

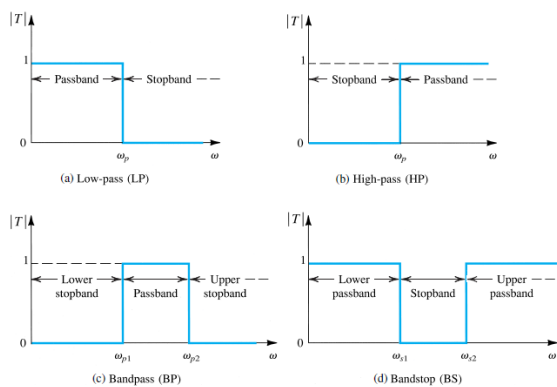


Figure 2. Ideal transmission characteristics of the four major filter types: (a) low-pass (LP), (b) high-pass (HP), (c) bandpass (BP), and (d) bandstop (BS) [5].

When discussing the filter specification, we usually concentrate our attention on the low-pass filter specification since this is the most easiest to understand..

The filter-design process begins with the filter user specifying the transmission characteristics required of the filter. Such a specification cannot be of the form shown in Fig. 2 above because physical circuits cannot realize these idealized characteristics. Instead, Fig. 3 shows realistic specifications for the transmission characteristics of a low-pass filter.

The specifications allow for deviation of the passband transmission from the ideal 0 dB, but place an upper bound, A_{max} (dB), on this deviation. This is because a physical circuit cannot provide constant transmission at all passband frequencies. Depending on the application, A_{max} typically ranges from 0.05 dB to 3 dB.

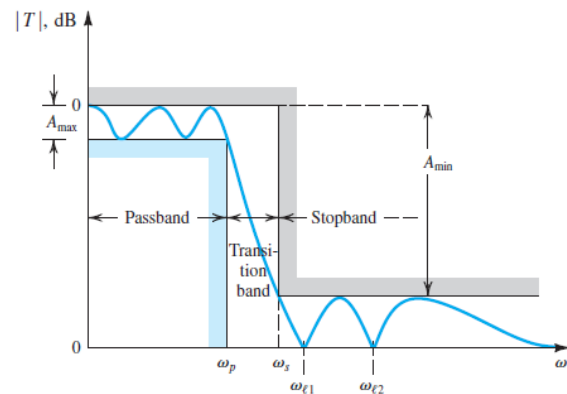


Figure 3. Specification of the transmission characteristics of a low-pass filter. The magnitude response of a filter that just meets specifications is also shown [5].

Also, because a physical circuit cannot provide zero transmission at all stopband frequencies, the specifications allow for some transmission over the stopband. However, the specifications require the stopband signals to be attenuated by at least A_{min} (dB) relative to the passband signals. Depending on the filter application, A_{min} can range from 20 dB to 100 dB.

The specifications of Fig. 3 provide for a band of frequencies over which the attenuation increases from near 0 dB to A_{min} because the transmission of a physical circuit cannot change abruptly at the edge of the passband. This transition band extends from the passband edge ω_p to the stopband edge ω_s . For convenience the passband transmission is specified to be 0 dB. The final filter, however, can be given a passband gain, if desired, without changing its selectivity characteristics.

To summarize, the transmission of a low-pass filter is thus specified by four parameters: the passband edge ω_p , the maximum allowed variation in passband transmission A_{max} , the stopband edge ω_s , and the minimum required stopband attenuation A_{min} .

The more tightly a filter is specified — that is, lower A_{max} , higher A_{min} , and / or a selectivity ratio $\frac{\omega_s}{\omega_p}$ closer to unity — the closer the response of the resulting filter will be to the ideal. However, the resulting filter circuit will be of higher order and thus more complex and expensive.

The next step is to find a transfer function whose magnitude meets the specification. The process of obtaining a transfer function that meets given specifications is known as filter approximation. Filter approximation is usually performed using computer programs or filter-design tables.

The filter transfer function $T(s)$ can be written as the ratio of two polynomials as

$$T(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \dots + a_0}{s^N + b_{N-1} s^{N-1} + \dots + b_0} \quad (6)$$

In Eq. 6, N is the filter order. In order the filter circuit to be stable, the degree of the numerator must be less than or equal to that of the denominator: $M \leq N$. The numerator and denominator coefficients, a_0, a_1, \dots, a_M and b_0, b_1, \dots, b_{N-1} , are real numbers. The polynomials in the numerator and denominator can be factored, and $T(s)$ can be expressed in the form

$$T(s) = \frac{a_M (s - z_1)(s - z_2) \dots (s - z_M)}{(s - p_1)(s - p_2) \dots (s - p_N)} \quad (7)$$

The numerator roots, z_1, z_2, \dots, z_M , are the transfer function zeros, or transmission zeros; and the denominator roots, p_1, p_2, \dots, p_N , are the transfer function poles, or the natural modes. Each transmission zero or pole can be either a real or a complex number. Complex zeros and poles, however, must occur in conjugate pairs.

This particular filter of Fig.3 can be seen to have zero transmission (infinite attenuation) at two stopband frequencies: ω_{l1} and ω_{l2} . The filter then must have transmission zeros at $s = +j\omega_{l1}$ and $s = +j\omega_{l2}$. However, since complex zeros occur in conjugate pairs, there must also be transmission zeros at $s = -j\omega_{l1}$ and $s = -j\omega_{l2}$. Thus the numerator polynomial of this filter will have the factors $(s + j\omega_{l1})(s - j\omega_{l1})(s + j\omega_{l2})(s - j\omega_{l2})$, which can be written as $(s^2 + \omega_{l1}^2)(s^2 + \omega_{l2}^2)$. For $s = j\omega$ (physical frequencies) the numerator becomes $(-\omega^2 + \omega_{l1}^2)(-\omega^2 + \omega_{l2}^2)$, which indeed is zero at $\omega = \omega_{l1}$ and $\omega = \omega_{l2}$. Note that the transmission decreases toward zero as ω approaches ∞ . Thus the filter must have one or more transmission zeros at $s = \infty$.

For a filter circuit to be stable, all its poles must lie in the left half of the s plane, and thus p_1, p_2, \dots, p_N must all have negative real parts. Figure 4 below shows typical pole and zero locations for the low-pass filter whose transmission function is depicted in Fig. 3 [5].

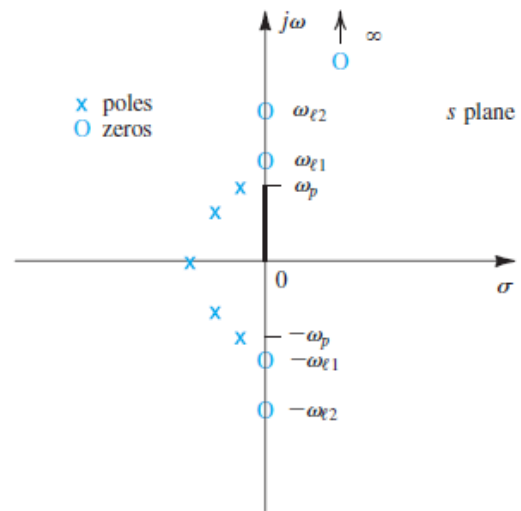


Figure 4. Pole-zero pattern for the low-pass filter whose transmission is sketched in Fig 3 [5].

We have assumed that this filter is of fifth order ($N = 5$). It has two pairs of complex-conjugate poles and one real-axis pole, for a total of five poles. All the poles lie in the vicinity of the passband, which is what gives the filter its high transmission at passband frequencies. The five transmission zeros are at $s = \pm j\omega_{l1}$, $s = \pm j\omega_{l2}$, and $s = \infty$. Thus, the transfer function for this filter is of the form

$$T(s) = \frac{a_4 (s^2 + \omega_{l1}^2)(s^2 + \omega_{l2}^2)}{s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0} \quad (8)$$

Another example, consider the low-pass filter whose transmission function is depicted in Fig. 5. This time, there are no finite values of ω at which the transmission is zero. Thus it is possible that all the transmission zeros of this filter are at $s = \infty$. The filter transfer function takes the form (see Eq. 6)

$$T(s) = \frac{a_0}{s^N + b_{N-1} s^{N-1} + \dots + b_0} \quad (9)$$

Such a filter is known as an all-pole filter. Typical pole-zero locations for a fifth-order all-pole low-pass filter are shown in Fig. 6.

A function that is frequently used in approximating the transmission characteristics of low-pass filters is the Butterworth filter, whose magnitude response is shown in Fig. 7.

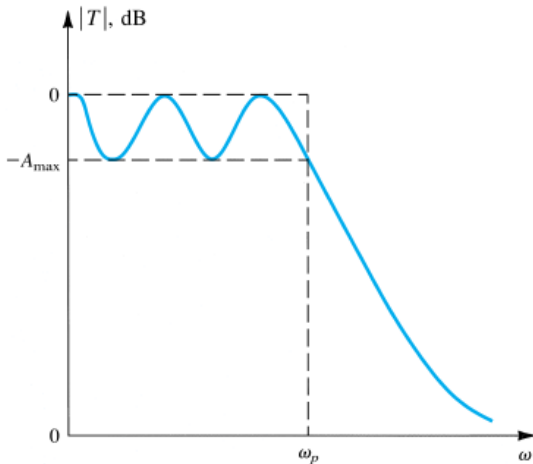


Figure 5. Transmission characteristics of a fifth-order low-pass filter having all transmission zeros at infinity [5]

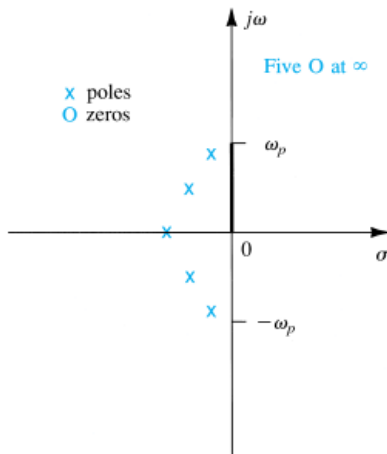


Figure 6. Pole-zero pattern for the filter in Figure 5 [5].

The Butterworth filter shows a monotonically decreasing transmission with all the transmission zeros at $\omega = \infty$, making it an all-pole filter. The magnitude function for an N th-order Butterworth filter with a passband edge ω_p is given by

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 \left(\frac{\omega}{\omega_p}\right)^{2N}}} \quad (10)$$

At $\omega = \omega_p$,

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2}} \quad (11)$$

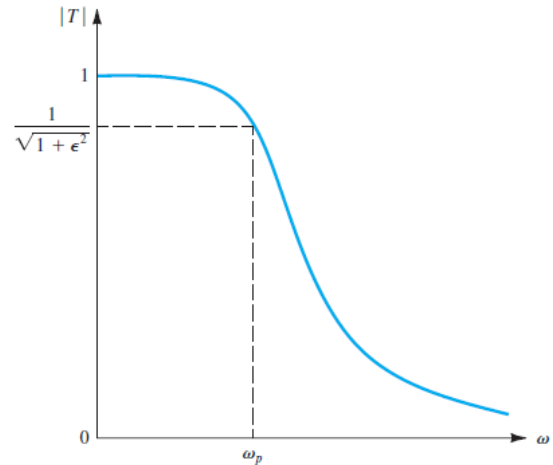


Figure 7. The Magnitude Response of a Butterworth Filter [5].

Thus, the parameter ϵ determines the maximum variation in passband transmission, A_{max} , according to

$$A_{max} = 20 \log \sqrt{1 + \epsilon^2} \quad (12)$$

Conversely, given A_{max} , the value of ϵ can be determined from

$$\epsilon = \sqrt{10^{A_{max}/10} - 1} \quad (13)$$

At the edge of the stopband, $\omega = \omega_s$, by substituting $\omega = \omega_s$ in Eq. (10), the attenuation of the Butterworth filter can be obtained. The result is given by

$$\begin{aligned} A(\omega_s) &= -20 \log \left[\frac{1}{\sqrt{1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}}} \right] \\ &= 10 \log \left[1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N} \right] \end{aligned} \quad (14)$$

To determine the filter order required, which is the lowest integer value of N that yields $A(\omega_s) \geq A_{min}$, Eq. 14 can be used

The graphical construction shown in Fig. 8(a) can then be used to determine the natural modes (the transfer function poles) of an N th-order Butterworth filter. The natural modes lie on a circle of radius $\omega_p(1/\epsilon)^{1/N}$ and are spaced by equal angles of π/N , with the first mode at an angle $\pi/2N$ from the $+j\omega$ axis. Fig. 8(b), (c), and (d) are for the natural modes of Butterworth filters of order $N = 2, 3$, and 4, respectively. Once the N natural modes p_1, p_2, \dots, p_N have been found, the transfer function can be written as

$$T(s) = \frac{K\omega_0^N}{(s - p_1)(s - p_2) \dots (s - p_N)} \quad (15)$$

where K is a constant equal to the required dc gain of the filter.

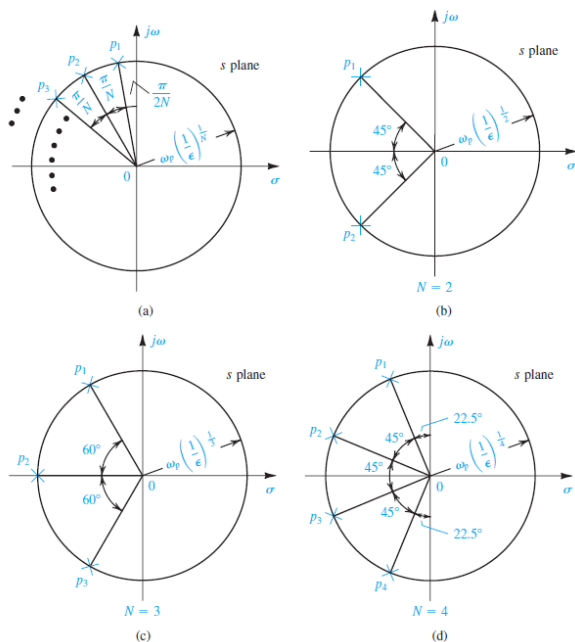


Figure 8. Graphical Construction for Determining the Poles of a Butterworth Filter of Order N . All the Poles Lie in the Left Half of the s Plane on a Circle of Radius $\omega_0 = \omega_p(1/\epsilon)^{1/N}$: (a) the General Case; (b) $N = 2$; (c) $N = 3$; (d) $N = 4$ [5].

To summarize, to find a Butterworth transfer function that meets transmission specifications of the form in Fig. 3, the following procedures were done: determine ϵ from Eq. (13), use Eq. (14) to determine the required filter order as the lowest integer value of N that results in $A(\omega_s) \geq A_{min}$, use Fig. 8(a) to determine the N natural modes (the transfer function poles), and use Eq. (15) to determine $T(s)$.

2. Methods

The Analog Filter Wizard is one of hands-on design tool for real operational amplifiers [6]. This tool assists the engineer and students in designing active RC filters. The design process consists of two steps. In Step 1, the filter type must be selected. Then the response of the filter is determined, meaning the attenuation and / or phase response of the filter is defined. In Step 2, the topology of the filter—how it is built—is defined [7].

2.1. Design Steps

Here, the filter response best suited for the design is determined by optimizing for pulse response, settling time, lowest cost, pass-band ripple, and stop-band attenuation with low-pass, high-pass, band-pass, and band-stop filter

types. In other word, we will determine the filter specifications.

Next, voltage supplies and components are chosen which includes values of resistor and capacitor for RC time constant and values for gain resistors.

2.1.1. Filter Type

First, a filter type will be selected. Since we are about to design a low pass filter, the *Low Pass* filter icon was clicked as shown in Fig. 9.

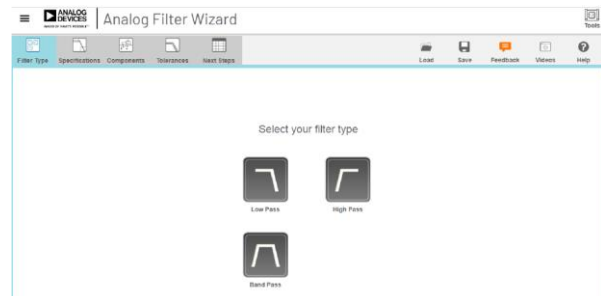


Figure 9. Filter Type Selection User Interface [8].

2.1.2. Filter Specification

In this filter specifications, the transmission characteristics required of the filter will be specified.

Passband is the range of frequencies that must be let through. This region is defined by entering the circuit gain at the passband frequencies, the frequency cutoff, and the attenuation at this frequency cutoff. Attenuation is relatif to the gain entered. Stopband is the range of frequencies to be rejected. The attenuation is relative to the circuit gain [8].

Next is the filter response option. Choosing a response is a trade-off between the number of components vs other desirable properties. This option lets us to select the type of filter response. This tool supports four reponse types which are Chebyshev, Butterworth, Butterworth Bessel, and Bessel. The filter response defines the transition from the pass band to the stop band [6].

2.1.3. Components

Next stage is determining the voltage supplies. Here, the voltage supplies for the op amp were chosen. These voltages will be used by the program to select an appropriate op amp when “pick for me” is chosen or narrow down the op amp choices if “I want to choose” is chosen. If a single supply voltage was chosen ($-V_s = 0$), the tools insert a mid-supply reference voltage generator into the circuit for us.

As the components were changed, there will be changes in the circuit view. The effect of component choices to the filter performance can then be seen. The software supports two different ways of picking components. Method 1: have the software pick components for us, and method 2: choose the components ourself.

This filter wizard supports the following circuit implementations: Sallen Key, Multiple Feedback, and First Order.

2.2. Design Example

Suppose a low-pass filter with the following specifications are going to be designed: a cut-off frequency of 1 MHz, stop band of 3 MHz, and a gain of 20 dB. The filter will have to be optimized for low noise and a $\pm 5 V$ supply. Use the default component tolerance and the amplifiers recommended by the tool.

2.2.1. Determining Filter Type

Since it is a low pass filter design, the *Low Pass* filter icon has to be clicked as shown in Fig. 9 above.

2.2.2. Determining Filter Specification

The filter which are about to design will have the maximum allowed variation in passband transmission, A_{max} , of $-3 dB$, a minimum required stopband attenuation, A_{min} , of $-35 dB$, and a gain of $20 dB$.

At $20 dB$ gain, the ratio for voltage (or current) is equal to $10 V/V$. A number 20 was entered at the the circuit gain. We enter 1 Mhz in the range of frequencies we want to let through and enter 3 Mhz in the range of frequencies we want to reject. In passband, we enter $-3 dB$ for attenuation and enter an attenuation $-35 dB$ in stopband. When the filter wizard tool was run, it gave the results of Fig. 10 for the magnitude (dB) versus frequency (Hz) view, and gave the results of Fig. 11 for the magnitude (V/V) versus frequency (Hz) view.

Note that the tool use the Butterworth filter response. It is a 2-stages, 4th order Butterworth.

2.2.3. Determining Components

At the voltage supplies, we enter $+5V$ and $-5V$ for $+V_s$ and $-V_s$ respectively. At the components, "pick for me" was chosen and we choose "low noise" for the optimization.



Figure 10. The Magnitude (dB) versus Frequency (Hz) View when the Filter Wizard Tool was Run

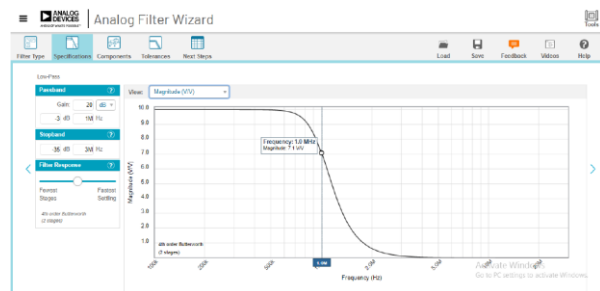


Figure 11. The Magnitude (V/V) versus Frequency (Hz) View when the Filter Wizard Tool was Run.

With the above components setting, when the tool was run the circuit view and the stages view were shown as in Fig. 12 and Fig. 13 respectively.

There are feedback resistor and input resistor at the non-inverting amplifier for both stages to accommodate the gain of $20 dB$. It is shown that the tool uses lower value resistors and higher value capacitors to keep noise contribution from resistors low.

This tool selects the ADA4807-2, an op- amp with a low spectral noise density (it has a voltage noise of $3.1 nV/\sqrt{Hz}$ at $100 kHz$ and a current noise of $0.7 pA/\sqrt{Hz}$ at $100 kHz$).

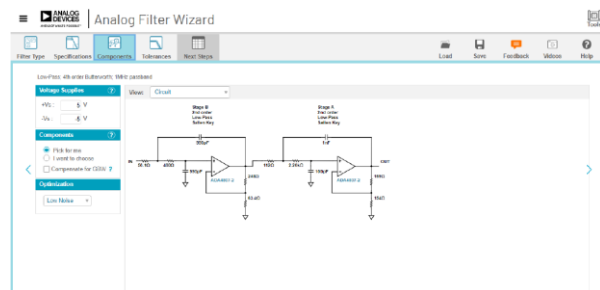


Figure 12. The Circuit View when the Filter Wizard Tool was Run, Showing Components Values.

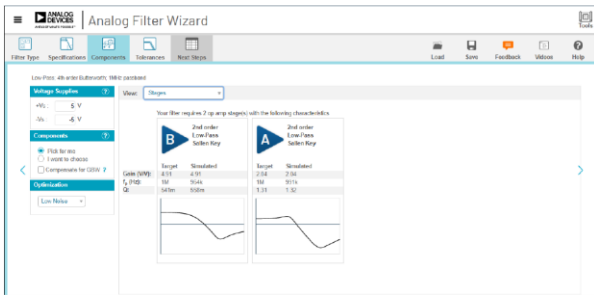


Figure 13. The Stages View when the Filter Wizard Tool was Run, Showing Gain, $f_{passband}$, and Q .

3. Results and Discussions

The design example consists of two stages, stage A and stage B, each of a type *2nd order Low-Pass Sallen Key*, which make it to become a two-stage, *4th order Butterworth filter*. We will first find out why this tool decided to realize our low pass filter with a *4th order filter*.

First, determine ϵ by using Eq. 13 as follows:

$$\epsilon = \sqrt{10^{A_{max}/10} - 1} = \sqrt{10^{0.3} - 1} = 0.99762834511$$

Next, use Eq. 14 to determine the required filter order as the lowest integer value of N that results in $A(\omega_s) \geq A_{min}$. If $N = 3$ was used, and substitute $\epsilon = 0.99762834511$, $\omega_s = 3 \text{ MHz}$, and $\omega_p = 1 \text{ MHz}$ we will have

$$A(\omega_s) = 10 \log \left[1 + 0.99762834511^2 \left(\frac{3}{1} \right)^{2 \times 3} \right] = 28.6126325216 \text{ dB}$$

If $N = 4$ was used instead, we will have

$$A(\omega_s) = 10 \log \left[1 + 0.99762834511^2 \left(\frac{3}{1} \right)^{2 \times 4} \right] = 38.1497410117 \text{ dB}$$

Thus, $N = 4$ is the lowest integer value of N that results in $A(\omega_s) \geq A_{min}$. This is the reason why the filter wizard decided to realize with a *4th order filter*.

A graphical construction will be used in determining the pole positions of the *4th-order Butterworth filter*, as shown in Fig. 14.

Note that the poles are spaced by equal angles of $\pi/N = \pi/4 = 45^\circ$, with the first pole at an angle $\pi/2N = \pi/8 = 22.5^\circ$ from the $+j\omega$ axis [5].

The first pole p_1 position is given by

$$p_1 = \omega_0(-\cos 22.5^\circ + j \sin 22.5^\circ) = \omega_0(-0.9239 + j 0.3827)$$

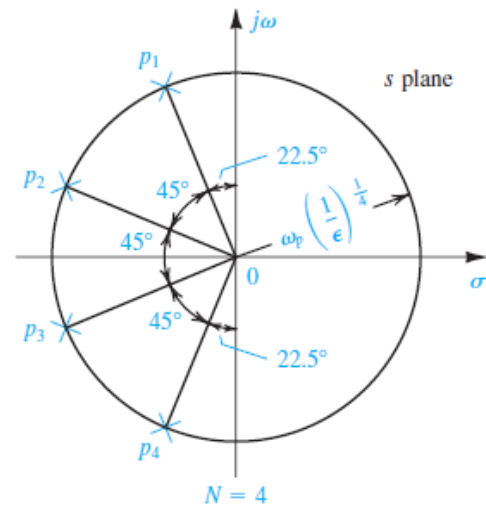


Figure 14. Graphical construction for determining the poles of a *4th Butterworth filter* [5].

Combining p_1 with its complex conjugate p_4 , $\omega_0(-0.9239 - j 0.3827)$, yields the factor

$$= \omega_0(-0.9239 + j 0.3827) \omega_0(-0.9239 - j 0.3827) = (s^2 + s1.8478\omega_0 + \omega_0^2)$$

in the denominator of the transfer function.

The same can be done for the second pole p_2 position:

$$p_2 = \omega_0(-\cos 67.5^\circ + j \sin 67.5^\circ) = \omega_0(-0.3827 + j 0.9239)$$

Combining p_2 with its complex conjugate p_3 , $\omega_0(-0.3827 - j 0.9239)$, yields the factor $= \omega_0(-0.3827 + j 0.9239) \omega_0(-0.3827 - j 0.9239) = (s^2 + s0.7654\omega_0 + \omega_0^2)$

in the denominator of the transfer function.

The complete transfer function is obtained using Eq. 15:

$$T(s) = \frac{10\omega_0^4}{(s^2 + s0.7654\omega_0 + \omega_0^2)(s^2 + s1.8478\omega_0 + \omega_0^2)}$$

Now, note at Fig. 15 that shows a stage of an active low-pass filter circuit [9]. Each stage in Fig. 12 is an active low-pass filter type circuit, like the one shown in Fig. 15. It is made up from a passive RC low pass filter and a non-inverting amplifier. The circuit will provide both RC low pass filter frequency response along with its increased output amplitude by the filter gain A_F . For a non-inverting op-amp, the filter gain will be:

$$\text{Filter Gain } A_F = \left(1 + \frac{R_2}{R_1} \right) \quad (16)$$

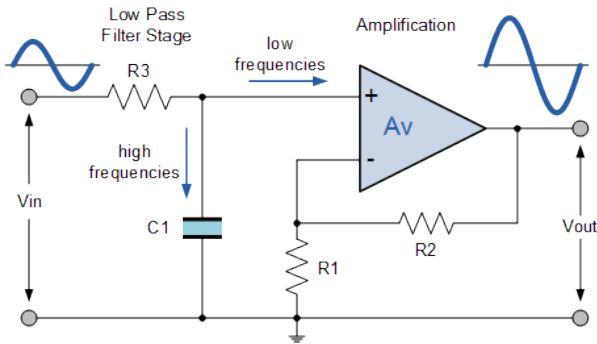


Figure 15. A Low-Pass Filter Consisting of a Resistor R_3 and a Capacitor C_1 has been Added to the Input of a Simple Non-Inverting Op Amp Circuit. The Result is an Active Low-Pass Filter Circuit [9].

The filter overall gain can be obtained as the result of the multiplication of the Stage B gain and the Stage A gain as follows:

$$\text{Overall Gain} = \left(1 + \frac{R_2}{R_1}\right)_{St.B} \times \left(1 + \frac{R_2}{R_1}\right)_{St.A} \quad (17)$$

By substituting the feedback resistors and input resistors values in Fig. 12 into Eq. 17, will give us

$$\begin{aligned} \text{Overall Gain} &= \left(1 + \frac{248 \Omega}{63.4 \Omega}\right)_{St.B} \times \left(1 + \frac{159 \Omega}{154 \Omega}\right)_{St.A} \\ &= (4.9117)_{St.B} \times (2.0325)_{St.A} = 9.9830 \approx 10 \text{ V/V} \end{aligned}$$

We can change those overall gain into dB, as follows:

$$\begin{aligned} \text{Ovrl Gain (dB)} &= 20 \log_{10} \text{Ovrl Gain (V/V)} \quad (18) \\ &= 20 \log 10 = 20 \text{ dB} \end{aligned}$$

Next, it is known that the voltage gain of an active low-pass filter is a function of frequency. That is:

$$\text{Voltage gain } (A_v) = \frac{V_{out}}{V_{in}} = \frac{\text{Overall Gain}}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}} \quad (19)$$

where:

Overall Gain = overall gain of the filter
 f = the frequency of the input signal in Hertz (Hz)
 f_c = the cut-off frequency in Hertz (Hz)

Thus, the operation of the low pass active filter can be verified from the frequency gain equation above as follows [10]: at very low frequencies, $f < f_c$ Voltage gain $(A_v) = \frac{V_{out}}{V_{in}} = \text{Overall Gain}$, at the cut-off frequency, $f = f_c$ Voltage gain $(A_v) = \frac{V_{out}}{V_{in}} =$

$\frac{\text{Overall Gain}}{\sqrt{2}}$, and at very high frequencies, $f > f_c$ Voltage gain $(A_v) = \frac{V_{out}}{V_{in}} < \text{Overall Gain}$

The result of the voltage gain equation can be described by Fig. 11. At very low frequencies, $f < f_c$, voltage gain $A_v = \frac{V_{out}}{V_{in}} = 10 \text{ V/V}$. At the cut-off frequency, $f = f_c = 1 \text{ MHz}$, voltage gain $A_v = \frac{V_{out}}{V_{in}} = \frac{\text{Overall Gain}}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.071 \text{ V/V}$ (shown as 7.1 V/V). At very high frequencies, $f > f_c$, voltage gain $A_v = \frac{V_{out}}{V_{in}} < \text{Overall Gain}$. Table 1 shows the voltage gain obtained as the frequencies changed from 200 kHz up to 2000 kHz.

Table 1. The Voltage Gain as a Function of Frequency (taken from Fig.11 when the Tool was Run)

f_c (kHz)	$\frac{V_{out}}{V_{in}}$ (V/V)
200.00	10
300.00	10
500.00	10
600.00	9.9
790.00	9.3
890.00	8.5
1000.00	7.1
1200.00	4.7
1400.00	2.7
1600.00	1.6
1800.00	1.0
2000.00	0.6

By entering the value $\frac{V_{out}}{V_{in}} = 7.071$ at 1 MHz, we obtained voltage gain $A_v \text{ (dB)} = 20 \log_{10}(7.071) = 16.9896 \text{ dB} \approx 17 \text{ dB}$. This voltage gain of 17 dB occurs at the pass band cutoff frequency of 1 MHz. Thus, when we enter 20 dB for the gain and attenuation of -3 dB, then the tool attempts to set the passband cutoff frequency (1 MHz) at 17 dB (i.e. $20 \text{ dB} - 3 \text{ dB} = 17 \text{ dB}$).

Each stage in Fig. 12 is a 2nd order Low-Pass Sallen Key. The circuit in Fig. 16 is known as the Sallen-Key amplifier configured as a low-pass filter [1].

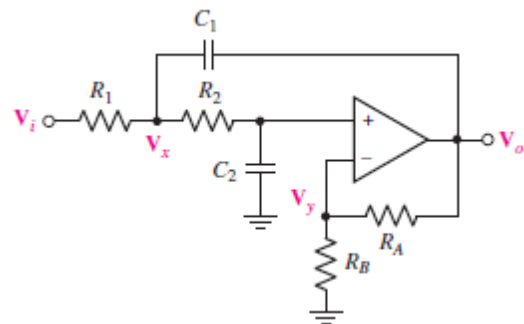


Figure 16. General Low-Pass Sallen Key Filter Circuit [1].

In connection with the circuit shown in Fig 16, the pass band cutoff frequency of 1 MHz can actually be calculated by using the circuit's components using the following equation [11]:

$$FSF \times f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \quad (20)$$

where $R_1, R_2, C_1,$ and C_2 are the values of the resistors and capacitors for each stage.

The value of FSF (Frequency Scaling Factor) can be obtained from Table 2, which is of magnitude 1 [11]. Therefore, for Stage B we have

$$f_{c(Stage\ B)} = \frac{1}{2\pi\sqrt{51.6\ \Omega \times 460\ \Omega \times 990\ pF \times 990\ pF}} \\ = 1000747.67679\ Hz \cong 1\ MHz$$

And for Stage A we have

$$f_{c(Stage\ A)} = \frac{1}{2\pi\sqrt{112\ \Omega \times 2.26\ k\Omega \times 1\ nF \times 100\ pF}} \\ = 1000361.34256\ Hz \cong 1\ MHz$$

Next, Fig. 13 shows the value of quality factor $Q = \frac{1}{2\zeta}$ for both stages. Quality factor Q is an equivalent design parameter to the filter order, N . The higher the Q value, the more a filter inclines to instability [11]. Stage A has a quality factor $Q = 1.3$, while Stage B has a quality factor $Q = 548\ m$.

For low-pass and high-pass filters, Q represents the pole quality and is defined as [12]:

$$Q = \frac{\sqrt{b_i}}{a_i} \quad (21)$$

Thus for Stage A, we have $Q = \frac{\sqrt{1}}{0.7654} = 1.3065 \cong 1.31$, and for Stage B, we have $Q = \frac{\sqrt{1}}{1.8478} = 0.54118 \cong 541\ m$ as shown in Fig. 13.

Table 2. Butterworth Filter Table [10]

FILTER ORDER	Stage 1		Stage 2		Stage 3		Stage 4		Stage 5	
	FSF	Q	FSF	Q	FSF	Q	FSF	Q	FSF	Q
2	1.000	0.7071								
3	1.000	1.0000	1.000							
4	1.000	0.5412	1.000	1.3065						
5	1.000	0.6180	1.000	1.6181	1.000					
6	1.000	0.5177	1.000	0.7071	1.000	1.9320				
7	1.000	0.5549	1.000	0.8019	1.000	2.2472	1.000			
8	1.000	0.5098	1.000	0.6013	1.000	0.8999	1.000	2.5628		
9	1.000	0.5321	1.000	0.6527	1.000	1.0000	1.000	2.8802	1.000	
10	1.000	0.5062	1.000	0.5612	1.000	0.7071	1.000	1.1013	1.000	3.1969

Besides described in terms of their frequency domain properties, filters can also be described in terms of their *time-domain* properties: rise time, overshoot, ringing, and settling time. This is of particular importance where steps

or pulses may be present. Figure 17 shows a typical low-pass filter step response. Here, *rise time* is the time required to go from 10% to 90% of the final value. The *delay time* is the time duration from the input step to the output reaching 50% of its final value [13]. The overshoot Q of a low-pass filter is the maximum excursion of the output from a unit input step function (see Fig. 18). The settling time τ_s is the time taken for the step response oscillation to settle to a value less than 5% of the maximum overshoot value, Q [14].

A system with high quality factor ($Q > 0.5$) is considered to be underdamped. Underdamped systems combine oscillation at a specific frequency with a decay of the amplitude of the signal.

The Step Response view of Fig. 19 shows the time-domain characteristics of the design filter. The first 2nd order Low-Pass filter (Stage A) has an underdamped $Q = 1.31$. The second 2nd order Low-Pass filter (Stage B) has an underdamped $Q = 0.541$.

Next, the ideal transfer function for the second order Sallen-Key low-pass filter of Fig. 16 is given by [1]:

$$T(s) = \frac{G}{s^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1-G}{R_2 C_2}\right)s + \frac{1}{R_1 R_2 C_1 C_2}} \quad (22)$$

where $G \equiv \frac{R_A + R_B}{R_B}$

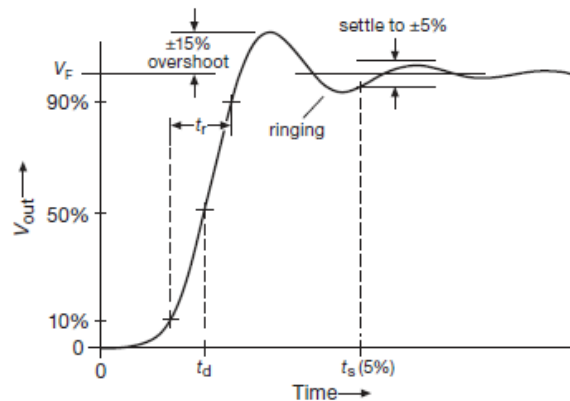


Figure 17. Time-Domain Filter Characteristics [13].

From Table 3 (and also from the Butterworth transfer function calculation above), we also expect that the transfer function for the 4th order Butterworth filter of Fig. 12 is given by:

$$T(s) = \frac{K\omega_0^4}{(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)} \quad (23) \\ \frac{K\omega_0^4}{s^4 + 2.6131s^3 + 3.4142s^2 + 2.6131s + 1}$$

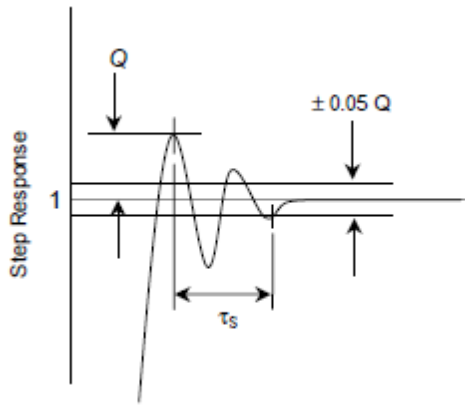


Figure 18. Definition of overshoot Q and settling time τ_s for a low pass filter [14].

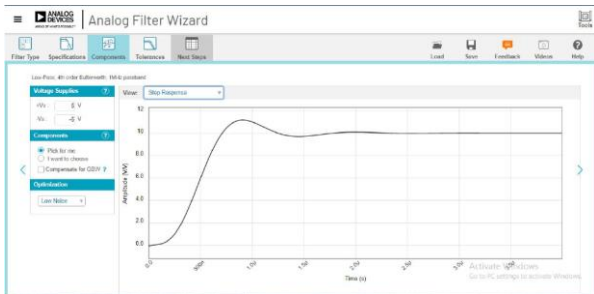


Figure 19. Time – Domain Properties when the Filter Wizard Tool was Run, Showing Step Response View.

where for the Stage A, $a_1 = 0.7654$ and $b_1 = 1.0000$, and for Stage B, $a_2 = 1.8478$ and $b_2 = 1.0000$.

Table 3. Butterworth Design Table [15]

Butterworth Coefficients					
n	i	a_i	b_i	$k_i = \frac{\omega_{ci}}{\omega_c}$	Q_i
1	1	1.0000	0.0000	1.000	—
2	1	1.4142	1.0000	1.000	0.71
3	1	1.0000	0.0000	1.000	—
	2	1.0000	1.0000	1.272	1.00
4	1	1.8478	1.0000	0.719	0.54
	2	0.7654	1.0000	1.390	1.31
5	1	1.0000	0.0000	1.000	—
	2	1.6180	1.0000	0.859	0.62
	3	0.6180	1.0000	1.448	1.62

Note that Eq. 23 actually consists of two 2nd order transfer function, which are

$$T(S)_{Stage A} = \frac{K\omega_0^2}{(s^2 + 0.7654s + 1)}$$

and

$$T(S)_{Stage B} = \frac{K\omega_0^2}{(s^2 + 1.8478s + 1)}$$

Recall that the general form of a 2nd order transfer function is as follows

$$T(s) = \frac{1}{s^2 + a_i s + b_i} \quad (24)$$

Comparing the denominator of Eq. 22 and Eq. 24, we have

$$a_i = \frac{1}{R_{1i}C_{1i}} + \frac{1}{R_{2i}C_{1i}} + \frac{1-G_i}{R_{2i}C_{2i}}, \quad \text{and} \quad b_i = \frac{1}{R_{1i}R_{2i}C_{1i}C_{2i}} \quad (25)$$

For the first 2nd order Low-Pass Sallen Key (Stage A), we substitute the components and G values into Eq. 25, as follows

$$a_1 = \frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-G}{R_2C_2}$$

$$a_1 = \frac{1}{112 \Omega \times 1 nF} + \frac{1}{2.26 k\Omega \times 1 nF} + \frac{1 - 2.0325}{2.26 k\Omega \times 100 pF}$$

$$a_1 = 8928571.42857 + 442477.876106 - 4568584.0708$$

$$a_1 = 4802465.233876$$

Dividing with $2\pi f_c$, we have

$$a_1 = \frac{4802465.233876}{2\pi f_c} = \frac{4802465.233876}{2\pi \times 1MHz}$$

$$= 0.76433608099 \approx 0.7654$$

And for the second 2nd order Low-Pass Sallen Key (Stage B), we substitute the components and G values into Eq. 25, as follows

$$a_2 = \frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-G}{R_2C_2}$$

$$a_2 = \frac{1}{56.1 \Omega \times 990 pF} + \frac{1}{460 \Omega \times 990 pF} + \frac{1 - 4.9117}{460 \Omega \times 990 pF}$$

$$a_2 = 18005365.5989 + 2195871.76109 - 8589591.56785$$

$$a_2 = 11611645.79214$$

Dividing with $2\pi f_c$, we have

$$a_2 = \frac{11611645.79214}{2\pi f_c} = \frac{11611645.79214}{2\pi \times 1MHz}$$

$$= 1.84805082525 \approx 1.8478$$

Note that this tool has picked the right components for both stages (Stage A and Stage B), so that results in the correct transfer function for every stage.

Finally, the noise performance of the designed filter now will be discussed. The spectral noise density view of our filter is shown in Fig. 20 when the filter wizard was run. At frequency of 100 kHz, the spectral noise density of the filter is $49 nV/\sqrt{Hz}$.

The noise produced by the op-amp ADA4807-2 is $3.1 nV/\sqrt{Hz}$ at 100 kHz [16]. The filter spectral noise density of magnitude $49 nV/\sqrt{Hz}$ above comes from input noise of the amplifier of $3.1 nV/\sqrt{Hz}$ plus the noise from the resistors.

The output noise voltage spectral density will be only caused by the resistors. Therefore, the tool uses lower value

resistors and higher value capacitors to keep noise contribution from resistors low.

At frequencies above 1 MHz, the noise was partially suppressed. Thus, the noise is shaped by the filter transfer function [16].

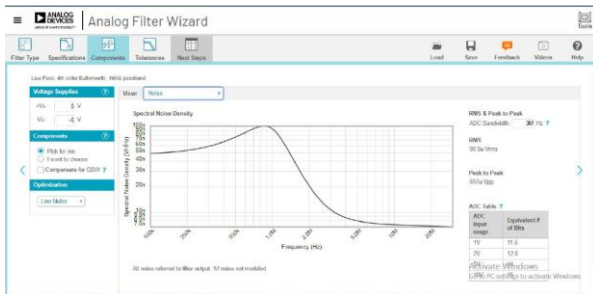


Figure 20. Spectral Noise Density View when the Filter Wizard Tool was Run.

4. Conclusions

In this research, an active RC filter circuit has been successfully created by using Analog Filter Wizard tool. From a filter design example, it can be shown that this Analog Filter Wizard has successfully perform the process of designing an active RC filter that satisfies a set of requirements. This tool is useful for student in helping them to design various type of an active RC filters by following an easy and simple step-by-step procedure, resulting in a complete filter circuit and also lets the students to design, optimize, and simulate complete multi-stage active filter solutions within minutes. The students are then able to compare the results created by this tool with the filter formulae and the results obtained from filter calculations.

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